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A NEW CLASS OF GENERALIZED MANLEY-ROWE RELATIONS

BY

HUGO ROSMAN

1. Established a quarter of a century ago, the Manley-Rowe (M.-R.) relations [1] have proved to be a very efficient tool for the study, especially as far as energy aspects are concerned, of nonlinear and parametric circuits. Since then a great many scientists have attempted to extend their domain of application and have established a series of generalizations for them.

In several previous papers [2]...[6] the author suggested a few possibilities of generalization of the M.-R. relations and in the following an attempt to a new and more comprehensive generalization is made with a view to an extensive range af applications.

2. We consider two scalar or vector functions, $f(t)$ and $g(t)$, double-periodic with respect to the same independent variable and having the same periods, $T_1=2\pi/\omega_1$, $T_2=2\pi/\omega_2$. Assuming that conditions for the expansion into a Fourier double-series are fulfilled, the functions $f(t)$, $g(t)$ can be written under the form :

$$(1)\qquad f(t)=\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\underline{F}_{m,n}\,e^{j(m\omega_1+n\omega_2)t},\qquad g(t)=\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\underline{G}_{m,n}\,e^{j(m\omega_1+n\omega_2)t},$$

where

$$(2)\qquad \begin{cases} \underline{F}_{m,n}=\dfrac{1}{4\pi^2}\displaystyle\int_0^{2\pi}\mathrm{d}(\omega_2 t)\int_0^{2\pi}f(t)\,e^{-j(m\omega_1+n\omega_2)t}\,\mathrm{d}(\omega_1 t), \\ \underline{G}_{m,n}=\dfrac{1}{4\pi^2}\displaystyle\int_0^{2\pi}\mathrm{d}(\omega_2 t)\int_0^{t}g(t)e^{-j(m\omega_1+n\omega_2)t}\,\mathrm{d}(\omega_1 t). \end{cases}$$

We consider the double-periodic functions :

$$(3)\qquad h(t)=f(t)\cos(\mu\omega_1+\nu\omega_2)t,\quad l(t)=g(t)\cos(\sigma\omega_1+\tau\omega_2)t,$$

$$(4)\qquad r(t)=f(t)\sin(\mu\omega_1+\nu\omega_2)t,\quad s(t)=g(t)\sin(\sigma\omega_1+\tau\omega_2)t,$$

having the same periods as the functions $f(t)$, $g(t)$. Obviously these functions, in their turn, can also be expanded into Fourier double-series :

$$(5)\qquad h(t)=\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\underline{H}_{m,n}\,e^{j(m\omega_1+n\omega_2)t},\qquad l(t)=\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\underline{L}_{m,n}\,e^{j(m\omega_1+n\omega_2)t}$$

and

$$(6)\qquad r(t)=\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\underline{R}_{m,n}\,e^{j(m\omega_1+n\omega_2)t},\qquad s(t)=\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\underline{S}_{m,n}e^{j(m\omega_1+n\omega_2)t},$$

respectively, with

$$(7)\qquad \begin{cases}\underline{H}_{m,n}=\dfrac{1}{4\pi^2}\displaystyle\int_0^{2\pi}\mathrm{d}(\omega_2 t)\int_0^{2\pi}h(t)e^{j(m\omega_1+n\omega_2)t}\,\mathrm{d}(\omega_1 t),\\ \underline{L}_{m,n}=\dfrac{1}{4\pi^2}\displaystyle\int_0^{2\pi}\mathrm{d}(\omega_2 t)\int_0^{2\pi}l(t)e^{-j(m\omega_1+n\omega_2)t}\,\mathrm{d}(\omega_1 t)\end{cases}$$

and

$$(8)\qquad \begin{cases}\underline{R}_{m,n}=\dfrac{1}{4\pi^2}\displaystyle\int_0^{2\pi}\mathrm{d}(\omega_2 t)\int_0^{2\pi}r(t)\,e^{-j(m\omega_1+n\omega_2)t}\,\mathrm{d}(\omega_1 t),\\ \underline{S}_{m,n}=\dfrac{1}{4\pi^2}\displaystyle\int_0^{2\pi}\mathrm{d}(\omega_2 t)\int_0^{2\pi}s(t)\,e^{-j(m\omega_1+n\omega_2)t}\,\mathrm{d}(\omega_1 t).\end{cases}$$

As it was shown in [6] the following relations take place:

$$(9)\qquad \underline{H}_{m,n}=\frac{1}{2}(\underline{F}_{m+\mu,n+\nu}+\underline{F}_{m-\mu,n-\nu}),\quad \underline{L}_{m,n}=\frac{1}{2}(\underline{G}_{m+\mu,n+\nu}+\underline{G}_{m-\mu,n-\nu}),$$

$$(10)\qquad \underline{R}_{m,n}=\frac{1}{2}(\underline{F}_{m+\mu,n+\nu}-\underline{F}_{m-\mu,n-\nu}),\quad \underline{S}_{m,n}=\frac{1}{2}(\underline{G}_{m+\mu,n+\nu}-\underline{G}_{m-\mu,n-\nu}).$$

3. We consider now the functions:

$$(11)\qquad \varphi(m,n)=\varphi(-m,-n),\quad \psi(m,n)=-\psi(-m,-n),$$

the first being even and the second odd with respect to the summation indices m, n. By aid of these functions we constitute the expressions:

$$(12)\qquad \begin{cases}\underline{A}=\displaystyle\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\psi(m,n)\,\mathcal{R}e\,(\underline{H}_{m,n}\,\underline{L}^*_{m,n}),\\ \underline{B}=\displaystyle\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\varphi(m,n)\,\mathcal{I}m\,(\underline{H}_{m,n}\,\underline{L}^*_{m,n})\end{cases}$$

and

$$(13)\qquad \begin{cases}\underline{C}=\displaystyle\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\psi(m,n)\,\mathcal{R}e\,(\underline{R}_{m,n}\,\underline{S}^*_{m,n}),\\ \underline{D}=\displaystyle\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\varphi(m,n)\,\mathcal{I}m\,(\underline{R}_{m,n}\,\underline{S}^*_{m,n}),\end{cases}$$

respectively. But, as it was shown in [4], the expressions (12) and (13) are equal to zero, i.e.

$$(14)\quad \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\psi(m,n)\mathcal{R}e(\underline{H}_{m,n}\underline{L}^{*}_{m,n})=0,\quad \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\varphi(m,n)\mathcal{I}m(\underline{H}_{m,n}\underline{L}^{*}_{m,n})=0$$

and

$$(15)\quad \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\psi(m,n)\mathcal{R}e(\underline{R}_{m,n}\underline{S}^{*}_{m,n})=0,\quad \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\varphi(m,n)\mathcal{I}m(\underline{R}_{m,n}\underline{S}^{*}_{m,n})=0.$$

By taking account of (9) relation (14_1) becomes:

$$(16)\quad \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\psi(m,n)[\mathcal{R}e(\underline{F}_{m+\mu,n+\nu}\underline{G}^{*}_{m+\sigma,n+\tau})+\mathcal{R}e(\underline{F}_{m+\mu,n+\nu}\underline{G}^{*}_{m-\sigma,n-\tau})+$$
$$+\mathcal{R}e(F_{m-\mu,n-\nu}G^{*}_{m+\sigma,n+\tau})+\mathcal{R}e(F_{m-\mu,n-\nu}G^{*}_{m-\sigma,n-\tau})]=0.$$

Also, by virtue of relations (10), expression (15_1) can be written:

$$(17)\quad \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\psi(m,n)[\mathcal{R}e(\underline{F}_{m+\mu,n+\nu}\underline{G}^{*}_{m+\sigma,n+\tau})-\mathcal{R}e(\underline{F}_{m+\mu,n+\nu}\underline{G}^{*}_{m-\sigma,n-\tau})-$$
$$-\mathcal{R}e(F_{m-\mu,n-\nu}G^{*}_{m+\sigma,n+\tau})+\mathcal{R}e(F_{m-\mu,n-\nu}G^{*}_{m-\sigma,n-\tau})]=0.$$

Relations (16) and (17) lead, for example, to

$$(18)\quad \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\psi(m,n)\mathcal{R}e(\underline{F}_{m+\mu,n+\nu}\underline{G}^{*}_{m+\sigma,n+\tau})+$$
$$+\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\psi(m,n)\mathcal{R}e(\underline{F}_{m-\mu,n-\nu}\underline{G}^{*}_{m-\sigma,n-\tau})=0.$$

4. By substituting m, n with $-m$, $-n$ into the second double-summation from (18) and taking into account on one hand (11_2) and on the other hand

$$(19)\quad \underline{F}_{-m-\mu,-n-\nu}=\underline{F}^{*}_{m+\mu,n+\nu},\quad \underline{G}_{-m-\sigma,-n-\tau}=\underline{G}^{*}_{m+\sigma,n+\tau}$$

we obtain

$$(20)\quad \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\psi(m,n)\mathcal{R}e(\underline{F}_{m+\mu,n+\nu}\underline{G}_{m+\sigma,n+\tau})=$$
$$=\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\psi(m,n)\mathcal{R}e(\underline{F}_{m-\mu,n-\nu}\underline{G}^{*}_{m-\sigma,n-\tau}).$$

Finally, the following remarkable relation is obtained:

$$(21)\quad \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\psi(m,n)\mathcal{R}e(\underline{F}_{m+\mu,n+\nu}\underline{G}^{*}_{m+\sigma,n+\tau})=0,\quad (\mu,\nu,\sigma,\tau=0,\pm1,\pm2,\ldots).$$

That may be considered as a new generalization of the M.-R. relations. In the particular case when $\psi(m,n)=m$ one can find the generalization of Kontorovitch's relation [8], established by the author in a previous paper [6].

By using a similar procedure one can also obtain the relation:

$$(22)\quad \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\varphi(m,n)\mathcal{I}m(\underline{F}_{m+\mu,n+\nu}\underline{G}^{*}_{m+\sigma,n+\tau})=0,\quad(\mu,\nu,\sigma,\tau=0,\pm1,\pm2,\ldots).$$

5. We consider a nonlinear one-port consisting of nondissipative, hysteresisless, reactive elements, in which nonperiodic phenomena do not take place, supplied by two sine-voltage sources having the frequencies f_1, f_2. In this case the voltage $u(t)$ across the terminals and the corresponding current $i(t)$ are double-periodic functions of time of the form (1). In other words:

$$(23)\quad u(t)=\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\underline{U}_{m,n}\,e^{j(m\omega_1+n\omega_2)t},\quad i(t)=\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\underline{I}_{m,n}\,e^{j(m\omega_1+n\omega_2)t},$$

where $\omega_1=2\pi f_1$, $\omega_2=2\pi f_2$. If we choose the odd function $\psi(m,n)$ of the particular form $\psi(m,n)=m$ and moreover consider $\mu=\sigma$, $\nu=\tau$, expression (21) leads to

$$(24)\quad \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}mP_{m+\mu,n+\nu}=0,\quad(\mu,\nu=0,\pm1,\pm2,\ldots),$$

where $P_{m+\mu,n+\nu}=\mathcal{R}e(U_{m+\mu,n+\nu}\underline{I}^{*}_{m+\mu,n+\nu})$ represents the active power exchanged between the harmonics of order $m+\mu$ and $n+\nu$. Expression (24) can be considered as a generalization of the „primitive" M.-R. relations [9].

6. We consider the function

$$(25)\quad a(t)=\frac{dg(t)}{dt},$$

also double-periodic with respect to t and having the periods $T_1=2\pi/\omega_1$, $T_2=2\pi/\omega_2$, so that

$$(26)\quad a(t)=\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\underline{A}_{m,n}\,e^{j(m\omega_1+n\omega_2)t},$$

with

$$(27)\quad \underline{A}_{m,n}=\frac{1}{4\pi^2}\int_0^{2\pi}d(\omega_2t)\int_0^{2\pi}a(t)\,e^{-j(m\omega_1+n\omega_2)t}\,d(\omega_1t).$$

From (1_2) and (26) we obtain:

$$(28)\quad \underline{A}_{m,n}=j(m\omega_1+n\omega_2)\underline{G}_{m,n}.$$

The double-periodic functions $f(t)$, $a(t)$ satisfy a relation of the kind (21), i.e.:

$$(29)\quad \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\psi(m,n)\frac{\mathcal{R}e(\underline{F}_{m+\mu,n+\nu}\,\underline{A}^{*}_{m+\sigma,n+\tau})}{(m+\sigma)\omega_1+(n+\tau)\omega_2}=0.$$

This time $f(t)$ corresponds to $u(t)$ and $a(t)$ to $i(t)$. Also we choose $\psi(m,n)=m$ and consider $\mu=\sigma$, $\nu=\tau$. Taking into account the above, expression (29) leads to:

$$(30)\quad \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\frac{mP_{m+\mu,n+\nu}}{(m+\mu)\omega_1+(n+\nu)\omega_2}=0,\quad(\mu,\nu=0,\pm1,\pm2,\ldots),$$

representing a new generalization of the M.-R. relations, which may be obtained as a particular case when $\mu=\nu=0$.

Considerations similar to the above enable us to establish other generalizations of the M.-R. relations.

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O NOUĂ CLASĂ DE RELAȚII MANLEY-ROWE GENERALIZATE

(Rezumat)

Se demonstrează că orice pereche de funcții, $f(t)$, $g(t)$, scalare sau vectoriale, dublu-periodice în raport cu t și având aceleași perioade, $T_1=2\pi/\omega_1$, $T_2=2\pi/\omega_2$, satisfac relații de tipul (21), (22), unde $\varphi(m,n)$, $\psi(m,n)$, sînt de forma (11).

În baza relației (21) se stabilește expresia (24), constituind o generalizare a relațiilor Manley-Rowe „primitive" [9] precum și expresia (30), reprezentînd o nouă generalizare a relațiilor Manley-Rowe.

representing a new generalization of the M-R relations, which may be obtained as a particular case when $\gamma = \nu = 0$.

Considerations similar to the above enable us to establish other generalizations of the M-K relations.

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ON THE SYSTEMS OF NONLINEAR DIFFERENTIAL EQUATIONS WITH TWO SMALL PARAMETERS

BY

EUGEN HNATIUC and MARIA BECEANU

1. The behaviour of a direct current electromagnet, considered as an electromechanical system of conversion of electric energy, can be described by a system of nonlinear equations of the form:

$$(1)\qquad \begin{cases} \ddot{y}(t)+2h\dot{y}(t)+\omega_0^2 y(t)=\dfrac{\psi_p}{ML'}\,\varphi(t)\left[1+\dfrac{\psi_{0\max}}{2\psi_p}\cdot\dfrac{1}{\psi_{0\max}}\,\varphi(t)\right], \\ \dot{\varphi}(t)+\dfrac{1}{T_0}\left[1-\dfrac{\delta_0}{\delta_0+a}\cdot\dfrac{1}{\delta_0}\,y(t)\right]\varphi(t)=\left.\dfrac{d\psi}{dt}\right|_{t=t_p}+\dfrac{R}{L'}\,\psi_p\,y(t), \end{cases}$$

with the initial conditions:

$$(2)\qquad y(0)=\dot{y}(0)=\varphi(0)=0.$$

If we introduce the notations:

$$(3)\qquad \frac{1}{T_0}=A,\ \frac{\psi_p}{ML'}=B,\ \frac{\psi_{0\max}}{2\psi_p}=\mu<1,\ \frac{1}{\psi_{0\max}}=C,\ \frac{\delta_0}{\delta_0+a}=\varepsilon<1,$$

$$\frac{1}{\delta_0}=D,\ \left.\frac{d\psi}{dt}\right|_{t=t_p}=E,\ \frac{R\psi_p}{L'}=G,$$

then the system (1) can be written:

$$(4)\qquad \begin{cases} \ddot{y}(t)+2h\dot{y}(t)+\omega_0^2 y(t)=B\varphi(t)[1+\mu C\varphi(t)], \\ \dot{\varphi}(t)+A[1-\varepsilon D y(t)]\varphi(t)=E+Gy(t), \end{cases}$$

with:

$$(5)\qquad t\in[0,\ T],\quad \mathcal{D}_{\varepsilon,\mu}:\{\mu\in[0,\ 1],\ \varepsilon\in[0,\ 1]\}.$$

In vector notation, the system of Eqs. (4), with initial conditions (2), is of the form:

$$(6)\qquad \dot{z}=\mathcal{A}z+\mathcal{B}+\mathcal{F}(z,\ \varepsilon,\ \mu),\quad z(0)=0,$$

where:

$$z=\begin{pmatrix} y \\ u \\ \varphi \end{pmatrix}, \quad \mathcal{A}=\begin{pmatrix} 0 & 1 & 0 \\ -\omega_0^2 & -2h & B \\ G & 0 & -A \end{pmatrix}, \quad \mathcal{B}=\begin{pmatrix} 0 \\ 0 \\ E \end{pmatrix},$$

$$\mathcal{F}(z,\ \varepsilon,\ \mu)=\begin{pmatrix} 0 \\ \mu BC\varphi^2 \\ \varepsilon ADy\varphi \end{pmatrix}, \quad \dot{y}(t)=u(t). \tag{7}$$

Having in view the physical meaning of the functions $y(t)$, $u(t)$ and $\varphi(t)$, respectively the displacement of mobile armature of an electromagnet, the velocity of motion and the variation of magnetic flux during the displacement, that these functions are bounded, we can write the conditions :

$$\mathcal{D}_z\colon \{|y|\leqslant Y,\ |u|\leqslant U,\ |\varphi|\leqslant \Phi\}. \tag{8}$$

2. Because the matrices $\mathcal{A}$ and $\mathcal{B}$ have constant elements, and the function $\mathcal{F}(z,\ \varepsilon,\ \mu)$ is continuous and satisfies the Lipschitz condition :

$$\begin{cases} \|\mathcal{F}(z_2,\ \varepsilon,\ \mu)-\mathcal{F}(z_1,\ \varepsilon,\ \mu)\|\leqslant L\|z_2-z_1\|, \\ \forall(z_1,\ \varepsilon,\ \mu)\in\mathcal{D},\ \forall(z_2,\ \varepsilon,\ \mu)\in\mathcal{D},\ \mathcal{D}=\mathcal{D}_z\times\mathcal{D}_{\varepsilon\mu}, \end{cases} \tag{9}$$

where :

$$\|z\|=(|y|^2+|u|^2+|\varphi|^2)^{1/2},\ L>0 \text{ const.}, \tag{10}$$

the system (6) admits an unique solution [1] :

$$z=z(t,\ \varepsilon,\ \mu),\ t\in[0,\ T_1). \tag{11}$$

We can observe that the function $\mathcal{F}(z,\ \varepsilon,\ \mu)$, defined by (7), admits continuous partial derivatives of all the orders with respect to y, u, φ, ε, μ in $\mathcal{D}$, then the right side of Eq. (6) is an analytic function.

In these conditions, the solution (11) of the system is an analytic function [1], [2]; the derivatives of the solution with respect to the small parameters ε, μ satisfy the system of equations in variations with respect to the parameters ε, μ.

Thus, it follows :

$$\begin{cases} \dot{y}_\varepsilon(t,\ \varepsilon,\ \mu)=x_\varepsilon(t,\ \varepsilon,\ \mu), \\ \dot{x}_\varepsilon(t,\ \varepsilon,\ \mu)=-2\omega_0(\varepsilon,\ \mu)\omega_{0\varepsilon}(\varepsilon,\ \mu)y(t,\ \varepsilon,\ \mu)-\omega_0^2(\varepsilon,\ \mu)\,y_\varepsilon(t,\ \varepsilon,\ \mu)- \\ \qquad -2hx_\varepsilon(t,\ \varepsilon,\ \mu)+B\varphi_\varepsilon(t,\ \varepsilon,\ \mu)+2\mu BC\varphi_\varepsilon(t,\ \varepsilon,\ \mu)\varphi(t,\ \varepsilon,\ \mu), \\ \dot{\varphi}_\varepsilon(t,\ \varepsilon,\ \mu)=Gy_\varepsilon(t,\ \varepsilon,\ \mu)-A\varphi_\varepsilon(t,\ \varepsilon,\ \mu)+ADy(t,\ \varepsilon,\ \mu)\varphi(t,\ \varepsilon,\ \mu)+ \\ \qquad +\varepsilon AD[y_\varepsilon(t,\ \varepsilon,\ \mu)\varphi(t,\ \varepsilon,\ \mu)+y(t,\ \varepsilon,\ \mu)\,\varphi_\varepsilon(t,\ \varepsilon,\ \mu)], \end{cases} \tag{12$_1$}$$

$$\begin{cases} \dot{y}_\mu(t,\ \varepsilon,\ \mu)=x_\mu(t,\ \varepsilon,\ \mu), \\ \dot{x}_\mu(t,\ \varepsilon,\ \mu)=-2\omega_0(\varepsilon,\ \mu)\omega_{0\mu}(\varepsilon,\ \mu)y(t,\ \varepsilon,\ \mu)-\omega_0^2(\varepsilon,\ \mu)y_\mu(t,\ \varepsilon,\ \mu)- \\ \qquad -2hx_\mu(t,\ \varepsilon,\ \mu)+B\varphi_\mu(t,\ \varepsilon,\ \mu)+BC\varphi^2(t,\ \varepsilon,\ \mu)+ \\ \qquad +2\mu BC\varphi_\mu(t,\ \varepsilon,\ \mu)\,\varphi(t,\ \varepsilon,\ \mu), \\ \dot{\varphi}_\mu(t,\ \varepsilon,\ \mu)=Gy_\mu(t,\ \varepsilon,\ \mu)-A\varphi_\mu(t,\ \varepsilon,\ \mu)+\varepsilon AD[y_\mu(t,\varepsilon,\mu)\varphi(t,\varepsilon,\mu)+ \\ \qquad +y(t,\ \varepsilon\ \mu)\,\varphi_\mu(t,\ \varepsilon,\ \mu)], \end{cases} \tag{12$_2$}$$

$$(12_3)\quad \begin{cases} \dot{y}_{\varepsilon\varepsilon}(t,\varepsilon,\mu)=x_{\varepsilon\varepsilon}(t,\varepsilon,\mu), \\ \dot{x}_{\varepsilon\varepsilon}(t,\varepsilon,\mu)=-2\{[\omega_{0\varepsilon}^2(\varepsilon,\mu)+\omega_0(\varepsilon,\mu)\omega_{0\varepsilon\varepsilon}(\varepsilon,\mu)]y(t,\varepsilon,\mu)+ \\ \quad +2\omega_0(\varepsilon,\mu)\omega_{0\varepsilon}(\varepsilon,\mu)y_\varepsilon(t,\varepsilon,\mu)\}-\omega_0^2(\varepsilon,\mu)y_{\varepsilon\varepsilon}(t,\varepsilon,\mu)- \\ \quad -2hx_{\varepsilon\varepsilon}(t,\varepsilon,\mu)+B\varphi_{\varepsilon\varepsilon}(t,\varepsilon,\mu)+2\mu BC[\varphi_\varepsilon^2(t,\varepsilon,\mu)+ \\ \quad +\varphi(t,\varepsilon,\mu)\varphi_{\varepsilon\varepsilon}(t,\varepsilon,\mu)], \\ \dot{\varphi}_{\varepsilon\varepsilon}(t,\varepsilon,\mu)=Gy_{\varepsilon\varepsilon}(t,\varepsilon,\mu)-A\varphi_{\varepsilon\varepsilon}(t,\varepsilon,\mu)+2AD[y_\varepsilon(t,\varepsilon,\mu)\varphi(t,\varepsilon,\mu)+ \\ \quad +y(t,\varepsilon,\mu)\varphi_\varepsilon(t,\varepsilon,\mu)]+\varepsilon AD[y_{\varepsilon\varepsilon}(t,\varepsilon,\mu)\varphi(t,\varepsilon,\mu)+ \\ \quad +2y_\varepsilon(t,\varepsilon,\mu)\varphi_\varepsilon(t,\varepsilon,\mu)+y(t,\varepsilon,\mu)\varphi_{\varepsilon\varepsilon}(t,\varepsilon,\mu)], \end{cases}$$

$$(12_4)\quad \begin{cases} \dot{y}_{\varepsilon\mu}(t,\varepsilon,\mu)=x_{\varepsilon\mu}(t,\varepsilon,\mu), \\ \dot{x}_{\varepsilon\mu}(t,\varepsilon,\mu)=-2\{\omega_{0\mu}(\varepsilon,\mu)\omega_{0\varepsilon}(\varepsilon,\mu)+\omega_0(\varepsilon,\mu)\omega_{0\varepsilon\mu}(\varepsilon,\mu)]y(t,\varepsilon,\mu)+ \\ \quad +\omega_0(\varepsilon,\mu)\omega_{0\varepsilon}(\varepsilon,\mu)y_\mu(t,\varepsilon,\mu)+\omega_0(\varepsilon,\mu)\omega_{0\mu}(\varepsilon,\mu)y_\varepsilon(t,\varepsilon,\mu)\}- \\ \quad -\omega_0^2(\varepsilon,\mu)y_{\varepsilon\mu}(t,\varepsilon,\mu)-2hx_{\varepsilon\mu}(t,\varepsilon,\mu)+B\varphi_{\varepsilon\mu}(t,\varepsilon,\mu)+ \\ \quad +2BC\{\varphi_\varepsilon(t,\varepsilon,\mu)\varphi(t,\varepsilon,\mu)+\mu[\varphi_{\varepsilon\mu}(t,\varepsilon,\mu)\varphi(t,\varepsilon,\mu)+ \\ \quad +\varphi_\varepsilon(t,\varepsilon,\mu)\varphi_\mu(t,\varepsilon,\mu)]\}, \\ \dot{\varphi}_{\varepsilon\mu}(t,\varepsilon,\mu)=Gy_{\varepsilon\mu}(t,\varepsilon,\mu)-A\varphi_{\varepsilon\mu}(t,\varepsilon,\mu)+AD[y_\mu(t,\varepsilon,\mu)\varphi(t,\varepsilon,\mu)+ \\ \quad +y(t,\varepsilon,\mu)\varphi_\mu(t,\varepsilon,\mu)]+\varepsilon AD[y_{\varepsilon\mu}(t,\varepsilon,\mu)\varphi(t,\varepsilon,\mu)+ \\ \quad +y_\varepsilon(t,\varepsilon,\mu)\varphi_\mu(t,\varepsilon,\mu)+y_\mu(t,\varepsilon,\mu)\varphi_\varepsilon(t,\varepsilon,\mu)+ \\ \quad +y(t,\varepsilon,\mu)\varphi_{\varepsilon\mu}(t,\varepsilon,\mu)], \end{cases}$$

$$(12_5)\quad \begin{cases} \dot{y}_{\mu\mu}(t,\varepsilon,\mu)=x_{\mu\mu}(t,\varepsilon,\mu), \\ \dot{x}_{\mu\mu}(t,\varepsilon,\mu)=-2\{[\omega_{0\mu}^2(\varepsilon,\mu)+\omega_0(\varepsilon,\mu)\omega_{0\mu\mu}(\varepsilon,\mu)]y(t,\varepsilon,\mu)+ \\ \quad +2\omega_0(\varepsilon,\mu)\omega_{0\mu}(\varepsilon,\mu)y_\mu(t,\varepsilon,\mu)\}-\omega_0^2(\varepsilon,\mu)y_{\mu\mu}(t,\varepsilon,\mu)- \\ \quad -2hx_{\mu\mu}(t,\varepsilon,\mu)+B\varphi_{\mu\mu}(t,\varepsilon,\mu)+4BC\varphi(t,\varepsilon,\mu)\varphi_\mu(t,\varepsilon,\mu)+ \\ \quad +2\mu BC[\varphi_{\mu\mu}(t,\varepsilon,\mu)\varphi(t,\varepsilon,\mu)+\varphi_\mu^2(t,\varepsilon,\mu)], \\ \dot{\varphi}_{\mu\mu}(t,\varepsilon,\mu)=Gy_{\mu\mu}(t,\varepsilon,\mu)-A\varphi_{\mu\mu}(t,\varepsilon,\mu)+\varepsilon AD[y_{\mu\mu}(t,\varepsilon,\mu)\varphi(t,\varepsilon,\mu)+ \\ \quad +2y_\mu(t,\varepsilon,\mu)\varphi_\mu(t,\varepsilon,\mu)+y(t,\varepsilon,\mu)\varphi_{\mu\mu}(t,\varepsilon,\mu)]. \end{cases}$$

The physical meaning of the pulsation $\omega_0(\varepsilon,\mu)$ imposes the following relation :

$$(13)\qquad \omega_0^2=\omega^2-\varepsilon\omega_1^2-\mu\omega_2^2-\varepsilon^2\omega_3^2-\varepsilon\mu\omega_4^2-\mu^2\omega_5^2-\ldots\ldots .$$

For $\varepsilon=0$, $\mu=0$ the systems of equations in variations (12) become :

$$(14_1)\quad \begin{cases} \dot{y}_\varepsilon(t,0,0)=x_\varepsilon(t,0,0), \\ \dot{x}_\varepsilon(t,0,0)=-2\omega_0(0,0)\,\omega_{0\varepsilon}(0,0)y(t,0,0)-\omega_0^2(0,0)y_\varepsilon(t,0,0)- \\ \quad -2hx_\varepsilon(t,0,0)+B\varphi_\varepsilon(t,0,0), \\ \dot{\varphi}_\varepsilon(t,0,0)=Gy_\varepsilon(t,0,0)-A\varphi_\varepsilon(t,0,0)+ADy(t,0,0)\varphi(t,0,0), \end{cases}$$

$$(14_2)\quad \begin{cases} \dot{y}_\mu(t,0,0)=x_\mu(t,0,0), \\ \dot{x}_\mu(t,0,0)=-2\omega_0(0,0)\omega_{0\mu}(0,0)y(t,0,0)-\omega_0^2(0,0)y_\mu(t,0,0)- \\ \qquad -2hx_\mu(t,0,0)+B\varphi_\mu(t,0,0)+BC\varphi^2(t,0,0), \\ \dot{\varphi}_\mu(t,0,0)=Gy_\mu(t,0,0)-A\varphi_\mu(t,0,0), \end{cases}$$

$$(14_3)\quad \begin{cases} \dot{y}_{\varepsilon\varepsilon}(t,0,0)=x_{\varepsilon\varepsilon}(t,0,0), \\ \dot{x}_{\varepsilon\varepsilon}(t,0,0)=-2\{[\omega_{0\varepsilon}^2(0,0)+\omega_0(0,0)\omega_{0\varepsilon\varepsilon}(0,0)]y(t,0,0)+ \\ \qquad +2\omega_0(0,0)\,\omega_{0\varepsilon}(0,0)y_\varepsilon(t,0,0)\}-\omega_0^2(0,0)y_{\varepsilon\varepsilon}(t,0,0)- \\ \qquad -2hx_{\varepsilon\varepsilon}(t,0,0)+B\varphi_{\varepsilon\varepsilon}(\mathrm{t},0,0), \\ \dot{\varphi}_{\varepsilon\varepsilon}(t,0,0)=Gy_{\varepsilon\varepsilon}(t,0,0)-A\varphi_{\varepsilon\varepsilon}(t,0,0)+ \\ \qquad +2AD[y_\varepsilon(t,0,0)\varphi(t,0,0)+y(t,0,0)\varphi_\varepsilon(t,0,0)], \end{cases}$$

$$(14_4)\quad \begin{cases} \dot{y}_{\varepsilon\mu}(t,0,0)=x_{\varepsilon\mu}(t,0,0), \\ \dot{x}_{\varepsilon\mu}(t,0,0)=-2\{[\omega_{0\mu}(0,0)\omega_{0\varepsilon}(0,0)+\omega_0(0,0)\omega_{0\varepsilon\mu}(0,0)]y(t,0,0)+ \\ \qquad +\omega_0(0,0)\omega_{0\varepsilon}(0,0)y_\mu(t,0,0)+\omega_0(0,0)\omega_{0\mu}(0,0)y_\varepsilon(t,0,0)\}- \\ \qquad -\omega_0^2(0,0)y_{\varepsilon\mu}(t,0,0)-2hx_{\varepsilon\mu}(t,0,0)+B\varphi_{\varepsilon\mu}(t,0,0)+ \\ \qquad +2BC\varphi_\varepsilon(t,0,0)\varphi(t,0,0), \\ \dot{\varphi}_{\varepsilon\mu}(t,0,0)=Gy_{\varepsilon\mu}(t,0,0)-A\varphi_{\varepsilon\mu}(t,0,0)+AD[y_\mu(t,0,0)\varphi(t,0,0)+ \\ \qquad +y(t,0,0)\varphi_\mu(t,0,0)], \end{cases}$$

$$(14_5)\quad \begin{cases} \dot{y}_{\mu\mu}(t,0,0)=x_{\mu\mu}(t,0,0), \\ \dot{x}_{\mu\mu}(t,0,0)=-2\{[\omega_{0\mu}^2(0,0)+\omega_0(0,0)\omega_{0\mu\mu}(0,0)]y(t,0,0)+ \\ \qquad +2\omega_0(0,0)\omega_{0\mu}(0,0)y_\mu(t,0,0)\}-\omega_0^2(0,0)y_{\mu\mu}(t,0,0)- \\ \qquad -2hx_{\mu\mu}(t,0,0)+B\varphi_{\mu\mu}(t,0,0)+4BC\varphi(t,0,0)\varphi_\mu(t,0,0), \\ \dot{\varphi}_{\mu\mu}(t,0,0)=Gy_{\mu\mu}(t,0,0)-A\varphi_{\mu\mu}(t,0,0). \end{cases}$$

By expanding $z(t,\varepsilon,\mu)$ in a powers series of the small parameters ε, μ it results:

$$(15)\qquad z(t,\varepsilon,\mu)=z_0(t)+\varepsilon z_1(t)+\mu z_2(t)+\varepsilon^2 z_3(t)+\varepsilon\mu z_4(t)+\mu^2 z_5(t)+\ldots,$$

where:

$$(16)\quad \begin{cases} z_1(t)=\dfrac{1}{1!}\,z_\varepsilon(t,0,0), \quad z_2(t)=\dfrac{1}{1!}\,z_\mu(t,0,0), \quad z_3(t)=\dfrac{1}{2!}\,z_{\varepsilon\varepsilon}(t,0,0), \\ z_4(t)=\dfrac{2}{2!}\,z_{\varepsilon\mu}(t,0,0), \quad z_5(t)=\dfrac{1}{2!}\,z_{\mu\mu}(t,0,0),\ldots, \end{cases}$$

and $z_0(t)$ is the solution of linear system of differential equations:

$$(17)\qquad \dot{z}(t)=\mathcal{A}z(t)+\mathcal{B}.$$

3. By virtue of the relationships (13), (15), ... (17), the solution of the system of nonlinear differential equations with two small parameters, ε, μ, is of the form:

$$(18)\quad \begin{cases} y(t,\varepsilon,\mu)=y_0(t)+\varepsilon y_1(t)+\mu y_2(t)+\varepsilon^2 y_3(t)+\varepsilon\mu y_4(t)+\mu^2 y_5(t)+\ldots, \\ \varphi(t,\varepsilon,\mu)=\varphi_0(t)+\varepsilon\varphi_1(t)+\mu\varphi_2(t)+\varepsilon^2\varphi_3(t)+\varepsilon\mu\varphi_4(t)+\mu^2\varphi_5(t)+\ldots \end{cases}$$

Changing (18) into system (4), taking into account (13) and comparing the coefficients of identical powers of ε and μ in the left and right members of the resulting egality, we get the following systems of linear differential equations with constant coefficients :

$$(19_1)\qquad \ddot{y}_0(t)+2h\dot{y}_0(t)+\omega^2 y_0(t)=B\varphi_0(t),\quad \dot{\varphi}_0(t)+A\varphi_0(t)=E+Gy_0(t)\ ;$$

$$(19_2)\qquad \begin{cases} \ddot{y}_1(t)+2h\dot{y}_1(t)+\omega^2 y_1(t)=\omega_1^2 y_0(t)+B\varphi_1(t), \\ \dot{\varphi}_1(t)+A\varphi_1(t)=ADy_0(t)\varphi_0(t)+Gy_1(t)\ ; \end{cases}$$

$$(19_3)\quad \ddot{y}_2(t)+2h\dot{y}_2(t)+\omega^2 y_2(t)=\omega_2^2 y_0(t)+BC\varphi_0^2(t)+B\varphi_2(t),\ \dot{\varphi}_2(t)+A\varphi_2(t)=Gy_2(t);$$

$$(19_4)\qquad \begin{cases} \ddot{y}_3(t)+2h\dot{y}_3(t)+\omega^2 y_3(t)=\omega_1^2 y_1(t)+\omega_3^2 y_0(t)+B\varphi_3(t), \\ \varphi_3(t)+A\varphi_3(t)=AD[y_0(t)\varphi_1(t)+y_1(t)\varphi_0(t)]+Gy_3(t)\ ; \end{cases}$$

$$(19_5)\qquad \begin{cases} \ddot{y}_4(t)+2hy_4(t)+\omega^2 y_4(t)=\omega_4^2 y_0(t)+\omega_2^2 y_1(t)+\omega_1^2 y_2(t)+2BC\varphi_0(t)\varphi_1(t)+B\varphi_4(t), \\ \dot{\varphi}_4(t)+A\varphi_4(t)=AD[y_2(t)\varphi_0(t)+y_0(t)\varphi_2(t)]+Gy_4(t)\ ; \end{cases}$$

$$(19_6)\qquad \begin{cases} \ddot{y}_5(t)+2hy_5(t)+\omega^2 y_5(t)=\omega_5^2 y_0(t)+\omega_2^2 y_2(t)+2BC\varphi_0(t)\varphi_2(t)+B\varphi_5(t), \\ \dot{\varphi}_5(t)+A\varphi_5(t)=Gy_5(t). \end{cases}$$

Supposing that $\omega_0=\omega_0(\varepsilon,\mu)$ is an analytic function with respect to ε, μ, it admits Taylor's series expansion :

$$(20)\qquad \begin{aligned} \omega_0(\varepsilon,\mu)=\omega_0(0,0)+\frac{\varepsilon}{1!}\omega_{0\varepsilon}(0,0)+\frac{\mu}{1!}\omega_{0\mu}(0,0)\frac{\varepsilon^2}{2!}\omega_{0\varepsilon\varepsilon}(0,0)+ \\ +\varepsilon\mu\omega_{0\varepsilon\mu}(0,0)+\frac{\mu^2}{2!}\omega_{0\mu\mu}(0,0)+\ldots, \end{aligned}$$

where :

$$(21)\qquad \omega_0(0,0)=\omega.$$

Thus, we get for $\omega_0^2(\varepsilon,\mu)$ the expression :

$$(22)\qquad \begin{aligned} \omega_0^2(\varepsilon,\mu)=\omega_0^2\ (0,0)+2\varepsilon\omega_0(0,0)\omega_{0\varepsilon}(0,0)+2\mu\omega_0(0,0)\omega_{0\mu}(0,0)+ \\ +\varepsilon^2[\omega_0(0,0)\omega_{0\varepsilon\varepsilon}(0,0)+\omega_{0\varepsilon}^2(0,0)]+2\varepsilon\mu[\omega_0(0,0)\omega_{0\varepsilon\mu}(0,0)+ \\ +\omega_{0\varepsilon}(0,0)\omega_{0\mu}(0,0)]+\mu^2[\omega_{0\mu\mu}(0,0)\omega_0(0,0)+\omega_{0\mu}^2(0,0)]+\ldots. \end{aligned}$$

If we use the notations :

$$(23)\qquad \begin{cases} \omega_0^2(0,0)=\omega^2,\ 2\omega_0(0,0)\omega_{0\varepsilon}(0,0)=-\omega_1^2,\ 2\omega_0(0,0)\omega_{0\mu}(0,0)=-\omega_2^2, \\ \omega_0(0,0)\omega_{0\varepsilon\varepsilon}(0,0)+\omega_{0\varepsilon}^2(0,0)=-\omega_3^2, \\ 2[\omega_0(0,0)\omega_{0\varepsilon\mu}(0,0)+\omega_{0\varepsilon}(0,0)\omega_{0\mu}(0,0)]=-\omega_4^2, \\ \omega_0(0,0)\ \omega_{0\mu\mu}\ (0,0)+\omega_{0\mu}^2(0,0)=-\omega_5^2\ , \end{cases}$$

then (22) become just (13).

We can observe that the systems of Eqs. (19) represent just the systems of Eqs. of the form (14) which being linear and with constant coefficients can be integrated.

The solution of the system of nonlinear differential Eqs. (1) can be approximated by (18).

Finding the solutions of the system (1) with an error of the order $O(\varepsilon^{m+1}, \mu^{m+1})$, one retains from the series (18) the terms of order m with respect ε, μ, inclusively.

Conclusions

If the approximate solution (18) of the system of nonlinear differential Eqs. (4) associated with the condition (13) is investigated, one observes that this represents an expansion of the Poincaré's method to the case of systems of nonlinear differential Eqs. with two small parameters.

The exposed method permits to find the approximate solution with a given error of nonlinear differential systems with two small parameters (4) by integration of a number of linear differential systems with constant coefficients of the form (19).

The solution of the linear systems represent the accurate terms for the approximate solution (18).

The mentioned method is applicable in technique, for approximate investigation of the behaviour of physical systems described by nonlinear systems of equations with two small parameters.

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ASUPRA REZOLVĂRII SISTEMELOR DE ECUAȚII DIFERENȚIALE NELINIARE CU DOI PARAMETRI MICI

(Rezumat)

Se prezintă o extindere a metodei micului parametru al lui Poincaré pentru cazul sistemelor de ecuații diferențiale cu doi parametri mici, care permite aproximarea convenabilă a soluțiilor acestora prin rezolvarea succesivă a unor sisteme liniare de ecuații diferențiale cu coeficienți constanți.

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ÜBER DEN FERRORESONANTEN, NICHTLINEAREN UND PARAMETRISCHEN *RLC* NEBENSCHLUSSKREIS

VON

MIRCEA NEMĒSCU

1. Einleitung

Es wird ein nichtlinearer, parametrischer *RLC* Nebenschlußkreis angenommen (Abb. 1), der mit einem Strom von $i=\sqrt{2}\,I\sin\omega t$ gespeist wird und dessen Parameterinduktor so gewählt ist, daß seine Schwankung sich im linearen Bereich bewegt. Die Charakteristik der als einheitliches Ganzes zu betrachtenden beiden seriengeschalteten Induktoren, die Hysteresis erscheinung vernachlässigend, entspricht der Darstellung aus Abb. 2, wo Kurve *1* dem kleinsten Induktanzwert des Parameterinduktors und Kurve *2* dessen Höchstwert entspricht. Die beiden seriengeschalteten Induktoren können als ein einziger nichtlinearer Parameterinduktor betrachtet werden,

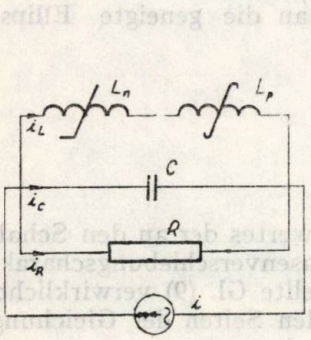

Abb. 1

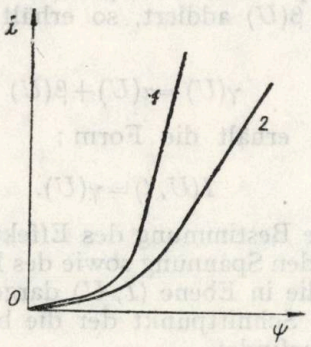

Abb. 2

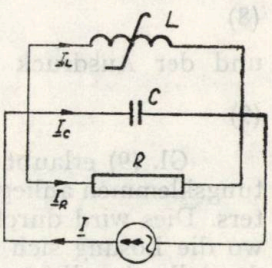

Abb. 3

dessen Charakteristik in der Zeit zwischen Kennlinien *1* und *2* schwankt. Mit dieser Bemerkung wird das der Schaltung entsprechende Schema zu dem im Abb. 3 dargestellten, wo *R* der Schaltung entsprechende, als gleichbleibend betrachtete, Widerstand darstellt.

2. Theoretische Betrachtungen

Wird der Induktor als inertial betrachtet, so gilt für die im Abb. 3 dargestellte Schaltung die Beziehung :

$$\underline{I}=\underline{I}_R+\underline{I}_L+\underline{I}_C. \tag{1}$$

Um die Betriebsweise der Schaltung zu erläutern, wird folgende grapho-analytische Methode angewandt, die auf der dynamischen Neuverteilung des Stromes beruht. In Effektivwerten kann Gleichung (1) folgendermassen geschrieben werden :

$$I^2=[I_L(U,t)-I_C(U)]^2+I_R^2, \tag{2}$$

oder, wenn man berücksichtigt, daß $I_C(U)=CU\omega$ und $I_R=U/R$, so kann Gl. (2) unter folgende Form gebracht werden :

$$I_L(U,t)=\pm\sqrt{I^2-(U/R)^2}+CU\omega. \tag{3}$$

Bezeichnet man :

$$\alpha(U)=\pm\sqrt{I^2-(U/R)^2} \text{ und } (5)\ \beta(U)=CU\omega, \tag{4}$$

so wird aus der Gl. (3) :

$$I(U,t)=\alpha(U)+\beta(U). \tag{6}$$

Gleichung (4) kann folgendermassen ausgedrückt werden :

$$\frac{\alpha^2}{I^2}+\frac{U^2}{(RI)^2}=1, \tag{7}$$

ein Ausdruck, der die punktierte Ellipse (Abb. 4) darstellt.

Wenn man dazu $\beta(U)$ addiert, so erhält man die geneigte Ellipse (Abb. 4) :

$$\gamma(U)=\alpha(U)+\beta(U) \tag{8}$$

und der Ausdruck (6) erhält die Form :

$$I(U,t)=\gamma(U). \tag{9}$$

Gl. (9) erlaubt die Bestimmung des Effektivwertes der an den Schaltungsklemmen anliegenden Spannung sowie des Phasenverschiebungscharakters. Dies wird durch die in Ebene (I, U) dargestellte Gl. (9) verwirklicht, wo die Lösung sich im Schnittpunkt der die beiden Seiten der Gleichung darstellenden Kurven befindet.

Wenn die Parameterinduktanz, die der Kurve $I_{L0}(U,t)$ entspricht, Kleinstwerte annimmt und der zugeführte Strom den Wert I beträgt, so haben die beiden Kurven $\gamma(U)$ und $I_{L0}(U,t)$ zwei Schnittpunkte, die den beiden Lösungen von Gl. (9) entsprechen, wobei die dem Tangenzpunkt entsprechende eine unbeständige Lösung darstellt (Abb. 4, Punkt P). Daher der Spannungssprung von U_0 zu U_1 in der Schaltung.

Der Charakter der Phasenverschiebung sei sie induktiver oder kapazitiver Art, wird durch den Wertevergleich von $I_L(U)$ und $I_C(U)$, die der betreffenden Spannung entsprechen, bestimmt. So erhält man für die Spannung U_0, $I_{C0} > I_{L0}$, eine Phasenverschiebung kapazitiven Charakters und für den Wert U_1 der Spannung, $I_{L1} > I_{C1}$, eine induktive Phasenverschiebung.

Man merkt, daß nach dem ferroresonanten Sprung die induktive Suszeptanz größer ist als die kapazitive Suszeptanz, somit bleibt ein Teil derselben unausgeglichen und die Klemmenspannung der Schaltung wird von dem entsprechenden Widerstand sowie von der unausgeglichenen induktiven Suszeptanz der Leitung bestimmt, da die Phasenverschiebung induktiver Art ist.

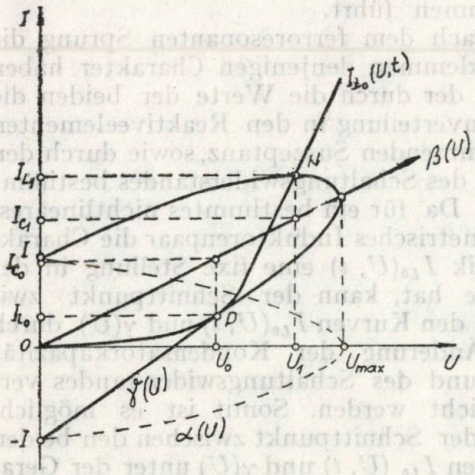

Abb. 4

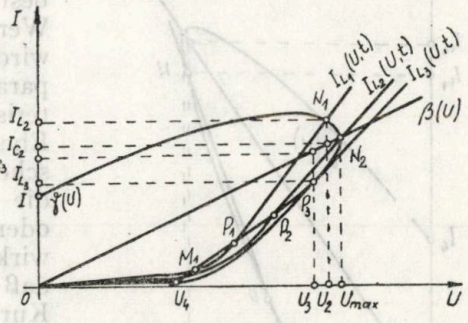

Abb. 5

Wird die Stromstärke konstant und gleich I gehalten, so wächst die Induktanz des Parameterinduktors. Somit fällt die induktive Suszeptanz und die Lage der Kennlinie $I_L(U, t)$ wird in der Ebene (I, U) verlegt und zwar kommt sie jetzt zu $I_{L1}(U, t)$ (Abb. 5) und schneidet die Kurve $\gamma(U)$ in drei Punkten. Die den Punkten M_1 und N_1 entsprechenden Lösungen sind stabil, während die dem Punkte P_1 entsprechende Lösung unbeständig ist. Das Kleinerwerden der induktiven Suszeptanz führt zum Anwachsen der Spannung bis zu U_2 (Abb. 5). Es findet somit eine dynamische Wiederaufteilung der Stromstärke unter den reaktiven Elementen der Schaltung statt, so daß $I_{L2} - I_{C2} < I_{L1} - I_{C1}$ zur Herabsetzung des induktiven Charakters der Spannungsphasenverschiebung führt. Verhält sich die parametrische Induktanz des Induktors so, daß die Charakteristik $I_L(U, t)$ durch den Punkt N_2 geht (Abb. 5), so erreicht die Klemmenspannung der Leitung ihren Höchstwert $U_{max} = RI$, und die beiden Suszeptanzen, die induktive und die kapazitive, gleichen sich gegenseitig aus. Von diesem Zeitpunkt an bewirkt jedes Anwachsen der Induktanz des Induktors ein Abfallen der induktiven Suszeptanz unter den Wert der kapazitiven

Suszeptanz, was zum Kleinerwerden der Spannung führt und den Charakter der Phasenverschiebung ändert, der damit kapazitiv wird.

Durch ein erneutes Anwachsen der Induktantz des Parameterinduktors kommt es dann dahin, daß die Charakteristik $I_L(U,t)$ zum unteren Teil der Ellipse $\gamma(U)$ tangent zu liegen kommt (Abb. 5, Punkt P_3). Die diesem Punkte entsprechende Lösung ist unstabil, es findet ein Spannungsrücksprung von U_3 nach U_4 statt und die Phasenverschiebung wird entschieden kapazitiv. Wenn nun die Induktanz des Parameterinduktors zu fallen beginnt, so nähert sich die Charakteristik $I_L(U, t)$ der anfänglichen Situation $I_{L0}(U, t)$, nach dem sich der Vorgang wiederholt.

Aus dem vorhin gesagten geht hervor, daß bei Induktanzänderung des Parameterinduktors in der Schaltung eine dynamische Neuaufteilung der Stromstärke stattfindet, was zur Änderung der Amplitude und Spannungsphase an den Schaltungsklemmen führt.

Es muß bemerkt werden, da nach dem ferroresonanten Sprung die Spannungsphase an den Schaltungsklemmen denjenigen Charakter haben wird, der durch die Werte der beiden die Stromverteilung in den Reaktiveelementen bestimmenden Suszeptanz, sowie durch den Wert des Schaltungswiderstandes bestimmt wird. Da für ein bestimmtes nichtlineares, parametrisches Induktorenpaar die Charakteristik $I_{L0}(U, t)$ eine fixe Stellung in der Ebene hat, kann der Schnittpunkt zwischen den Kurven $I_{L0}(U, t)$ und $\gamma(U)$ durch die Änderung der Kondensatorkapazität oder/und des Schaltungswiderstandes verwirklicht werden. Somit ist es möglich, daß der Schnittpunkt zwischen den beiden Kurven I_{L0} (U, t) und $\gamma(U)$ unter der Geraden $\beta(U)$ (Abb. 6) zu liegen kommt. Es wird bemerkt, daß in dieser Lage $I_{C4}(U) > I_{L4}(U)$, was den kapazitiven Charakter der Spannungsphase bestimmt. Das erklärt sich dadurch, daß sowohl vor als auch nach dem ferroresonanten Sprung die kapazitive Suszeptanz größer ist als die induktive. Somit bleibt ein Teil der kapazitiven Suszeptanz unausgeglichen, während die Spannung ihren kapazitiven Charakter beibehält.

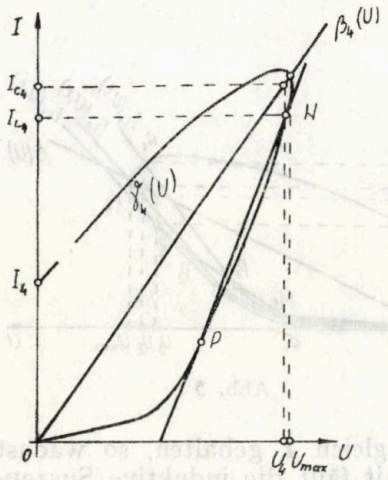

Abb. 6

Daraus folgt, daß es möglich ist, ferroresonante Spannungssprünge zu erzielen, ohne den Charakter der Spannungsphase zu beinflußen. Das Verhalten der Schaltung bei späteren Schwankungen des Parameterinduktors ist dem vorhin erläuterten Falle gleich.

2 Experimentalergebnisse

Die Versuche wurden anhand eines nichtlinearen, parametrischen *RLC* Nebenschlußkreises durchgeführt, für den die Charakteristik $i(\psi)$

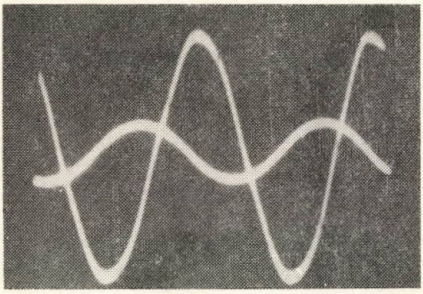

Abb. 8 *a*

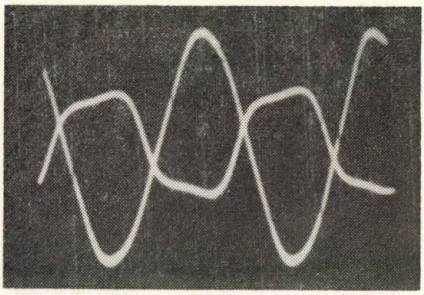

Abb. 8 *b*

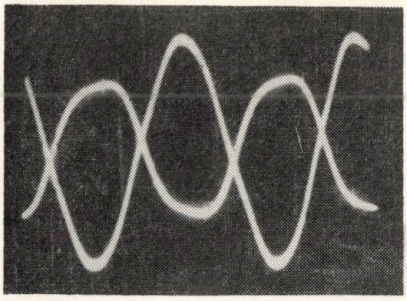

Abb. 9

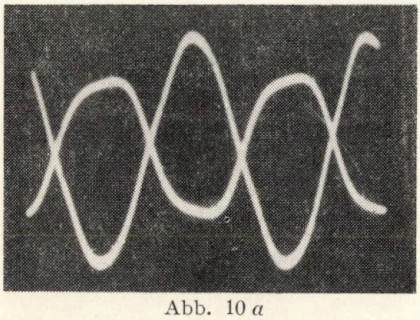

Abb. 10 *a*

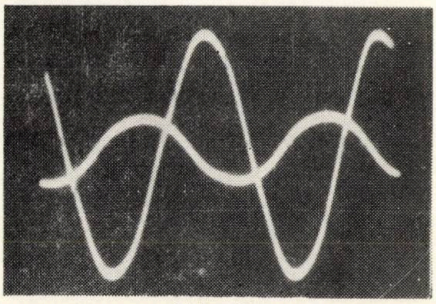

Abb. 10 *b*

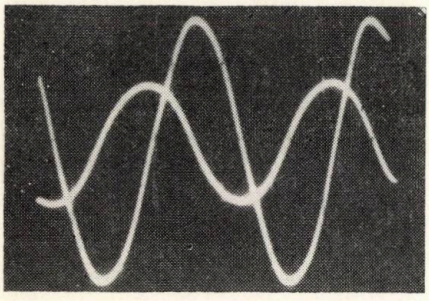

Abb. 11 *a*

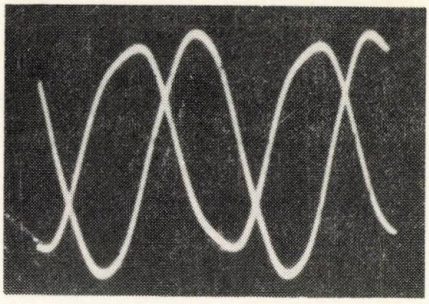

Abb. 11 *b*

der beiden, als einheitliches Ganzes betrachteten Induktoren im Abb. 7 dargestellt ist. Die Kurve *1* entspricht dem Mindestwert der Induktanz des Parameterinduktors und die Kurve *2* dem Höchstwert dieser Induktanz.

Die Abb. 8 *a* und *b* stellen die Oszillogramme für Stromstärke und Spannung vor und nach dem ferroresonanten Sprung dar, während die Parameterinduktanz minimal ist ($I=11{,}5$ mA, $C=6$ μF, $U_0=10{,}5$ V und $U_1=16{,}3$ V). Durch das Kleinerwerden der induktiven Suszeptans wird

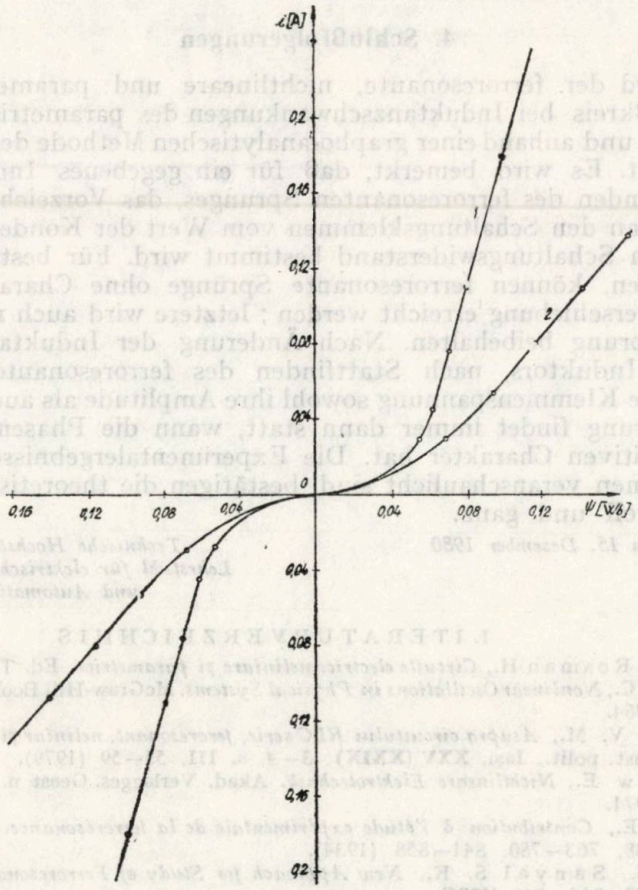

Abb. 7

auch eine. Verringerung des induktiven Charakters der Spannungsphasenverschiebung und ein Zuwachs dφ Spannungsamplitude bewirkt. So erhält die Spannung den Wert $U_{max}=20{,}6$ V und steht in Gegenphase mit der Stromstärke (Abb. 9). Ein späteres Anwachsen der Induktanz führt zur Spannungssenkung ($U=20{,}1$ V) und zu deren Phasenänderung, indem letztere einen kapazitiven Charakter erhält.

Im Abb. 10 *a* und *b* sind die Stromstärke- und Spannungsoszillogramme vor und nach dem Spannungsrücksprung dargestellt ; man kann dabei

das Beibehalten des kapazitiven Phasenverschiebungscharakters beobachten. Wächst die Kapazität des Schaltungskondensators auf $C=24$ μF, so findet der ferroresonante Sprung bei einer Stromstärke von $I=92$ mA statt, bei einer Spannung von $U=17{,}6$ V bis $U_1=28{,}4$ V, ohne daß sich das Vorzeichen der Spannungsphase dabei ändert. Abb. 11 *a* und *b* stellen die Stromstärke- und Spannungsoszillogramme vor und nach dem ferroresonanten Sprung dar, wobei man das Beibehalten des Vorzeichens der Spannungsphase bemerken kann.

4. Schlußfolgerungen

Es wird der ferroresonante, nichtlineare und parametrische *RLC* Nebenschlußkreis bei Induktanzschwankungen des parametrischen Induktors studiert und anhand einer grapho-analytischen Methode dessen Betriebsweise erklärt. Es wird bemerkt, daß für ein gegebenes Induktorenpaar, nach Stattfinden des ferroresonanten Sprunges, das Vorzeichen der Spannungsphase an den Schaltungsklemmen vom Wert der Kondensatorkapazität und dem Schaltungswiderstand bestimmt wird. Für bestimmte Werte dieser Größen, können ferroresonante Sprünge ohne Charakteränderung der Phasenverschiebung erreicht werden ; letztere wird auch nach dem kapazitiven Sprung beibehalten. Nach Änderung der Induktanz des parametrischen Induktors, nach Stattfinden des ferroresonanten Sprunges, verändert die Klemmenspannung sowohl ihre Amplitude als auch die Phase ; der Rücksprung findet immer dann statt, wann die Phasenverschiebung einen kapazitiven Charakter hat. Die Experimentalergebnisse, die in den Oszillogrammen veranschaulicht sind, bestätigen die theoretischen Voraussetzungen voll und ganz.

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ASUPRA CIRCUITULUI *RLC* DERIVAŢIE, FEROREZONANT, NELINIAR ŞI PARAMETRIC

(Rezumat)

Este studiat circuitul *RLC* derivaţie, ferorezonant, neliniar şi parametric, la variaţia inductorului parametric, punîndu-se în evidenţă regimurile de funcţionare prin aplicarea unei metode grafo-analitice. Se remarcă faptul că pentru o pereche de inductoare date, după producerea saltului ferorezonant semnul fazei tensiunii la bornele circuitului este determinat de valorile capacităţii condensatorului şi rezistenţei circuitului. Pentru anumite valori ale acestora se pot obţine salturi ferorezonante fără schimbarea caracterului defazajului, acesta menţinîndu-şi caracterul capacitiv.

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EXISTENCE THEOREMS OF THE NORMAL FORM FOR ELECTRICAL NETWORKS WITH NONLINEAR COUPLINGS

BY

ADRIAN IOINOVICI

1. Introduction

Since there exist many networks that cannot be represented in the normal form, one of the basic problems in nonlinear circuits is to find sufficient conditions for networks which admit such a representation.

Let $\mathcal{N}$ be a nonlinear network containing independent and controlled sources, resistances, capacitances, inductances.

Let $\mathcal{T}$ be any normal tree of $\mathcal{N}$ chosen as in [1] and let each edge of the network graph for $\mathcal{N}$ represent a circuit element. It is advantageous to be partitioned the set of network branches into eight disjoint subsets, namely :

1. Tree-independent voltage sources E,
2. Tree-capacitances Ct,
3. Tree-resistances Rt,
4. Tree-inductances Lt,
5. Link-capacitances Cl,
5. Link-resistances Rl,
7. Link-inductances Ll,
8. Link-independent current sources J.

Based on the previous notations, the current and voltage vectors $(\mathbf{i}, \mathbf{v})$ as well as the fundamental cut-set matrix $\mathbf{Q}$ will be partitioned. Then, the governing equations can be written in the following form[1] :

$$\mathbf{i}_E = -\mathbf{Q}_{EC}\mathbf{i}_{Cl} - \mathbf{Q}_{ER}\mathbf{i}_{Rl} - \mathbf{Q}_{EL}\mathbf{i}_{Ll} - \mathbf{Q}_{EJ}\mathbf{i}_J, \tag{1}$$

$$\mathbf{i}_{Ct} = -\mathbf{Q}_{CC}\mathbf{i}_{Cl} - \mathbf{Q}_{CR}\mathbf{i}_{Rl} - \mathbf{Q}_{CL}\mathbf{i}_{Ll} - \mathbf{Q}_{CJ}\mathbf{i}_J, \tag{2}$$

$$\mathbf{i}_{Rt} = -\mathbf{Q}_{RR}\mathbf{i}_{Rl} - \mathbf{Q}_{RL}\mathbf{i}_{Ll} - \mathbf{Q}_{RJ}\mathbf{i}_J, \tag{3}$$

$$\mathbf{i}_{Lt} = -\mathbf{Q}_{LL}\mathbf{i}_{Rl} - \mathbf{Q}_{LJ}\mathbf{i}_J, \tag{4}$$

$$\mathbf{v}_{Cl} = \mathbf{Q}^t_{EC}\mathbf{v}_E + \mathbf{Q}^t_{CC}\mathbf{v}_{Ct}, \tag{5}$$

$$\mathbf{v}_{Rl} = \mathbf{Q}^t_{ER}\mathbf{v}_E + \mathbf{Q}^t_{CR}\mathbf{v}_{Ct} + \mathbf{Q}^t_{RR}\mathbf{v}_{Rt}, \tag{6}$$

$$\mathbf{v}_{Ll} = \mathbf{Q}^t_{EL}\mathbf{v}_E + \mathbf{Q}^t_{CL}\mathbf{v}_{Ct} + \mathbf{Q}^t_{RL}\mathbf{v}_{Rt} + \mathbf{Q}^t_{LL}\mathbf{v}_{Lt}, \tag{7}$$

$$\mathbf{v}_J = \mathbf{Q}^t_{EJ}\mathbf{v}_E + \mathbf{Q}^t_{CJ}\mathbf{v}_{Ct} + \mathbf{Q}^t_{RJ}\mathbf{v}_{Rt} + \mathbf{Q}^t_{LJ}\mathbf{v}_{Lt}. \tag{8}$$

Let the circuit elements be characterized by the functions :

$$\mathbf{q}_{Ct}=\mathbf{f}_{Ct}(\mathbf{v}_{Ct}), \qquad \mathbf{q}_{Cl}=\mathbf{f}_{Cl}(\mathbf{v}_{Cl}), \tag{9}$$

$$\varphi_{Lt}=\mathbf{f}_{Lt}(\mathbf{i}_{Lt}, \mathbf{i}_{Ll}), \qquad \varphi_{Ll}=\mathbf{f}_{Ll}(\mathbf{i}_{Lt}, \mathbf{i}_{Ll}), \tag{10}$$

$$\mathbf{f}_R(\mathbf{i}_{Rt}, \mathbf{v}_{Rt}, \mathbf{i}_{Rl}, \mathbf{v}_{Rl})=0. \tag{11}$$

The network can be represented in the normal form if the next equations (called R, L, C, respectively) admit unique solutions [1]:

$$\mathbf{q}_{Ct}+\mathbf{Q}_{CC}\mathbf{q}_{Cl}=\mathbf{f}_{Ct}(\mathbf{v}_{Ct})+\mathbf{Q}_{CC}\mathbf{f}_{Cl}(\mathbf{Q}^t_{EC}\mathbf{v}_E+\mathbf{Q}^t_{CC}\mathbf{v}_{Ct}), \tag{12}$$

$$\varphi_{Ll}-\mathbf{Q}^t_{LL}\varphi_{Lt}=\mathbf{f}_{Ll}(-\mathbf{Q}_{LL}\mathbf{i}_{Ll}-\mathbf{Q}_{LJ}\mathbf{i}_J, \mathbf{i}_{Ll})-\mathbf{Q}^t_{LL}\mathbf{f}_{Lt}(-\mathbf{Q}_{LL}\mathbf{i}_{Ll}-\mathbf{Q}_{LJ}\mathbf{i}_J, \mathbf{i}_{Ll}), \tag{13}$$

$$\mathbf{f}_R(-\mathbf{Q}_{RR}\mathbf{i}_{Rl}-\mathbf{Q}_{RL}\mathbf{i}_{Ll}-\mathbf{Q}_{RJ}\mathbf{i}_J, \mathbf{v}_{Rt}, \mathbf{i}_{Rl}, \mathbf{Q}^t_{ER}\mathbf{v}_E+\mathbf{Q}^t_{CR}\mathbf{v}_{Ct}+\mathbf{Q}^t_{RR}\mathbf{v}_{Rt})=0. \tag{14}$$

2. Main Existence Theorems

We have studied here the equations R, L, C for the real nonlinear networks which contain linear elements too.

Theorem 1[r] *If $\mathcal{T}$ is chosen so that the link-resistances are linear time-invariant, the tree-resistances nonlinear, the linear controlled sources represent chords and the nonlinear controlled sources tree-branches, (11) becomes:*

$$\mathbf{i}_{Rl}=\mathbf{G}_{lt}\mathbf{v}_{Rt}+\mathbf{G}_{ll}\mathbf{v}_{Rl}, \qquad \mathbf{i}_{Rt}=\mathbf{f}_{Rt}(\mathbf{v}_{Rt}, \mathbf{v}_{Rl}), \tag{15}$$

or, more detailed:

$$i_{Rt_i}=f_{Rt_{i1}}(v_{Rt_1})+\ldots+f_{Rt_{in}}(v_{Rt_n})+h_{Rt_{i1}}(v_{Rl_1})+\ldots+h_{Rt_{im}}(v_{Rl_m}),\ (i=1, 2,\ldots,n), \tag{15'}$$

where n represents the number of resistive tree-branches and m – the number of resistive chords.

In this case the equation R admits a unique solution, if the following hypotheses are satisfied:

a) Each $f_{Rt_{i1}},\ldots,f_{Rt_{in}}$, $(i=1, 2,\ldots,n)$, is a maximal monotone graph mapping R^1 onto R^1.

b) Each $h_{Rt_{i1}},\ldots, h_{Rt_{im}},(i=1, 2,\ldots, n)$, is a Lipschitz function, with the constants $L_1,\ldots, L_m$.

c) The matrix:

$$\mathbf{S}=\mathbf{Q}_{RR}\mathbf{G}_{ll}\mathbf{Q}^t_{RR}+\mathbf{Q}_{RR}\mathbf{G}_{lt} \tag{16}$$

is positive-definite:

$$<\mathbf{S}\mathbf{v}_{Rt}, \mathbf{v}_{Rt}>\geqslant\omega\,\|\mathbf{v}_{Rt}\|\ ^{1}), \tag{17}$$

where ω is a positive number.

$$\text{d)}\quad \omega>\sum_{j=1}^{m} L_j\ \Big|\Big/\overline{n\sum_{i=1}^{n} t_{ji}^2}, \tag{18}$$

where t_{ji}, $(i=1,2,\ldots,n\ ;\ j=1, 2,\ldots,m)$, are the elements of the matrix:

---

[1]) $\|\ldots\|$ represent the usual Euclidean norm.

(19) $$\mathbf{T}=\mathbf{Q}_{RR}^{t}.$$

P r o o f. Using, succesively, (3), (15), (6), the equation R finally becomes:

$$\mathbf{f}_{Rt}(\mathbf{v}_{Rt},\ \mathbf{Q}_{RR}^{t}\mathbf{v}_{Rt}+\mathbf{Q}_{ER}^{t}\mathbf{v}_{E}+\mathbf{Q}_{CR}^{t}\mathbf{v}_{Ct})+(\mathbf{Q}_{RR}\mathbf{G}_{tt}\mathbf{Q}_{RR}^{t}+\mathbf{Q}_{RR}\mathbf{G}_{tt})\mathbf{v}_{Rt}=$$
$$=-\mathbf{Q}_{RR}\mathbf{G}_{tt}\mathbf{Q}_{ER}^{t}\mathbf{v}_{E}-\mathbf{Q}_{RR}\mathbf{G}_{tt}\mathbf{Q}_{CR}^{t}\mathbf{v}_{Ct}-\mathbf{Q}_{RL}\mathbf{i}_{Lt}-\mathbf{Q}_{RJ}\mathbf{i}_{J},$$

or, according to (16) and (19):

(20) $$\mathbf{f}_{Rt}(\mathbf{v}_{Rt},\mathbf{T}\mathbf{v}_{Rt}+\mathbf{t})+\mathbf{S}\mathbf{v}_{Rt}=\mathbf{s},$$

where ;

(21) $$\mathbf{t}=\mathbf{Q}_{ER}^{t}\mathbf{v}_{E}+\mathbf{Q}_{CR}^{t}\mathbf{v}_{Ct},$$

(22) $$\mathbf{s}=-\mathbf{Q}_{RR}\mathbf{G}_{tt}\mathbf{Q}_{ER}^{t}\mathbf{v}_{E}-\mathbf{Q}_{RR}\mathbf{G}_{tt}\mathbf{Q}_{CR}^{t}\mathbf{v}_{Ct}-\mathbf{Q}_{RL}\mathbf{i}_{Lt}-\mathbf{Q}_{RJ}\mathbf{i}_{J}.$$

According to (15′), (20) becomes:

(23) $$f_{Rt_{i_1}}(v_{Rt_1})+\ldots+f_{Rt_{i_n}}(v_{Rt_n})+h_{Rt_{i_1}}(t_{11}v_{Rt_1}+\ldots+t_{1n}v_{Rt_n}+t_1)+\ldots+$$
$$+h_{Rt_{i_m}}(t_{m1}v_{Rt_1}+\ldots+t_{mn}v_{Rt_m}+t_m)+s_{i1}v_{Rt_1}+\ldots+s_{in}v_{Rt_n}=s_i,\ (i=1,2,\ldots,n),$$

where t_j, $(j=1, 2,\ldots,m)$, and s_i, $(i=1, 2,\ldots, n)$, represent the elements of the vectors **t** and **s**.

Having in view [2], the hypotheses a)...d) are sufficient to assure a unique solution of (23), namely:

(24) $$\mathbf{v}_{Rt}=\mathbf{g}_{Rt}(\mathbf{v}_{Ct},\ \mathbf{i}_{Lt},\ \mathbf{v}_{E},\ \mathbf{i}_{J}).$$

Using (6) and (15_1) it also results the unique solution:

(25) $$\mathbf{i}_{Rt}=\mathbf{g}_{Rt}(\mathbf{v}_{Ct},\mathbf{i}_{Lt},\mathbf{v}_{E},\mathbf{i}_{J}).$$

T h e o r e m 2. *If $\mathcal{T}$ is chosen so that the tree-resistances are linear and time-invariant, the link-resistances are nonlinear, the linear controlled sources represent tree-branches and the nonlinear controlled sources chords, (11) becomes*:

(26) $$\mathbf{v}_{Rt}=\mathbf{R}_{tt}\mathbf{i}_{Rt}+\mathbf{R}_{tl}\mathbf{i}_{Rt},\qquad \mathbf{v}_{Rl}=\mathbf{f}_{Rl}(\mathbf{i}_{Rl},\mathbf{i}_{Rt}),$$

or, more detailed:

(26′) $$v_{Rl_i}=f_{Rl_{i_1}}(i_{Rl_1})+\ldots+f_{Rl_{i_n}}(i_{Rl_n})+h_{Rl_{i_1}}(i_{Rt_1})+\ldots+h_{Rl_{i_m}}(i_{Rt_m}),\ (i=1,2,\ldots,n),$$

where n denotes the number of resistive chords and m the number of resistive tree-branches.

Then, the equation R admits the unique solutions (25) and (24) if the same hypotheses a)...d) of Theorem 1 are satisfied, where (16), (17), (19) become:

(16′) $$\mathbf{S}=\mathbf{Q}_{RR}^{t}\mathbf{R}_{tt}\mathbf{Q}_{RR}-\mathbf{Q}_{RR}^{t}\mathbf{R}_{tl},\qquad (17')\quad <\mathbf{S}\mathbf{i}_{Rl},\ \mathbf{i}_{Rl}>\geqslant\omega\,\|\mathbf{i}_{Rl}\|,$$

(19′) $$\mathbf{T}=-\mathbf{Q}_{RR}.$$

Remark 1. If the matrix S is nonsingular, one can apply the Banach fixed point theorem to (23), but the set of hypotheses founded thus is more restrictive than the conditions a)...d).

Remark 2. Other types of controlled sources and inductive couplings (Eq. *L*) may be considered in a similar manner.

Remark 3. The theorems are also valid if the dependent sources are controlled by voltages of capacitive branches, currents of inductive branches, voltages of independent voltage sources or currents of independent current sources.

Proof. The additional terms appear in the vector **s**.

3. Particular Cases

Reducing the generality of the previous theorems one can express the constraints regarding the existence of the normal form in terms of the nature of the characteristic curves of the network elements and the network topology.

Theorem 3. *Let the same circuit as in Theorem 1 be considered, with two additional conditions: a) there are no nonlinear couplings between the resistive tree-branches and b) the currents of different resistive tree-branches are not functions of the voltage of the same resistive chords. Then, using (6), (19) and (21), (15′) becomes:*

$$(27)\quad \begin{aligned} i_{Rt_i} = f_{Rt_{ii}}(0.v_{Rt_1}+\ldots+1.v_{Rt_i}+\ldots+0.v_{Rt_n}+0)+h_{Rt_{i1}}\ (t_{11}v_{Rt_1}+\ldots+ \\ +t_{1n}v_{Rt_n}+t_1)+\ldots+h_{Rt_{im}}(t_{m1}v_{Rt_1}+\ldots+t_{mn}v_{Rt_m}+t_m)\ ,\ (i=1,\,2,\ldots,n). \end{aligned}$$

But the tree-branch i can represent either a resistance or a controlled source. So, (27) can be written:

$$(28)\quad i_{Rt_i}=\rho_{Rt_i}(\overline{t}_{i1}v_{Rt_1}+\overline{t}_{i2}v_{Rt_2}+\ldots+\overline{t}_{in}v_{Rt_n}+\overline{t}_i),\quad (i=1,\,2,\ldots,\,n),$$

where ρ_{Rt} represents one of the functions f_{Rt} or h_{Rt}.

Eq. (28) can be also written:

$$(29)\quad \mathbf{i}_{Rt}=\rho_{Rt}(\overline{\mathbf{T}}\mathbf{v}_{Rt}+\overline{\mathbf{t}}).$$

Using (29), (3), (15) and (6), with the notations (16) and (22), the equation R becomes:

$$(30)\quad \rho_{Rt}(\overline{\mathbf{T}}\mathbf{v}_{Rt}+\overline{\mathbf{t}})+\mathbf{S}\mathbf{v}_{Rt}=\mathbf{s}.$$

If the matrix $\overline{\mathbf{T}}$ is nonsingular, using the notation:

$$(31)\quad \mathbf{u}=\overline{\mathbf{T}}\mathbf{v}_{Rt}+\overline{\mathbf{t}}$$

Eq. (30) becomes:

$$(32)\quad \rho_{Rt}(\mathbf{u})+\mathbf{S}\overline{\mathbf{T}}^{-1}\mathbf{u}=\mathbf{s}+\mathbf{S}\overline{\mathbf{T}}^{-1}\overline{\mathbf{t}}.$$

Having in view [3], the next hypotheses are sufficient to assure a unique solution of (32):

(i) Each $f_{Rt_{i1}},\ldots, f_{Rt_{in}}, h_{Rt_{i1}},\ldots, h_{Rt_{im}}$, $(i=1, 2,\ldots,n)$, is a continue, strictly monotone increasing function mapping R^1 into R^1.

(ii) The elements s_{ij} of the matrix S satisfy the inequality :

$$s_{ii} > \sum_{\substack{i=1 \\ i\neq j}}^{n} | s_{ij} |, \quad (j=1, 2,\ldots,n). \tag{33}$$

(iii) The elements $\bar{t}_{ij}$ of the matrix $\bar{\mathbf{T}}$ satisfy the inequality :

$$\bar{t}_{ii} > \sum_{\substack{i=1 \\ i\neq j}}^{n} | \bar{t}_{ij} |, \quad (j=1, 2,\ldots,n). \tag{34}$$

The following condition must be added :

(iiii) The matrix $\bar{\mathbf{T}}$ is nonsingular.

R e m a r k 1. The matrix $\bar{\mathbf{T}}$ is obtained by filling in the i row either by the set $(0, 0,\ldots, 1,\ldots, 0, 0)$ (n elements) if the i tree-branch corresponds to a resistance or by one of the sets $(t_{j1}, t_{j2},\ldots,t_{jn})$, $(j=1, 2,\ldots,m)$, if the i tree-branch corresponds to a source controlled by v_{Rl_j}, $(j=1, 2,\ldots,m)$.

R e m a r k 2. Other particular cases, corresponding to other general theorems, can be studied in a similar manner.

4. Example

Consider the network of Fig. 1 ; the graph is represented in Fig. 2. The functions f_4 and f_5 are continuous, strictly monotone increasing, mapping R^1 onto R^1. In this case :

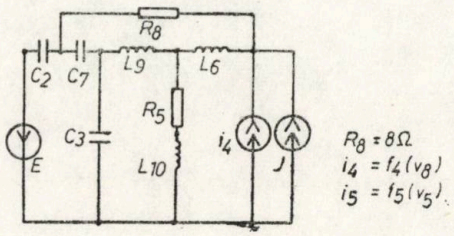

Fig. 1

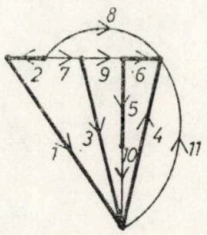

Fig. 2

$$Q_{RR}=\begin{bmatrix}1\\0\end{bmatrix}; \quad i_8=0\cdot v_4+0\cdot v_5+\frac{1}{8}v_8;$$

$$G_{it}=[0 \quad 0]; \qquad G_{ii}=\begin{bmatrix}\frac{1}{8}\end{bmatrix}.$$

According to (16) and (19), it results :

$$S=\begin{bmatrix}1/8 & 0\\ 0 & 0\end{bmatrix}; \quad T=[1 \quad 0]; \quad \bar{T}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

and all the hypotheses (i)...(iiii) are fulfilled.

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TEOREME DE EXISTENŢĂ A FORMEI NORMALE A ECUAŢIILOR CIRCUITELOR ELECTRICE CU CUPLAJE NELINIARE

(Rezumat)

Se propun cîteva teoreme de existenţă a soluţiilor unice ale ecuaţiilor R, L, C, (12)...(14) care asigură existenţa formei normale a ecuaţiilor circuitului electric. În unele cazuri particulare teoremele referitoare la ecuaţia R conţin condiţiile (i)...(iiii) uşor de verificat prin simplă inspecţie.

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SYMBOLIC REPRESENTATION OF SIGNALS ON HYPERSPACES

I. SYMBOLIC REPRESENTATION OF MODULATED SIGNALS

BY

DANIEL CONDURACHE

1. Introduction

For the first time, the notion of symbolic representation of a real, derivable function of a scalar argument by means of a finite dimension algebra element on the real number field, was introduced in [1]. A certain element of the algebra represents symbolically a function belonging to a certain family if, whichever the function be, the element corresponding to its derivative results from the multiplication of the element corresponding to the function with a unique element of the algebra playing the role of a derivative operator[1]).

The functions which may be represented symbolically, as this procedure appears in [1], are those satisfying differential equations with linear-constant coefficients. Applications of this representation in Mechanical Vibration and Seismology have there been offered. Three of the [1] established correspondences were used in [2] studying the parameters of some feedback control systems. This proceeding, though utilizable for any order algebras, has only been applied to some classes of functions leading to small order algebras because of very laborious calculi.

The aim of this paper — the first in a series dedicated to the study of a new symbolic representation of signals — is to demonstrate that this proceeding may be extended to the symbolic representation of a large class of analogical signals passing beyond the difficult calculi appeared in applications. This extension is due to the character of direct sum algebras of representing a reunion of hypercomplex orthogonal spaces. At the same time, some original, first notified aspects, applicable in problems concerning the analysis and synthesis of electrical network, transmitted message, etc. are rendered evident.

2. General Results

We shall denote with $f_i \to \hat{w}_i$ the possibility to represent symbolically a real function f_i, by the element $\hat{w}_i$ of an algebra. Excepting basis elements, the elements of the algebras will all be affected by the „$\wedge$" sign. In [1] it

[1]) For instance, the harmonic signal $a(t)=A \sin(\omega t+\varphi)$ is represented symbolically on the second order complex algebra (complex field) by an element

$$\hat{a}=A\cos\varphi+jA\sin\varphi,\quad j^2=-1,$$

with the derivation operator $\hat{d}=j\omega$.

is demonstrated that the sum of two functions representable symbolically, each of them, on finite algebras on the real number field, is a function which may be represented symbolically on the direct sum of the two algebras. The demonstration that follows becomes possible by means of mathematical induction :

Theorem. *If n real valued functions of a scalar variable* $f_k(t)$, $k=\overline{1,n}$, *are represented symbolically on the finite algebras over the real number field* $\mathcal{A}_{n_k}$, *then the function* $f(t)=\sum_{k=1}^{n} f_k(t)$ *may be represented symbolically by an element belonging to the direct sum of the algebras* $\mathcal{A}_{n_k}$. *Both the derivation operator and the respective element are obtained without supplementary calculi.*

Let n_k be the order of algebra $\mathcal{A}_{n_k}$ and :

$$\varepsilon_i^{(n_k)}\varepsilon_j^{(n_k)}=\sum_{j=1}^{n_k}\gamma_{ij\nu}\varepsilon_\nu^{(n_k)} \tag{1}$$

its multiplication table, where $\varepsilon_i^{(n_k)}$, $i=\overline{1,n_k}$, denote the elements of a basis in algebra $\mathcal{A}_{n_k}$ and $\gamma_{ij\nu}$ its constants of structure.

The element $\hat{w}_{n_k}\in\mathcal{A}_{n_k}$, given by :

$$\hat{w}_{n_k}=\sum_{\nu=1}^{n_k} a_\nu^{(n_k)}\varepsilon_\nu^{(n_k)},\quad a_\nu^{(n_k)}\in\mathbf{R}, \tag{2}$$

with the derivation operator :

$$\hat{d}_{n_k}=\sum_{\nu=1}^{n_k}\delta_\nu^{(n_k)}\varepsilon_\nu^{(n_k)},\quad \delta_\nu^{(n_k)}\in\mathbf{R}, \tag{3}$$

represents symbolically the function $f_k(t)$ on algebra $\mathcal{A}_{n_k}$ [1].

Let $\mathcal{A}$ be the direct sum algebra of algebras $\mathcal{A}_{n_k}$, $k=\overline{1,n}$. The order of algebra $\mathcal{A}$ is :

$$\sum_{k=1}^{n} n_k=N. \tag{4}$$

With ε_ν, $\nu=1$, N, denoting base elements in algebra $\mathcal{A}$, an element :

$$\hat{w}=\sum_{\nu=1}^{N} a_\nu\,\varepsilon_\nu,\ a_\nu\in\mathbf{R}, \tag{5}$$

with the derivation operator :

$$\hat{d}=\sum_{\nu=1}^{N}\delta_\nu\,\varepsilon_\nu,\ \delta_\nu\in\mathbf{R}, \tag{6}$$

where a_ν, $\delta_\nu\in\mathbf{R}$, are to be subsequently specified, represents symbolically the function $f(t)=\sum_{k=1}^{n} f_k(t)$ over algebra $\mathcal{A}$.

Let us consider the following finite sets:

$$M=\left\{1,\ 2,\ 3,\ldots,\ n_1,\ldots,\ n_1+n_2,\ldots,\ \sum_{k=1}^{n} n_k\right\}, \tag{7}$$

$$M_p=\left\{1+\sum_{k=1}^{p-1} n_k,\ 2+\sum_{k=1}^{p-1} n_k,\ldots,\ n_p+\sum_{k=1}^{p-1} n_k\right\},\quad \forall p=\overline{1,n}. \tag{8}$$

The following relations became obvious:

$$\bigcap_{p=1}^{n} M_p=\emptyset,\ \bigcup_{p=1}^{n} M_p=M. \tag{9}$$

From the same reasons than in [1], it results:

$$\delta_\nu=\delta_\nu^{(n_p)\,n-\sum_{k=1}^{p} n_k}, \tag{10}$$

$$a_\nu=a_\nu^{(a_p)\,n-\sum_{k=1}^{p} n_k};\quad \forall \nu\in M_p,\ p=\overline{1,n}. \tag{11}$$

The element (5) and its corresponding derivation operator are thus obtained without further calculi having in view the representation of the functions $f_k(t)$ on algebras $\mathcal{A}_{n_k}$.

Let ε_ν, $\nu=\overline{1,N}$, be elements of a certain basis in algebra $\mathcal{A}$. Their multiplication table [3] is:

$$\varepsilon_i\,\varepsilon_j=\delta_{kp}\,e_i^{(n_k)}e_j^{(n_p)};\ i\in M_k,\ j\in M_p,\ k,p=\overline{1,n}, \tag{12}$$

where δ_{kp} is K r o n e c k e r symbol. An element $\hat{w}\in\mathcal{A}$ may be written as:

$$\hat{w}=\sum_{\nu=1}^{N} a_\nu\varepsilon_\nu=\sum_{\nu\in M} a_\nu\varepsilon_\nu=\sum_{k=1}^{n}\sum_{\nu\in M_k} a_\nu\varepsilon_\nu. \tag{13}$$

Denoting by:

$$\varepsilon'_\nu=\varepsilon_{\nu-\sum_{k=1}^{p} n_k}^{(n_p)\,n-\sum_{k=1}^{p} n_k},\ \forall\ \nu\in M_p,\ p=\overline{1,n}, \tag{14}$$

then ε'_ν, $\nu=\overline{1,N}$, satisfy the multiplication table (12) only if:

$$\varepsilon_\nu^{(n_k)}\varepsilon_\nu^{(n_p)}=0;\ k\neq p;\ k,p=\overline{1,n},\ \nu=\overline{1,n_k}. \tag{15}$$

Taking into account both (11), (14) and (15), then (13) becomes:

$$\hat{w}=\sum_{k=1}^{n}\sum_{k=1}^{n} a_\nu^{(n_k)}\,\varepsilon_\nu^{(n_k)}=\sum_{k=1}^{n}\hat{w}_{n_k}, \tag{16}$$

where $\hat{w}_{nk}$ is given by relation (2).

Conditions (15) may be given up if the element (16) is written as:

$$\hat{w}=\sum_{k=1}^{n}\hat{w}_{n_k}\mathbf{E}_k, \tag{17}$$

where $\mathbf{E}_k$, $k=\overline{1,n}$, are symbols with the properties:

$$\mathbf{E}_k\mathbf{E}_p=\begin{cases}\mathbf{E}_k & \text{if } p=k,\\ 0 & \text{if } p\neq k.\end{cases} \tag{18}$$

In similar circumstances, the expression of the derivation operator in algebra $\mathcal{A}$ is:

$$\hat{\mathrm{d}}=\sum_{k=1}^{n}\hat{\mathrm{d}}_{n_k}\mathbf{E}_k, \tag{19}$$

with $\hat{\mathrm{d}}_{n_k}$ the derivation operator in algebra $\mathcal{A}_{n_k}$.

Remarks.

1. As a result of relations (17) and (18), for algebraic operations with elements of algebra $\mathcal{A}$, it is enough to know the multiplication tables of algebras $\mathcal{A}_{n_k}$, $k=\overline{1,n}$.

2. A certain element $\hat{w}\in\mathcal{A}$ may now be considered as a *figurative point* belonging to a hyperspace $\mathcal{A}$. This hyperspace is a result of the reunion of n hypercomplex spaces $(\mathcal{A}_{n_k},\ k=\overline{1,n})$ on which the functions $f_k(t)$ are represented symbolically. The n hypercomplex spaces are orthogonal (15) and have a unique point in common which is the zero element of the algebras. In what follows $\hat{w}_{nk}$ will be denoted as component of the figurative point $\hat{w}$ in the hypercomplex space $\mathcal{A}_{n_k}$.

If algebras $\mathcal{A}_{n_k}$ are associative and have the unity element among the ones of the basis, an exponential form [1] may be established for a certain element $\hat{w}_{n_k}$ in $\mathcal{A}_{n_k}$:

$$\hat{w}_{n_k}=\prod_{\nu=1}^{n_k}\exp\left[x_{\nu k}\varepsilon_{\nu}^{(n_k)}\right], \tag{20}$$

where the real quantities $x_{\nu k}$, $\nu=\overline{1,n_k}$, depend on the real ones $a_{\nu}^{(n_k)}$ involved in (2). With $\varepsilon_1^{(n_k)}$ as unity element, in algebra $\mathcal{A}_{n_k}$, $\exp x_{1k}$ requires the role of *norm* for the element $\hat{w}_{n_k}$[1].

We now denote:

$$\|\hat{w}_{n_k}\|=\exp x_{1k},\ \forall k=\overline{1,n}. \tag{21}$$

By *definition*,

$$\|\hat{w}\|_r=\sum_{k\in\mathcal{P}}\|\hat{w}_{n_k}\| \tag{22}$$

is the *r order quasi-norm* of a certain element $\hat{w}\in\mathcal{A}$ where:

$$\mathcal{P}\subseteq\{1,2,\ldots,n\},\ \operatorname{card}\mathcal{P}=r. \tag{23}$$

We also denote:

$$\overline{\mathcal{P}}=\{1,2,\ldots,n\}\setminus\mathcal{P}. \tag{24}$$

The n order quasi-norm shall be named *norm of the element* $\hat{w}\in\mathcal{A}$ and denoted as $\|\hat{w}\|$. We shall also *define* an element $\hat{w}\in\mathcal{A}$, as *r-order quasi-divisor of zero* if:

(25) $$\|\hat{w}\|_r = 0.$$

The n order quasi-divisors of zero are *divisors of zero* in the hyperspace $\mathcal{A}$.

3. A Symbolic Representation of Modulated Signals

Let $s(t)$ be an amplitude modulated signal[2]) with an arbitrary modulating signal, the harmonic carrier being amortized:

(26) $$s(t) = Se^{\sigma t}[1+f(t)]\sin(\omega t+\varphi).$$

In [4] it is demonstrated that, whichever the function $f(t)$ is, analytically or graphically given, it should be approximated with an arbitrary small error, as:

(27) $$f_a(t) = \sum_{k=1}^{n} m_k e^{\sigma_k t}\sin(\omega_k t+\psi_k).$$

Under these circumstances, the signal $s(t)$ may be written as:

(28) $$s(t) = \sum_{k=0}^{n} f_k(t),$$

where:

(29) $$f_0(t) = S\,e^{\sigma t}\sin(\omega t+\varphi),$$

(30) $$f_k(t) = m_k S\,e^{(\sigma+\sigma_k)t}\sin(\omega_k t+\psi_k)\sin(\omega t+\varphi),\quad \forall k=\overline{1,n}.$$

The function (29) may be represented symbolically [1] on the second order complex algebra by means of an element like:

(31) $$\hat{w}_0 = S\,e^{j\varphi}\leftrightarrow S\,e^{\sigma t}\sin(\omega t+\varphi),$$

with 4^{th} derivation operator and multiplication tables respectively:

(32) $$\hat{d}_0 = \delta+j\omega\ ;\ j^2=-1,\ 1.j=j\cdot 1=j.$$

The functions $f_k(t)$, $k=\overline{1,n}$, can be symbolically represented [1] on the 4^{th} order complex algebra by an element having the form:

(33) $$\hat{w}_k = m_k S\,e^{i\psi_k}e^{j\varphi}\leftrightarrow m_k\ S\,e^{(\sigma+\sigma_k)t}\sin(\omega_k t+\psi_k)\sin(\omega t+\varphi),$$

with the derivation operator and multiplication table respectively:

(34) $$\hat{d}_k = \sigma+\sigma_k+\omega_k i+\omega j\ ;\ i^2=j^2=-1,\ (ij)^2=(ji)^2=1.$$

If we consider the results obtained in §2, the signal (26) may be represented symbolically by an element situated on the hyperspace $\mathcal{H}$ resulted from the reunion of the complex field with n hypercomplex orthogonal 4^{th} order spaces:

(35) $$\hat{w}^* = S\,e^{j\varphi}\mathbf{E}_0+\sum_{k=1}^{n} m_k S\,e^{i\psi_k}e^{j\varphi}\mathbf{E}_k,$$

---

[2]) A. M.

with the derivation operator:

$$\hat{d}=(\sigma+j\omega)\mathbf{E}_0+\sum_{k=1}^{n}(\sigma+\sigma_k+i\omega_k+j\omega)\mathbf{E}_k, \tag{36}$$

with $\mathbf{E}_k$, $k=\overline{0,\ n}$, denoted by (18).

A certain element belonging to the hyperspace $\mathcal{H}$ may be written as:

$$\hat{w}=(a_1+a_2j)\mathbf{E}_0+\sum_{p=1}^{n}(a_{1p}+a_{2p}i+a_{3p}j+a_{4p}ij)\mathbf{E}_p, \tag{37}$$

or, in an exponential form:

$$\hat{w}=A\,e^{j\varphi}\mathbf{E}_0+\sum_{p=1}^{n}(A_{1p}+ijA_{2p})\,e^{i\beta_p}\,e^{j\alpha_p}\mathbf{E}_p. \tag{38}$$

The transition from (38) to (37) is possible with aid of the multiplication table (34_2) and of Euler's relation $e^{j\beta}=\cos\beta+j\sin\beta$, $j^2=-1$.

The quantities from (37) and (38) are linked by the following relations:

$$\begin{cases} A=\sqrt{a_1^2+a_2^2},\ \operatorname{tg}\alpha=\dfrac{a_2}{a_1}, \\ A_{1p},\ A_{2p}=\dfrac{1}{2}\left[\sqrt{(a_{1p}+a_{4p})^2+(a_{2p}-a_{3p})^2}\pm\sqrt{(a_{1p}-a_{4p})^2+(a_{2p}+a_{3p})^2}\right], \\ \operatorname{tg}\beta_p=\dfrac{a_{4p}A_{1p}-a_{1p}A_{2p}}{a_{3p}A_{1p}+a_{2p}A_{2p}}\left(=\dfrac{a_{2p}A_{1p}+a_{3p}A_{2p}}{a_{1p}A_{1p}-a_{4p}A_{2p}}\right), \\ \operatorname{tg}\alpha_p=\dfrac{a_{3p}A_{1p}+a_{2p}A_{2p}}{a_{1p}A_{1p}-a_{4p}A_{2p}}\left(=\dfrac{a_{4p}A_{1p}-a_{1p}A_{2p}}{a_{2p}A_{1p}+a_{3p}A_{2p}}\right). \end{cases} \tag{39}$$

We may also establish the relations [1]:

$$\|a_1+a_2j\|=\sqrt{a_1^2+a_2^2}=A,\ \|a_{1p}+a_{2p}i+a_{3p}j+a_{4p}ij\|=\sqrt{A_{1p}^2-A_{2p}^2}, \tag{40}$$

so that the *r-order quasi-norm* of the element (32) becomes:

$$\|\hat{w}\|_r=\varepsilon A+\sum_{p\in\mathcal{P}}\sqrt{A_{1p}^2-A_{2p}^2},\ \operatorname{card}\mathcal{P}=r-\varepsilon, \tag{41}$$

where ε may be 0 or 1.

The r-order quasi-divisors of zero are characterized by:

$$\varepsilon A=0,\ A_{1p}^2=A_{2p}^2,\ \forall p\in\mathcal{P},\ \operatorname{card}\mathcal{P}=r-\varepsilon. \tag{42}$$

Relations (42) are satisfied only if:

$$\begin{cases} \varepsilon A=0, \\ A_{1p}=A_{2p}, \end{cases}\Leftrightarrow\begin{cases} \varepsilon a_1=\varepsilon a_2=0, \\ a_{1p}+a_{4p}=0, \\ a_{2p}-a_{3p}=0, \end{cases}\quad\forall p\in\mathcal{P},\ \operatorname{card}\mathcal{P}=r-\varepsilon, \tag{43}$$

or:

$$(44)\qquad \begin{cases} \varepsilon A=0, \\ A_{1p}=-A_{2p}, \end{cases} \Leftrightarrow \begin{cases} \varepsilon a_1=\varepsilon a_2=0, \\ a_{1p}-a_{4p}=0, \\ a_{2p}+a_{3p}=0, \end{cases} \quad \forall p\in\mathcal{P},\ \text{card}\,\mathcal{P}=r-\varepsilon.$$

The equivalences are obvious thanks to relation (39). We shall name the elements satisfying relations (43), respectively (44), *r-order quasi-divisors of zero of 1st, (2nd) species.*

With $r=n$ ($\varepsilon=1$) in relations (43), (44), the conditions for divisors of zero of 1st (2nd) species in the hyperspace $\mathcal{H}$ are obtained.

By *definition*, a certain element $\hat{w}\in\mathcal{H}$ is denoted as *r-order arbitrary element* if :

$$(45)\qquad \begin{cases} A_{2p}=0 \\ \alpha_p=\alpha \end{cases} \Leftrightarrow \frac{a_1}{a_2}=\frac{a_{1p}}{a_{3p}}=\frac{a_{2p}}{a_{4p}},\ \forall p\in\mathcal{P},\ \text{card}\,\mathcal{P}=r-\varepsilon.$$

The n order arbitrary elements will be denoted as *special type elements*. We may notice that the signal (26) is represented in the hyperspace $\mathcal{H}$ by a special type element.

The following designations will be hence used :

$$(46)\qquad \begin{cases} C_s-\text{the class of special type elements in the hyperspace } \mathcal{H}. \\ C_r,(r=\overline{1,n-1})- \text{ the class of } r\text{-order arbitrary elements in the hyperspace } \mathcal{H}. \\ O'(O'')\ -\text{the class of 1st (2nd) species divisors of zero in the hyperspace } \mathcal{H}. \\ O'(O''),\ (r=\overline{1,n-1})\ -\text{the class of } r\text{-order quasi-divisors of zero of 1st (2nd) species on the hyperspace } \mathcal{H}. \end{cases}$$

The relation :

$$(47)\qquad \hat{w}_1\sim\hat{w}_2$$

is introduced in the hyperspace $\mathcal{H}$ if $\hat{w}_1$ and $\hat{w}$ satisfy the same kind of relation (43), (44) or (45). It is easy to verify that (47) is an equivalence relation in the hyperspace $\mathcal{H}$. The equivalence classes determined by (47) are (46).

This way, a complete partition of the hyperspace $\mathcal{H}$ into partition classes was realised. Those partition classes enjoy some remarkable properties.

4. Properties of the Partition Classes

a) *The Class of r-Order Arbitrary Elements.* An element belonging to this class in denoted as :

$$(48)\qquad \hat{w}_r=Ae^{j\alpha}\mathbf{E}_0+\sum_{p\in\mathcal{P}}A_{1p}\,e^{i\beta_p}e^{j\alpha}\mathbf{E}_p+\sum_{p\in\overline{\mathcal{P}}}(A_{1p}+ijA_{2p})e^{i\beta_p}e^{j\alpha_p}\mathbf{E}_p,$$

where card $\mathcal{P}=r$ and the signification of $\overline{\mathcal{P}}$ is given by (24).

Taking into account that :

$$(49)\qquad ijA_{2p}e^{i\beta_p}e^{j\alpha_p}=A_{2p}e^{i(\beta_p+\pi/2)}e^{j(\alpha_p+\pi/2)}$$

and the correspondences (31), (33), the following one is obtained:

$$\hat{w}_r \leftrightarrow A e^{\sigma t}\left[1+\sum_{p\in\mathcal{P}} \mu_p e^{\sigma_p t} \sin(\omega_p t+\beta_p)\right]\sin(\omega t+\alpha)+ \tag{50}$$
$$+\sum_{p\in\bar{\mathcal{P}}} \mathcal{A}_p(t)\sin[\omega t+\beta_p+\theta_p(t)],$$

with the derivation operator (36) and:

$$\mu_p=\frac{A_{1p}}{A},\quad \forall p\in\mathcal{P}, \tag{51}$$

$$\begin{cases} \mathcal{A}_p(t)=e^{(\sigma+\sigma_p)t}\sqrt{A_{1p}^2\sin^2(\omega_p t+\beta_p)+A_{2p}^2\cos^2(\omega_p t+\beta_p)}, \\ \operatorname{tg}\theta_p(t)=\dfrac{A_{2p}}{A_{1p}}\operatorname{ctg}(\omega_p t+\beta_p),\ \forall p\in\overline{\mathcal{P}}. \end{cases} \tag{51'}$$

We thus obtained:

Proposition 1. *Any r-order arbitrary element in the hyperspace $\mathcal{H}$ represents symbolically, with the derivation operator (36), a signal resulted from the superposition of a purely amplitude modulated signal with $n-r$ amplitude and phase modulated signals.*

For $r=n$, we obtain:

Proposition 2. *Any special type element in the hyperspace $\mathcal{H}$ represents symbolically, with the derivation operator (36), a purely amplitude modulated signal with an amortized carrier.*

Remark. Special type elements in the hyperspace $\mathcal{H}$, having $a_1=a_2=0$, represent symbolically, with the derivation operator (36), *purely amplitude modulated signals with a supressed harmonic carrier.*

b) *The Class of r-Order Quasi-Divisors of Zero.* With the notations:

$$a_{1p}=a_p,\ a_{2p}=b_p,\ \forall p\in\mathcal{P},\ \operatorname{card}\mathcal{P}=r, \tag{52}$$

and taking into account (43), (44), an element of this class may be written as:

$$\hat{w}_{0r}=\sum_{p\in\mathcal{P}}(a_p+b_p\mathrm{i}\pm b_p\mathrm{j}\mp a_p\mathrm{ij})\mathbf{E}_p+\sum_{p\in\bar{\mathcal{P}}}(A_{1p}+\mathrm{ij}A_{2p})e^{\mathrm{i}\beta_p}e^{\mathrm{j}\alpha_p}\mathbf{E}_p. \tag{53}$$

Minus and plus symbols in (53) correspond to the quasi-divisors of 1st (2nd) species. Having in view the correspondence [1]:

$$a_p+b_p\mathrm{i}\pm b_p\mathrm{j}\mp a_p\mathrm{ij}\leftrightarrow A_p e^{(\sigma+\sigma_p)t}\sin[(\omega\pm\omega_p)t+\gamma_p], \tag{54}$$

with the operation derivator (34), where:

$$A_p=\sqrt{a_p^2+b_p^2},\ \operatorname{tg}\gamma_p=-\frac{a_p}{b_p},\quad \forall p\in\mathcal{P}, \tag{55}$$

and the correspondences (31), (33), one obtains:

$$\hat{w}_{or} \leftrightarrow \sum_{p\in\mathcal{P}} A_p e^{(\sigma+\sigma_p)t} \sin[(\omega\pm\omega_p)t+\gamma_p]+$$

(56)

$$+\sum_{p\in\overline{\mathcal{P}}} e^{(\sigma+\sigma_p)t}[A_{1p}\sin(\omega_p t+\beta_p)\sin(\omega t+\alpha_p)+A_{2p}\cos(\omega_p t+\beta_p)\cos(\omega t+\alpha_p)].$$

Transforming the products of trigonometric functions involved in (56) into sums and denoting:

$$A_p=\mu_p\frac{A}{2},\ \gamma_p=\alpha_p+\beta_p(=\alpha_p-\beta_p),\ \forall p\in\mathcal{P}\ ; \tag{57}$$

(57′) $A_{1p}+A_{2p}=\mu_p A$, $(A_{1p}-A_{2p}=\mu_p A)$, $A_{1p}-A_{2p}=k_p A(A_{1p}+A_{2p}=k_p A)$, $\forall p\in\overline{\mathcal{P}}$,

where the relations between brackets correspond to quasi-divisors of 2nd species, (56) becomes for 1st species quasi-divisors:

$$\hat{w}'_{0r}\leftrightarrow\sum_{p=1}^{n}\frac{\mu_p A}{2}e^{(\sigma+\sigma_p)t}\sin[(\omega+\omega_p)t+\alpha_p+\beta_p]+$$

(58)

$$+\sum_{p\in\overline{\mathcal{P}}}\frac{k_p A}{2}e^{(\sigma+\sigma_p)t}\sin[(\omega-\omega_p)t+\alpha_p-\beta_p]$$

and

$$\hat{w}''_{0r}\leftrightarrow\sum_{p=1}^{n}\frac{\mu_p A}{2}e^{(\sigma+\sigma_p)t}\sin[(\omega-\omega_p)t+\alpha_p-\beta_p]+$$

(59)

$$+\sum_{p\in\overline{\mathcal{P}}}\frac{k_p A}{2}e^{(\sigma+\sigma_p)t}\sin[(\omega+\omega_p)t+\alpha_p+\beta_p]$$

for 2nd species quasi-divisors. We obtain this way:

Proposition 3. *Any r-order quasi-divisor of zero of 1st (2nd) species in the hyperspace $\mathcal{H}$ represents symbolically, with the derivation operator (36), a purely amplitude modulated signal with a single lower (upper) sideband*[3]) *and vestigial sideband*[4]) *with n−r components.*

For $r=n$, it results:

Proposition 4. *The divisors of zero of 1st (2nd) species in the hyperspace $\mathcal{H}$ represent symbolically, with the derivation operator (36), an amplitude modulated signal with a single lower (upper) sideband.*

Remarks.

1. The r-order quasi-divisors of zero, $\varepsilon=1$, represent symbolically signals with both sidebands and unsupressed carrier.

2. To obtain the linear systems response to excitations which may be represented symbolically in the hyperspace $\mathcal{H}$, the substitution of variables with the expression of the derivation operator takes place in Laplace transfer function. We thus obtain a hypercomplex transfer function which we denote as *transferance*.

The hypercomplex response of the system may be obtained by multiplication of the hypercomplex element representing symbolically the excitation with the transferance. As the partition classes determined in the

[3]) S.S.B.
[4]) V.S.B.

hyperspace $\mathcal{H}$ cover it (s. § 3), propositions 1...4 permit to obtain immediate response in time domain.

Due to the algorithmic character of calculi, many new device are to be foreseen using electronic computers, i.e. : the determination of systems response in time domain, original proceedings concerning the behaviour of systems under excitations which may be represented on this hyperspace, etc.

All the above aspects will be subject to subsequent works.

5. Conclusions

1. In the same hyperspace in which a purely amplitude-modulated signal is represented symbolically, any representative belonging to the set **S** may be represented with the same derivation operator. The set **S** is formed by signals obtained by means of the application of any linear operator to the signal (26). An original, many-sided procedure appliable to A.M., S.S.B., V.S.B. signals as well as to a large class of amplitude and phase modulated signals has been rendered here evident. The above device is useful in problems concerning analysis of electrical network circuits, transmitted message etc.

2. Propositions allowing the immediate specification of the signal class represented symbolically by a certain element in the hyperspace $\mathcal{H}$ have there been established.

3. In a hyperspace in which modulated signals are represented symbolically, the divisors (quasi-divisors) play a significant role, characterizing the class of S.S.B. (V.S.B.) from the point of view of the there given representation.

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REPREZENTAREA SIMBOLICĂ A SEMNALELOR PE HIPERSPAȚII

I. Reprezentarea simbolică a semnalelor modulate

(Rezumat)

Se demonstrează posibilitatea reprezentării simbolice a unei largi clase de semnale analogice pe un hiperspațiu rezultat prin reuniunea a n spații hipercomplexe ortogonale. Se pune în evidență un procedeu original de tratare unitară a semnalelor modulate în amplitudine, modulate în amplitudine cu purtătoarea suprimată, modulate în amplitudine cu bandă laterală unică, modulate în amplitudine cu o bandă laterală parțial suprimată, precum și a unei clase de semnale modulate simultan în amplitudine și fază.

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APARAT PENTRU MĂSURAREA PARAMETRILOR BOBINELOR ŞI CONDENSATOARELOR LINIARE (*RLC*-metru)

DE

ADRIAN CREŢU

1. Introducere

Într-o serie de lucrări anterioare [1], [2], [4], s-au propus şi studiat mai multe metode şi scheme de măsurare a parametrilor bobinelor şi condensatoarelor liniare, bazate pe detectarea stării de rezonanţă din circuitele electrice de curent alternativ, liniare, cu ajutorul unor redresoare sensibile la fază (RSF) asociate cu dispozitive magnetoelectrice cu rol de indicatoare de nul.

Precizia de măsurare şi, în general, caracteristicile metrologice ale aparatelor (*RLC*-metre) posibil de construit pe baza acestor metode, sînt determinate, pe de o parte, de componentele circuitului rezonant *RLC*, iar, pe de altă parte, de tipul şi performanţele RSF utilizat (se ştie că există o mare diversitate de scheme electronice de RSF [5]).

RLC-metrul propus şi studiat în prezenta lucrare este destinat măsurării parametrilor bobinelor şi condensatoarelor liniare cu factori de calitate reduşi ($Q>1$), în gama frecvenţelor audio şi joase.

2. Schema electrică

Schema electrică a *RLC*-metrului propus în cele ce urmează este reprezentată în fig. 1.

Pentru ca sensibilitatea de măsurare a aparatului să rămînă ridicată, practic constantă, într-un domeniu de măsurare suficient de larg, este necesar ca el să poată funcţiona la mai multe frecvenţe de lucru fixe, f_j, ($j=1, 2, 3,...$), din gama audio, stabilite pe baza limitelor subdomeniilor de măsurare adoptate iniţial.

Circuitul principal al aparatului, alimentat cu t.e.m. $\underline{E}$, de frecvenţă f_j, de la un generator de semnal sinusoidal, audio, GS, cuprinde : circuitul rezonant alcătuit din condensatorul variabil C şi bobina de măsurat L, R (înlocuită cu bobina auxiliară L_0, r_0 cînd aparatul serveşte pentru măsurarea capacităţilor C_x), primarul L_{1j}, r_{1j} al inductorului mutual M_j, ($j=1, 2, 3,...$), miliampermetrul mA şi potenţiometrul liniar P_1.

Modificarea frecvenţei de lucru, f_j, a *RLC*-metrului se efectuează cu ajutorul comutatorului K_3 cu şase secţiuni : K_3^1, K_3^2,..., K_3^6.

Comutatorul K_1, cu patru secţiuni : K_1^1, K_1^2..., K_1^4, permite modificarea structurii circuitului rezonant al aparatului, în funcţie de parametrul de măsurat, după cum urmează :

a) poziția L, destinată măsurării bobinelor liniare, corespunde înscrierii (prin contactul K_1^2), în circuitul principal al aparatului, a condensatorului variabil C și a bobinei de măsurat L, R ;

b) poziția C_{xs}, corespunzătoare măsurării parametrilor condensatoarelor liniare avînd capacități C_x mai mari decît capacitatea maximă C_{max} a condensatorului de acord C, asigură înscrierea, prin K_1^3, a condensatorului C, a bobinei auxiliare L_0 și a condensatorului de măsurat C_x ;

c) poziția C_{xp}, destinată măsurării parametrilor condensatoarelor avînd capacități $C_x < C_{max}$, corespunde înserierii, prin K_1^1 și K_1^4, a bobinei auxiliare L_0 cu grupul paralel format de condensatoarele C și C_x ;

d) poziția 0 permite șuntarea circuitului rezonant, prin K_1^1, în scopul anulării derivei de zero a indicatorului de nul IN cu ajutorul potențiometrului de simetrizare P_s.

„Detectorul stării de rezonanță" al *RLC*-metrului este un redresor sensibil la fază, electronic, cu durata de conducție reglabilă (RSFEDR), de concepție proprie [3], [5], al cărui contact K (puntea simetrică cu diode $ABCD$) este comandat de un semnal dreptunghiular I_{cd} furnizat de un circuit basculant monostabil (CBM), controlat prin tensiune (cu potențiometrul P_{Tc}), realizat în principal din tranzistoarele complementare T_1 și T_2 și generatorul de curent constant T_3. Comanda CBM se efectuează cu impulsuri negative, de scurtă durată, aplicate pe baza tranzistorului T_1, obținute prin derivarea impulsurilor dreptunghiulare produse de comutatorul cu cuplaj colector—bază constituit din tranzistoarele T_4 și T_5.

La bornele circuitului de măsurare al RSFEDR, format din contactul K, rezistența adițională R_a și indicatorul de nul IN (galvanometru magnetoelectric sau microampermetru, în funcție de sensibilitatea și precizia de măsurare a *RLC*-metrului construit), se aplică, prin intermediul comutatorului K_v, fie tensiunea U obținută la bornele a, b' ale circuitului rezonant (poziția I), fie t.e.m. continuă E_v (poziția II), produsă de o baterie uscată, necesară pentru reglarea și verificarea duratei de conducție T_c a RSFEDR, prin „metoda dublei deviații" [5].

Comanda RSFEDR este efectuată, în raport cu parametrul măsurat, fie de semnalul sinusoidal $\underline{I}_c$ aflat în cuadratură cu curentul $\underline{I}_1$ din circuitul principal al *RLC*-metrului (comutatorul K_2 pe poziția L, C_x), atunci cînd trebuie măsurate inductanța bobinei L sau capacitatea condensatorului C_x, fie de semnalul $\underline{I}'_c$ adus aproape în fază cu curentul $\underline{I}_1$, în cazul cînd trebuie măsurată rezistența R a circuitului rezonant (K_2 pe poziția R).

Într-adevăr, în prima etapă de măsurare, cînd comutatorul K_2 se află pe poziția L, C_x, secundarul L_{2j}, r_{2j} al inductorului mutual M_j este conectat practic pe rezistența liniară R_c de valoare mult mai mare decît rezistența R_i, neliniară, de intrare, a grupului T_5-D_5. Cum condensatorul C_{cj}, ($j=1, 2, 3...$), asigură compensarea integrală a efectului reactanțelor inductive din circuitul de comandă al RSFEDR (de exemplu : $\omega_j L_{2j}$ etc.), curentul $\underline{I}_c$ va fi în fază cu t.e.m. $\underline{E}_2 = -j\omega_j M_j \underline{I}_1$ indusă în secundarul inductorului M_j, deci în cuadratură cu curentul $\underline{I}_1$ din circuitul principal.

În cea de a doua etapă de măsurare (K_2 pe poziția R), rezistența liniară R_c este substituită prin reactanța capacitivă $X_c = 1/(2\pi f_j C_{Rj})$ a condensatorului C_{Rj}, ($j=1, 2, 3,...$), ales astfel încît $I'_c \approx I_c$. Drept urmare, semnalul $\underline{I}'_c$, care comandă RSFEDR în noua situație, va fi defazat în raport cu curentul $\underline{I}_1$ cu un unghi φ'_c a cărui valoare rezultă din expresia :

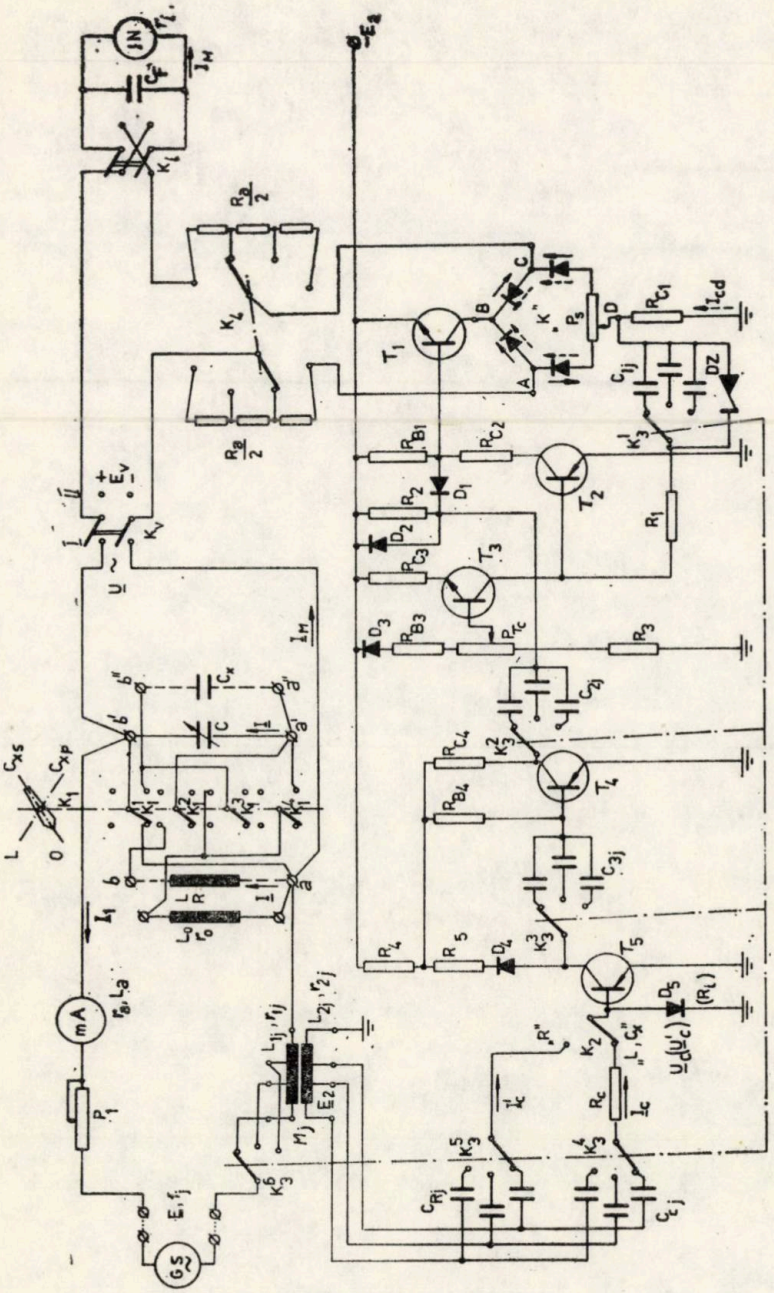

Fig. 1

$$\cos\varphi_c' = \frac{\omega_j L_{2j} - 1/(\omega_j C_{Rj})}{\sqrt{(r_{2j}+R_i)^2+[\omega_j L_{2j}-1/(\omega_j C_{Rj})]^2}} \approx 1, \tag{1}$$

deoarece, în practică:

$$\frac{1}{\omega_j C_{Rj}} \gg \omega_j L_{2j}, \quad \frac{1}{\omega_j C_{Rj}} \gg (r_{2j}+R_i). \tag{2}$$

3. Ecuațiile de funcționare ale *RLC*-metrului

În cadrul schemei din fig. 1, RSFEDR sesizează de fapt defazajul β al tensiunii $\underline{U}$ la bornele circuitului *RLC* în raport cu curentul $\underline{I}_1$ din circuitul principal și *nu* defazajul φ al acestei tensiuni în raport cu curentul $\underline{I}$ din circuitul rezonant *RLC*, de detectat.

În momentul producerii rezonanței și *numai în acest moment*, deoarece circuitul *RLC* devine pur activ ($\underline{Z}_{RLC}=R$), la fel ca și circuitul de măsurare al RSFEDR (de impedanță practic egală cu R_a), cele două unghiuri devin egale, anulîndu-se simultan:

$$\beta=(\widehat{\underline{U};\underline{I}_1})=\varphi=(\widehat{\underline{U};\underline{I}})=0. \tag{3}$$

Rezultă că RSFEDR servește pentru detectarea concordanței de fază a trei semnale sinusoidale, și anume: $\underline{U}$, $\underline{I}$ și $\underline{I}_1$. Justețea acestei afirmații este confirmată și de diagrama fazorială trasată în fig. 2, pentru situația în care circuitul *RLC* al *RLC*-metrului se află în vecinătatea rezonanței ($\omega_j L \neq 1/(\omega_j C_0)$; $\beta \neq \varphi \neq 0$). Se constată ușor că pe măsură ce circuitul *RLC* se apropie de rezonanță ($C \to C_0$), unghiurile β și φ se micșorează, tinzînd spre zero ($\beta, \varphi \to 0$).

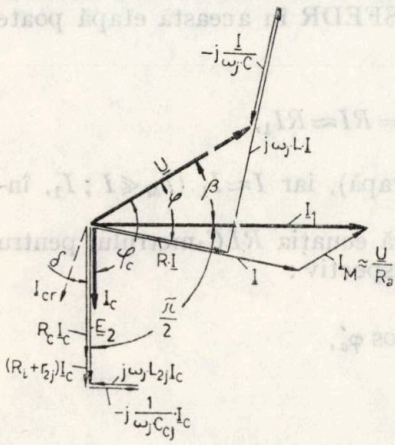

Fig. 2

La funcționarea în curent alternativ sinusoidal, ecuația RSFEDR are expresia generală [5], [6]:

$$\alpha = S_I \frac{1}{2K_f} \cdot \frac{U}{R_a} \sin\frac{\omega T_c}{2} \sin\left(\frac{\omega T_c}{2} - \varphi_c\right), \tag{4}$$

în care: α este deviația dispozitivului magnetoelectric IN asociat cu RSFEDR; S_I — sensibilitatea la curent a dispozitivului IN; $K_f = 1{,}11$ — factorul de formă al semnalelor sinusoidale; T_c — durata de conducție a RSFEDR, iar $\varphi_c = (\widehat{\underline{U};\underline{I}_c})$ — faza comutării contactului acestui dispozitiv.

În cazul particular al *RLC*-metrului din fig. 1, în prima etapă de măsurare (K_2 pe poziția L, C_x), parametrii RSFEDR au valorile:

$$\begin{cases} T_c = \dfrac{T}{2} \text{ și, corespunzător, } \omega T_c = \pi, \\ \varphi_c = (\widehat{\underline{U};\underline{I}_c}) = (\widehat{\underline{U};\underline{I}_1}) + (\widehat{\underline{I}_1;\underline{I}_c}) = \beta + \dfrac{\pi}{2}, \text{ unde } \beta \gtrless 0 \text{ (v. fig. 2).} \end{cases} \tag{5}$$

Ținînd cont de (5), din (4) rezultă ecuația *RLC*-metrului, pentru prima etapă de măsurare, respectiv:

$$\alpha = S_I \frac{1}{2K_f} \cdot \frac{U}{R_a} \sin \beta. \tag{6}$$

Pe baza ecuației (6) se constată că la rezonanță (cînd $\omega_f L = 1/(\omega_f C_0)$ și $\beta = \varphi = 0$) și numai în acest moment, deviația α a indicatorului IN devine egală cu zero. Din condiția de rezonanță se obține imediat valoarea inductanței L de măsurat, respectiv:

$$L = \frac{1}{\omega_f^2 C_0} = \frac{1}{(2\pi f_f)^2 C_0}, \tag{7}$$

unde $1/(2\pi f_f)^2 = \text{const.}$

Dacă, în prima etapă, se urmărește măsurarea unor capacități C_x, se aplică metoda de substituție, binecunoscută, a dublului acord [2].

În cea de a doua etapă (K_2 pe poziția R), cînd aparatul servește pentru măsurarea rezistenței de pierderi R a circuitului rezonant, parametrii RSFEDR se modifică în sensul că:

$$T_c = \frac{T}{2} \quad \text{iar} \quad \varphi_c = (\underset{\frown}{U}; \underset{\frown}{I_c}) = \varphi_c', \tag{8}$$

valoarea unghiului φ_c' rezultînd din (1). Totodată valoarea efectivă a tensiunii U aplicate circuitului de măsurare al RSFEDR în această etapă poate fi aproximată prin:

$$U = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega_f C_0}\right)^2}\, I = RI \approx RI_1, \tag{9}$$

deoarece $\omega_f L = 1/(\omega_f C_0)$ (încă din prima etapă), iar $I \approx I_1$ ($I_M \ll I; I_1$, întrucît $R_a \gg R$).

Ținînd cont de (8) și (9), din (4) rezultă ecuația *RLC*-metrului pentru cazul funcționării sale ca „ohmmetru", respectiv:

$$\alpha \approx S_I \frac{1}{2K_f} \cdot \frac{RI_1}{R_a} \cos \varphi_c', \tag{10}$$

din care se obține imediat:

$$R \approx \frac{2K_f R_a}{S_I I_1} \cdot \frac{1}{\cos \varphi_c'} \alpha. \tag{11}$$

În condițiile practice (2), expresia (11) se simplifică, devenind în final:

$$R \approx \frac{2K_f R_a}{S_I I_1} \alpha = K_R \alpha, \tag{12}$$

în care K_R este constantă numai dacă și curentul I_1 este menținut la o anumită valoare, adoptată inițial (cu ajutorul potențiometrului liniar P_1).

4. Calculul sensibilității *RLC*-metrului

Sensibilitatea relativă globală a *RLC*-metrului se definește prin limita raportului dintre variația absolută $\Delta\alpha$ a deviației indicatorului de nul IN și variația relativă $\Delta C/C_0$ a parametrului de acord C cînd ΔC tinde la zero (circuitul *RLC* tinde spre rezonanță), respectiv:

$$S_\alpha = \lim_{\Delta C \to 0} C_0 \frac{\Delta\alpha}{\Delta C} = C_0 \frac{d\alpha}{dC} = \frac{d\alpha}{dK_c}, \tag{13}$$

în care $K_c = C/C_0$ reprezintă raportul dintre capacitatea C a condensatorului de acord și valoarea sa $C_0 = 1/(\omega_j^2 L)$ din momentul rezonanței.

Pentru calculul sensibilității S_α se utilizează ecuația (6) a *RLC*-metrului, în care unghiul $\beta = (\widehat{\underline{U}; \underline{I}_1})$ rezultă din [1]);

$$\beta = \operatorname{arctg} \frac{\omega_j L - \frac{1}{\omega_j C}}{R\left(1+\frac{R}{R_a}\right) + \frac{1}{R_a}\left(\omega_j L - \frac{1}{\omega_j C}\right)^2}. \tag{14}$$

Expresia (14) poate fi scrisă și sub forma mai convenabilă:

$$\beta = \operatorname{arctg} \frac{Q_j\left(1-\frac{1}{K_c}\right)}{1+\frac{R}{R_a}\left[1+Q_j^2\left(1-\frac{1}{K_c}\right)^2\right]}, \tag{15}$$

în care $Q_j = \omega_j L/R$ este factorul de calitate al circuitului rezonant.

Întrucît în vecinătatea rezonanței este satisfăcută egalitatea $I \approx I_1$ ($I_M \ll I, I_1$; $R_a \gg R$), tensiunea U poate fi exprimată sub forma:

$$U = \sqrt{R^2 + \left(\omega_j L - \frac{1}{\omega_j C}\right)^2}\, I \approx R I_1 \sqrt{1 + Q_j^2\left(1-\frac{1}{K_c}\right)^2}. \tag{16}$$

Ținînd cont de (15) și (16), ecuația (6) a *RLC*-metrului devine în final:

$$\alpha = S_I \frac{I_1}{2K_f} \cdot \frac{R}{R_a} \cdot \frac{Q_j\left(1-\frac{1}{K_c}\right)}{\sqrt{\left(1+\frac{R}{R_a}\right)^2 + \frac{R^2}{R_a^2} Q_j^2\left(1-\frac{1}{K_c}\right)^2}}. \tag{17}$$

Din (13), ținînd cont de (17), în ipoteza acordării circuitului *RLC* la $I_1 = \text{const}$, se obține:

---

[1]) Unghiul β este de fapt argumentul impedanței echivalente $\underline{Z}_e$ a circuitului rezultat prin cuplarea în paralel a circuitului *RLC* cu circuitul de măsurare al RSFEDR, de impedanță practic egală cu R_a.

$$(18)\qquad S_\alpha|_{I_1=\text{const.}} = S_I \frac{I_I}{2K_f} \cdot \frac{Q_j}{1+R_a/R} \cdot \frac{1}{K_c^2}\left[1+\frac{Q_j^2}{(1+R_a/R)^2}\left(1-\frac{1}{K_c}\right)^2\right]^{-\frac{3}{2}}.$$

Curbele $\alpha=f(K_c)$ și $S_\alpha=f(K_c)$ au fost trasate pentru Q_j, I_1, $R/R_a=$ =const., în fig. 3. Din analiza lor se desprind următoarele observații cu importanță practică :

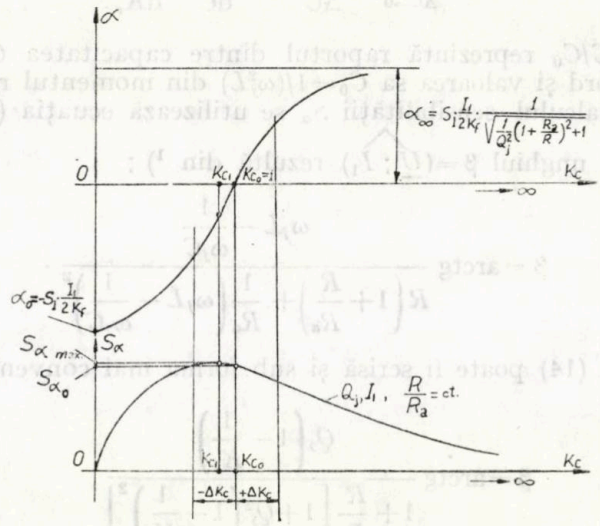

Fig. 3

a) Sensibilitatea relativă globală S_α, calculată la $I_1=$const., trece prin valoarea sa maximă $S_{\alpha\max}$ într-un punct de abscisă :

$$(19)\qquad K_{c_1} = \frac{1+3\sqrt{1+\frac{8}{9Q_j^2}\left(\frac{R_a}{R}+1\right)^2}}{4\left[1+\frac{1}{Q_j^2}\left(\frac{R_a}{R}+1\right)^2\right]} < K_{C_0}=1,$$

situat cu atît mai aproape de punctul de rezonanță $K_{C_0}=1$ $(C=C_0)$, cu cît factorul de calitate Q_j, $(j=1, 2, 3,...)$, este mai ridicat.

b) Valoarea S_{α_0} a sensibilității relative S_α în momentul rezonanței, rezultată din (18) prin substituirea parametrului de acord K_c cu valoarea sa $K_{C_0}=1$, se obține din expresia :

$$(20)\qquad S_{\alpha_0} = S_I \frac{I_1}{2K_f} \cdot \frac{Q_j}{1+R_a/R} = S_I \frac{I_1}{2K_f} \cdot \frac{2\pi f_j L}{R+R_a}.$$

c) Sensibilitatea S_α rămîne practic constantă, apropiată de valoarea sa maximă, pentru un domeniu relativ larg de valori ale parametrului de acord K_c situate în vecinătatea rezonanței.

d) Sensibilitatea $(S_\alpha)_{-\Delta K_c}$ la dezacordarea circuitului rezonant prin micșorarea parametrului K_c este mai mare decît sensibilitatea $(S_\alpha)_{+\Delta K_c}$ la dezacordarea sa prin creșterea lui K_c. Diferența dintre cele două valori scade cu creșterea factorului de calitate Q_j.

e) În timp ce sensibilitatea relativă S_φ a circuitului rezonant *RLC*, considerat independent (fără a fi interconectat cu RSFEDR), dată de expresia :

$$S_\varphi = \lim_{\Delta C \to 0} C_0 \frac{\Delta \varphi}{\Delta C} = \frac{d\varphi}{dK_c} = \frac{Q_j}{K_c^2 + Q_j^2 (K_c - 1)^2}, \tag{21}$$

devine egală în momentul rezonanței $(K_c = K_{C_0} = 1)$ chiar cu factorul său de calitate $(S_{\varphi_0} = Q_j)$, sensibilitatea relativă globală S_{α_0} a *RLC*-metrului depinde de mai mulți factori care, evident, diminuează ponderea valorii lui Q_j. Din (20) rezultă că prin utilizarea unui indicator IN cu sensibilitatea S_I ridicată și menținerea curentului I_1 și a raportului R/R_a în limite convenabile, se pot realiza sensibilități S_{α_0} de valori relativ mari chiar și în cazul măsurării unor bobine și condensatoare cu factori de calitate Q_j reduși (la frecvențe audio și chiar joase).

5. Analiza și calculul erorilor *RLC*-metrului

5.1. Surse de erori

Analiza funcționării *RLC*-metrului permite identificarea următoarelor erori posibile : a) erori datorită impreciziilor constructive ale elementelor circuitului rezonant ; b) erori de determinare ; c) erori sistematice datorită abaterii de la cuadratură a semnalelor $\underline{I}_c$ și $\underline{I}_1$.

5.2. Calculul erorilor de construcție

Erorile care vor afecta valoarea măsurată a inductanței L, introduse de impreciziile de construcție ale elementelor schemei de măsurare, rezultă prin aplicarea calculului clasic expresiei (7), respectiv :

$$\left[\frac{\Delta L}{L}\right]_{c\%} = \frac{\Delta C}{C} 100 + 2 \frac{\Delta f_j}{f_j} 100, \tag{24}$$

în care : $\Delta C/C$ este eroarea relativă maximă a condensatorului de acord C iar $\Delta f_j/f_j$ — eroarea de cunoaștere a frecvenței de lucru f_j.

În cazul măsurării capacității C_x prin metoda dublului acord, erorile de măsurare vor fi determinate practic numai de eroarea $\Delta C/C$.

5.3. Calculul erorilor de determinare

Eroarea de determinare a *RLC*-metrului, numeric egală cu pragul său de sensibilitate, poate fi obținută pe două căi și anume :

a) Teoretic, printr-un calcul simplu, din relația $(\Delta L/L)_{d\%} = (\Delta \alpha_d / S_{\alpha_0})_{\max} 100 = (\Delta \alpha_d / S_{\alpha_0 \min}) 100$, în care $\Delta \alpha_d = 0{,}2 \ldots 0{,}5$ diviziuni, iar sensibilitatea minimă $S_{\alpha_0 \min}$ se obține din (20), unde $Q_j = Q_{j\min}$ și $R_a/R = (R_a/R)_{\max}$.

b) Experimental, prin determinarea variației relative $\Delta C_d/C_0$ a capacității de acord C care caracterizează pragul de sensibilitate a *RLC*-metrului : $(\Delta L/L)_{d\%} = (\Delta C_d/C_0) 100$.

5.4. Calculul erorilor sistematice datorită abaterii de la cuadratură a semnalelor $\underline{I}_c$ și $\underline{I}_1$

Conform ecuației (4), deviația α a indicatorului IN se va anula exact în momentul producerii rezonanței, numai dacă $T_c = T/2$, iar faza φ_c a comutării contactului RSFEDR este riguros egală cu $\pi/2$ (v. rel. (5)); aceasta presupune însă existența unei cuadraturi perfecte între semnalele $\underline{I}_c$ și $\underline{I}_1$.

În practică, oricît de corect s-ar alege condensatorul de compensare C_{cj}, defazajul real dintre semnalele $\underline{I}_1$ și $\underline{I}_c$ va fi egal cu $\pi/2+\delta$, în care abaterea δ de la cuadratura celor două semnale (pusă în evidență și în fig. 2, în care semnalul real de comandă $\underline{I}_{cr}$ s-a trasat cu linie întreruptă) este pozitivă sau negativă, după cum $\underline{I}_c$ este defazat în întîrziere, respectiv în avans față de t.e.m. $\underline{E}_2$ aplicată circuitului de comandă al RSFEDR.

Datorită abaterii δ faza reală a comutării contactului RSFEDR va avea valoarea (fig. 2):

$$\varphi_{cr} = (\underline{U}\,\hat{;}\,\underline{I}_{cr}) = (\underline{U}\,\hat{;}\,\underline{I}_1) + (\underline{I}_1\,\hat{;}\,\underline{I}_{cr}) = \beta + \delta + \frac{\pi}{2}. \tag{25}$$

Corespunzător, ecuația (6) a *RLC*-metrului devine:

$$\alpha = S_I \frac{1}{2K_f} \cdot \frac{U}{R_a} \sin(\beta+\delta). \tag{26}$$

Din (26) rezultă că deviația α nu se va anula exact în momentul rezonanței (cînd $\beta = \varphi = 0$), cum s-a presupus în cazul ideal, ci în momentul în care $\beta = -\delta$, respectiv cînd, ținînd cont de (14), este satisfăcută egalitatea:

$$\operatorname{tg}\beta = \frac{\omega_j L_m - \dfrac{1}{\omega_j C_0}}{R\left(1+\dfrac{R}{R_a}\right) + \dfrac{1}{R_a}\left(\omega_j L_m - \dfrac{1}{\omega_j C_0}\right)^2} = -\operatorname{tg}\delta, \tag{27}$$

în care cu L_m s-a notat valoarea măsurată a inductanței L.

Întrucît, în practică, abaterea δ are valori relativ mici ($\delta < 3...5°$), este evident că:

$$\frac{1}{R_a}\left(\omega_j L_m - \frac{1}{\omega_j C_0}\right)^2 \ll R\left(1+\frac{R}{R_a}\right). \tag{28}$$

Din (27), ținînd cont de (28), rezultă:

$$L_m \approx \frac{1}{\omega_j^2 C_0} - \frac{R}{\omega_j}\left(1+\frac{R}{R_a}\right)\operatorname{tg}\delta = L - \frac{R}{\omega_j}\left(1+\frac{R}{R_a}\right)\operatorname{tg}\delta, \tag{29}$$

unde $L = 1/(\omega_j^2 C_0)$ este valoarea exactă a inductanței bobinei de măsurat.

Eroarea sistematică relativă de măsurare a inductanței L va rezulta din:

$$\left(\frac{\Delta L}{L}\right)_{\delta\%} = \frac{L_m - L}{L}\,100, \tag{30}$$

care, ținînd cont de (29), conduce la:

$$(31) \qquad \left(\frac{\Delta L}{L}\right)_{\delta\%} = -100\frac{R}{\omega_f L}\left(1+\frac{R}{R_a}\right)\operatorname{tg}\delta.$$

Dacă se are în vedere că, în practică, raportul $R/R_a \ll 1$, iar $\omega_f L/R = Q_f$, expresia (31) devine în final:

$$(32) \qquad \left(\frac{\Delta L}{L}\right)_{\delta\%} \approx -100\frac{R}{2\pi f_f L}\operatorname{tg}\delta = -\frac{100}{Q_f}\operatorname{tg}\delta.$$

Eroarea sistematică (32) poate fi compensată prin aplicarea de corecții asupra valorilor măsurate L_m. Abaterea δ, necesară pentru trasarea curbelor de corecții, se determină experimental prin conectarea, în locul circuitului rezonant RLC al aparatului (între bornele a și b'), a unei rezistențe R_n, reglabilă între zero și valoarea maximă R_{max} admisă pentru rezistența R a circuitului rezonant. Modulul și semnul abaterii δ vor rezulta din:

$$(33) \qquad \delta \approx \arcsin\frac{R_a}{R_n}\cdot\frac{2K_f}{S_I I_1}\alpha,$$

obținută din (26), în care $\beta=0$, iar $U \approx R_n I_1$; $(I_M \ll I_1)$.

6. Rezultate experimentale

RLC-metrul construit și încercat este caracterizat prin: cinci frecvențe de lucru fixe (100, 330, 1 000, 3 300 și 10 000 Hz); domeniile de măsurare: $L=0{,}25.10^{-3}...100$ H, divizat în cinci subgame; $C_x=0...100$ μF și $R=0...1\,000$ Ω. Alimentarea circuitului principal al RLC-metrului ($E=25$ V, $I_1=10$ mA) s-a efectuat de la un generator de semnal sinusoidal de tipul TR-0161 (gama de frecvențe 20 Hz...200 kHz; gama de tensiuni 0...100 V; factorul de distorsiune <0,2%), iar pentru acordul circuitului rezonant s-a utilizat o cutie de capacități, în decade (0...1,1111 μF), de tip DK-5, în clasa de precizie 0,5.

Pentru verificarea performanțelor aparatului s-au măsurat parametrii L, R ai unor bobine etalon (cutie de inductanțe, în decade, de tip P567-MUCπ și un grup de bobine fixe, toate în clasa de precizie 0,2) și parametrii unor cutii de capacități, în decade, de tip DK-5 și DK-50 (0...10. 111 μF, clasa de precizie 0,5, cu excepția decadei 10×1 μF care este în clasa 1).

S-a constat că valorile măsurate L_m ale inductanțelor L sînt afectate de erori mai mici de 2,5%; după aplicarea corecțiilor, calculate pe baza expresiei (32), valorile măsurate corectate L_{mc} rămîn afectate de erori mai mici de 1% (în ele se includ erorile de construcție ale condensatorului de acord și erorile datorită instabilității frecvenței generatorului de semnal). Pentru bobine avînd factori de calitate $Q>1$ s-a constatat, atît teoretic (pe baza relației (20)), cît și experimental, că erorile de sensibilitate sînt neglijabile comparativ cu erorile de construcție.

Erorile care afectează valorile măsurate, R_m, ale rezistenței R sînt mai mici de 2,5%, fiind practic egale cu erorile de măsurare ale curentului I_1.

În cazul măsurării capacităților C_x erorile obținute experimental sînt practic egale cu erorile de construcție ale condensatorului de acord.

7. Concluzii

Aparatul propus şi studiat în lucrare este destinat măsurării parametrilor bobinelor şi condensatoarelor liniare, cu factori de calitate reduşi ($Q>1$), în gama frecvenţelor audio şi joase. La baza funcţionării acestui aparat stă o metodă de măsurare (brevetată în R.S.R. [1]) care se distinge prin faptul că detectarea regimului de rezonanţă, dintr-un circuit *RLC* serie, se realizează cu un indicator de nul realizat prin asocierea unui dispozitiv de măsură magneto-electric cu un redresor sensibil la fază electronic.

Prin utilizarea, la construirea *RLC*-metrului propus, a unui redresor sensibil la fază electronic, cu durata de conducţie reglabilă (de asemenea, brevetat în R.S.R. [3]), s-au obţinut performanţe comparabile cu cele realizate de aparatele clasice bazate pe metode de zero (de exemplu : punţile de c.a. echilibrate etc.). Astfel, *RLC*-metrul descris în lucrare oferă posibilitatea utilizării metodelor de rezonanţă, avantajoase din unele puncte de vedere în raport cu metodele de punte (lipsa convergenţei, existenţa unui singur parametru reglabil de circuit etc.) şi în cazul măsurării bobinelor şi condensatoarelor cu factor de calitate scăzut.

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APPAREIL POUR LA MESURE DES PARAMÈTRES DES BOBINES ET DES CONDENSATEURS LINÉAIRES (*RLC*-MÈTRE)

(Résumé)

L'appareil proposé et étudié dans cette Note est destiné à la mesure des paramètres des bobines et des condensateurs linéaires, ayant des petits facteurs de qualité ($Q>1$), dans le domaine des fréquences audio et basses.

Le fonctionnement de l'appareil est basé sur la détection de la résonance dans les circuits linéaires de courant alternatif, à l'aide des redresseurs sensibles à la phase, associés aux dispositifs magnétoélectriques, fonctionnant en tant qu'indicateurs de zéro. Utilisant dans la construction de l'appareil, un redresseur électronique, sensible à la phase, au temps de conduction réglable, conçu par l'auteur, on a obtenu des performances comparables à ceux realisées par les ponts de c.a. équilibrés, en audiofréquence.

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CALCULUL ŞI ANALIZA ERORILOR TRANSFORMATORULUI DE CURENT COMPENSAT ELECTRONIC

DE

ELENA CREŢU, ADRIAN CREŢU şi ADRIANA PRISĂCARU

1. Introducere

În lucrările [1], [2], [4]...[7] s-au propus şi analizat diferite scheme practice destinate compensării electronice a erorilor transformatoarelor de curent.

În cele ce urmează se determină expresiile analitice ale erorilor de raport şi de unghi ale transformatorului de curent compensat electronic. Analiza acestor expresii permite obţinerea unor concluzii de interes practic deosebit referitoare la dependenţa eficacităţii compensării atît de parametrii dispozitivelor de compensare electronică utilizate cît şi de parametrii transformatorului compensat.

2. Calculul erorilor transformatorului de curent compensat electronic

În schema de principiu a procedeului electronic de compensare, reprezentată în fig. 1, semnalul de intrare în amplificatorul electronic AE se culege de pe înfăşurarea de indicator N_3. Înainte de cuplarea dispozitivului electronic de compensare, t.e.m. $\underline{E}_2$ indusă în înfăşurarea secundară a transformatorului de curent are valoarea :

$$\underline{E}_2=(\underline{Z}_S+\underline{Z}_2)\,\underline{I}_2=\underline{Z}_{2t}\,\underline{I}_2, \tag{1}$$

în care : $\underline{Z}_s$, $\underline{Z}_2$ şi $\underline{Z}_{2t}$ reprezintă impedanţa de sarcină, impedanţa înfăşurării secundare şi, respectiv, impedanţa totală a circuitului secundar, iar $\underline{I}_2$ — intensitatea curentului secundar.

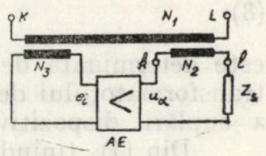

Fig. 1

Totodată, valoarea efectivă E_2 a t.e.m. secundare este dată de relaţia cunoscută :

$$E_2=4{,}44\,fN_2SB_m, \tag{2}$$

în care : B_m este amplitudinea inducţiei magnetice în miezul transformatorului de curent necompensat, f — frecvenţa de lucru a transformatorului de curent, S — secţiunea miezului iar N_2 — numărul de spire ale înfăşurării secundare.

Pentru a calcula t.e.m. $\underline{E}_{2\alpha}$ indusă în înfășurarea secundară, după cuplarea dispozitivului electronic de compensare, se aplică ecuația de tensiuni a lui K i r c h h o f f circuitului secundar al transformatorului de curent, respectiv :

$$\underline{E}_{2\alpha}+\underline{U}_{\alpha}=\underline{Z}_{2t}\,\underline{I}_{2\alpha}+\underline{Z}_{e}\underline{I}_{2\alpha}, \tag{3}$$

în care :

$$\underline{U}_{\alpha}=\underline{A}\,\frac{N_3}{N_2}\,\underline{E}_{2\alpha} \tag{4}$$

reprezintă tensiunea de compensare furnizată de amplificatorul electronic.

$$\underline{A}=\frac{\underline{U}_{\alpha}}{\underline{E}_{i\alpha}}=\frac{\underline{U}_{\alpha}}{\frac{N_3}{N_2}\underline{E}_2}$$

— factorul de amplificare al amplificatorului, iar Z_e — impedanța de ieșire a acestuia.

Dacă se notează :

$$\underline{k}_Z=\frac{\underline{Z}_{2t}+\underline{Z}_e}{\underline{Z}_{2t}}=\frac{\underline{Z}_{2t\alpha}}{\underline{Z}_{2t}} \tag{5}$$

și se ține cont de (4), ecuația (3) devine :

$$(1+\underline{A}N_3/N_2)\,\underline{E}_{2\alpha}=\underline{k}_Z\underline{Z}_{2t}\underline{I}_{2\alpha}. \tag{6}$$

Soluționînd în raport cu $\underline{E}_{2\alpha}$ se obține :

$$\underline{E}_{2\alpha}=\frac{\underline{k}_Z\underline{Z}_{2t}\underline{I}_{2\alpha}}{1+\underline{A}N_3/N_2}=\frac{\underline{k}_Z\underline{Z}_{2t}\underline{I}_2\underline{I}_{2\alpha}/\underline{I}_2}{1+\underline{A}N_3/N_2}\approx\frac{\underline{k}_Z\underline{E}_2}{\underline{A}N_3/N_2}, \tag{7}$$

întrucît $\underline{I}_{2\alpha}\approx\underline{I}_2$ iar $AN_3/N_2\gg 1$.

Modificarea t.e.m. secundare de la valoarea E_2, dată de relația (2), la valoarea :

$$E_{2\alpha}=4{,}44fN_2SB_{m\alpha} \tag{8}$$

este determinată de variația amplitudinii inducției magnetice din miezul transformatorului de curent de la valoarea B_m la valoarea $B_{m\alpha}$ ca urmare a cuplării dispozitivului electronic de compensare.

Din (7), ținînd cont de (2) și (8), rezultă :

$$B_{m\alpha}=\frac{k_Z}{AN_3/N_2}\,B_m. \tag{9}$$

Prin urmare, modificarea amplitudinii inducției magnetice în miezul transformatorului de curent compensat electronic depinde de parametrii amplificatorului electronic (amplificarea A și, prin k_Z, de impedanța de ieșire Z_e), de raportul N_3/N_2 dintre numărul de spire ale înfășurării de in-

dicator și secundare, de impedanța totală a circuitului secundar (prin k_Z) și este independentă de anumiți parametri constructivi ca : lungimea circuitului magnetic, secțiunea lui etc.

În cazul practic în care impedanța de ieșire, Z_e, a amplificatorului electronic este neglijabilă în raport cu impedanța totală, Z_{2t}, a circuitului secundar al transformatorului de curent, din (5) rezultă $k_Z \approx 1$, iar relația (9) devine :

$$B'_{m\alpha} \approx \frac{B_m}{A\, N_3/N_2}. \tag{9'}$$

Relațiile (9) și (9') indică deplasarea punctului de funcționare pe caracteristica de magnetizare a miezului, ca urmare a cuplării dispozitivului electronic de compensare. Schimbarea punctului de funcționare conduce la modificarea curentului de magnetizare de la valoarea I_μ la $I_{\mu\alpha}$, a permeabilității magnetice de la μ la μ_α, a unghiului de pierderi de la α la α_α și deci a erorilor de raport și de unghi de la ε_i la $\varepsilon_{i\alpha}$ și, respectiv, de la δ_i la $\delta_{i\alpha}$.

Pentru calculul erorilor transformatorului de curent compensat electronic se pleacă de la expresiile cunoscute [4] ale erorilor transformatorului de curent :

$$\varepsilon_i = \frac{N_1 I_0}{N_1 I_1} \sin(\alpha+\psi_2) \quad \text{și} \quad \delta_i = \frac{N_1 I_0}{N_1 I_1} \cos(\alpha+\psi_2), \tag{10, 11}$$

în care $N_1 I_0$ reprezintă solenația de funcționare la gol, $N_1 I_1$ — solenația primară, iar ψ_2 — defazajul dintre t.e.m. secundară E_2 și curentul I_2.

Se exprimă solenația $N_1 I_0$ funcție de amplitudinea B_m a inducției magnetice din miezul transformatorului de curent [3] :

$$N_1 I_0 = \frac{N_1 I_\mu}{\cos\alpha} = \frac{Hl}{\cos\alpha} = \frac{l}{\cos\alpha} \cdot \frac{B_m}{\sqrt{2}\mu}, \tag{12}$$

în care l reprezintă lungimea circuitului magnetic iar H — valoarea efectivă a intensității cîmpului magnetic.

Înlocuind în relațiile (10) și (11) pe $N_1 I_0$ prin valoarea sa dată de (12), se obține :

$$\varepsilon_i = \frac{1}{N_1 I_1} \cdot \frac{l}{\cos\alpha} \cdot \frac{B_m}{\sqrt{2}\mu} \sin(\alpha+\psi_2) \tag{13}$$

și

$$\delta_i = \frac{1}{N_1 I_1} \cdot \frac{l}{\cos\alpha} \cdot \frac{B_m}{\sqrt{2}\mu} \cos(\alpha+\psi_2). \tag{14}$$

Ținînd cont de schimbarea punctului de funcționare pe caracteristica de magnetizare și de modificarea impedanței totale a circuitului secundar, ca urmare a cuplării dispozitivului electronic, erorile transformatorului de curent compensat devin :

$$\varepsilon_{i\alpha} = \frac{1}{N_1 I_1} \cdot \frac{l}{\cos\alpha_\alpha} \cdot \frac{B_{m\alpha}}{\sqrt{2}\mu_\alpha} \sin(\alpha_\alpha+\psi_{2\alpha}), \tag{15}$$

și

$$\delta_{i\alpha}=\frac{1}{N_1 I_1}\cdot\frac{l}{\cos\alpha_\alpha}\cdot\frac{B_{m\alpha}}{\sqrt{2}\,\mu_\alpha}\cos(\alpha_\alpha+\psi_{2\alpha}), \tag{16}$$

în care $\psi_{2\alpha}$ reprezintă unghiul de defazaj dintre $E_{2\alpha}$ și $I_{2\alpha}$.

Prin raportarea relațiilor (15) la (13) și (16) la (14) și ținînd cont de (9) rezultă :

$$\frac{\varepsilon_{i\alpha}}{\varepsilon_i}=\frac{\cos\alpha}{\cos\alpha_\alpha}\cdot\frac{\mu}{\mu_\alpha}\cdot\frac{kz}{AN_3/N_2}\cdot\frac{\sin(\alpha_\alpha+\psi_{2\alpha})}{\sin(\alpha+\psi_2)} \tag{17}$$

și

$$\frac{\delta_{i\alpha}}{\delta_i}=\frac{\cos\alpha}{\cos\alpha_\alpha}\cdot\frac{\mu}{\mu_\alpha}\cdot\frac{kz}{AN_3/N_2}\cdot\frac{\cos(\alpha_\alpha+\psi_{2\alpha})}{\cos(\alpha+\psi_2)}. \tag{18}$$

Final, după efectuarea tuturor calculelor, expresiile (17) și (18) devin :

$$\frac{\varepsilon_{i\alpha}}{\varepsilon_i}=\frac{\mu}{\mu_\alpha}\cdot\frac{kz}{AN_3/N_2}\cdot\frac{\cos\psi_{2\alpha}}{\cos\psi_2}\cdot\frac{\operatorname{tg}\alpha_\alpha+\operatorname{tg}\psi_{2\alpha}}{\operatorname{tg}\alpha+\operatorname{tg}\psi_2}, \tag{19}$$

respectiv :

$$\frac{\delta_{i\alpha}}{\delta_i}=\frac{\mu}{\mu_\alpha}\cdot\frac{kz}{AN_3/N_2}\cdot\frac{\cos\psi_{2\alpha}}{\cos\psi_2}\cdot\frac{1-\operatorname{tg}\alpha_\alpha\operatorname{tg}\psi_{2\alpha}}{1-\operatorname{tg}\alpha\operatorname{tg}\psi_2}. \tag{20}$$

Întrucît :

$$kz\frac{\cos\psi_{2\alpha}}{\cos\psi_2}=\frac{Z_{2t\alpha}\cos\psi_{2\alpha}}{Z_{2t}\cos\psi_2}=\frac{R_{2t\alpha}}{R_{2t}}=1+\frac{R_e}{R_{2t}}, \tag{21}$$

relațiile (19) și (20) pot fi scrise și sub forma :

$$\frac{\varepsilon_{i\alpha}}{\varepsilon_i}=\frac{\mu}{\mu_\alpha}\cdot\frac{1}{AN_3/N_2}\left(1+\frac{R_e}{R_{2t}}\right)\frac{\operatorname{tg}\alpha_\alpha+\operatorname{tg}\psi_{2\alpha}}{\operatorname{tg}\alpha+\operatorname{tg}\psi_2} \tag{22}$$

și

$$\frac{\delta_{i\alpha}}{\delta_i}=\frac{\mu}{\mu_\alpha}\cdot\frac{1}{AN_3/N_2}\left(1+\frac{R_e}{R_{2t}}\right)\frac{1-\operatorname{tg}\alpha_\alpha\operatorname{tg}\psi_{2\alpha}}{1-\operatorname{tg}\alpha\operatorname{tg}\psi_2}. \tag{23}$$

Expresiile (22) și (23) permit calculul erorilor transformatorului de curent compensat electronic funcție de valorile erorilor transformatorului necompensat. În acest scop trebuie cunoscuți parametrii de funcționare ai transformatorului necompensat : ε_i, δ_i, Z_{2t}, ψ_2, N_2, N_3, valoarea permeabilității magnetice μ și a unghiului de pierderi α, corespunzătoare punctului de funcționare pe caracteristica de magnetizare a miezului, care depinde de natura și proprietățile acestuia și parametrii dispozitivului electronic folosit în scopul compensării : amplificarea A în tensiune și impedanța de ieșire $Z_e=R_e+jX_e$.

Pentru cazul particular în care impedanța de ieșire Z_e a amplificatorului electronic este neglijabilă în raport cu impedanța totală $Z_{2t}=R_{2t}+jX_{2t}$ a circuitului secundar al transformatorului de curent ($R_e \ll R_{2t}$ și $X_e \ll X_{2t}$), se poate considera :

(24) $$1+\frac{R_e}{R_{2t}}\approx 1$$

și

(25) $$\operatorname{tg}\psi_{2\alpha}=\frac{X_{2t}+X_e}{R_{2t}+R_e}\approx\frac{X_{2t}}{R_{2t}}=\operatorname{tg}\psi_2.$$

Ținînd cont de (24) și (25), expresiile (22) și (23) devin :

(22′) $$\frac{\varepsilon'_{i\alpha}}{\varepsilon_i}=\frac{\mu}{\mu'_\alpha}\cdot\frac{1}{AN_3/N_2}\cdot\frac{\operatorname{tg}\alpha'_\alpha+\operatorname{tg}\psi_2}{\operatorname{tg}\alpha+\operatorname{tg}\psi_2}$$

și

(23′) $$\frac{\delta'_{i\alpha}}{\delta_i}=\frac{\mu}{\mu'_\alpha}\cdot\frac{1}{AN_3/N_2}\cdot\frac{1-\operatorname{tg}\alpha'_\alpha\operatorname{tg}\psi_2}{1-\operatorname{tg}\alpha\operatorname{tg}\psi_2},$$

în care permeabilitatea magnetică μ'_α și unghiul de pierderi α'_α corespund punctului de funcționare de pe caracteristica de magnetizare a materialului feromagnetic al miezului, determinat de inducția magnetică $B'_{m\alpha}$ calculată cu relația simplificată (9′).

În acest caz practic ($Z_e \ll Z_{2t}$) rapoartele erorilor $\varepsilon'_{i\alpha}/\varepsilon_i$ și $\delta'_{i\alpha}/\delta_i$ pot fi calculate cu relațiile (22′) și (23′) cunoscînd numai caracteristicile materialului feromagnetic al miezului, parametrii amplificatorului electronic utilizat și caracterul sarcinii secundare, modificările erorilor nedepinzînd de parametrii constructivi ai miezului, de domeniul de măsurare sau de modulul impedanței de sarcină.

Pe baza expresiilor (22′) și (23′), pentru orice tip de tablă de transformator, la valori diferite ale inducției punctului de funcționare a transformatorului necompensat și pentru anumite valori ale lui $\cos\psi_2$, se pot trasa curbele de variație ale rapoartelor $\varepsilon'_{i\alpha}/\varepsilon_i$ și $\delta'_{i\alpha}/\delta_i$ funcție de amplificarea în tensiune a dispozitivului electronic folosit. Din aceste curbe, cunoscînd erorile ε_i și δ_i ale transformatorului necompensat, se pot obține rapid erorile $\varepsilon'_{i\alpha}$ și $\delta'_{i\alpha}$ corespunzătoare unei anumite amplificări în tensiune a dispozitivului electronic utilizat pentru compensare.

Ca exemplu practic, în fig. 2 *a* și *b* s-au reprezentat curbele rapoartelor erorilor $\varepsilon'_{i\alpha}/\varepsilon_i=f(A)$ și $\delta'_{i\alpha}/\delta_i=f(A)$, calculate cu relațiile (22′) și (23′), ale unui transformator de curent cu puterea nominală $S_n=5$ VA, compen-

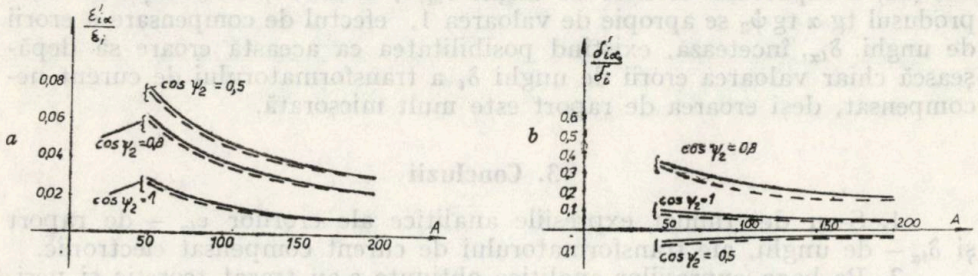

Fig. 2

sat cu amplificatoare electronice avînd $Z_e \ll Z_{2t}$, cînd $\cos\psi_2 = 0{,}8$; 0,5 și 1 ; curbele trasate cu linie plină corespund inducției maxime $B_m = 0{,}18$ T, iar cele trasate cu linie întreruptă, inducției maxime $B_m = 0{,}14$ T.

Miezul transformatorului este executat din tablă de transformator, laminată la rece, tip M 6X, cu grosimea de 0,5 mm, iar înfășurarea de indicator are același număr de spire ca și înfășurarea secundară ($N_3 = N_2$).

Pentru a ilustra efectul compensării electronice la diverse sarcini, pe baza relațiilor exacte (22) și (23), în fig. 3 *a* și *b* s-au reprezentat curbele $\varepsilon'_{i\alpha}/\varepsilon_i = f(Z_{2t})$ și $\delta'_{i\alpha}/\delta_i = f(Z_{2t})$, pentru același transformator, în situația în care amplificarea dispozitivului de compensare are valoarea $A = 50$, impedanța $\underline{Z}_e = (0{,}07 + j\,0{,}05)\,\Omega$, iar B_m și $\cos\psi_2$ au aceleași valori ca și în cazul fig. 2.

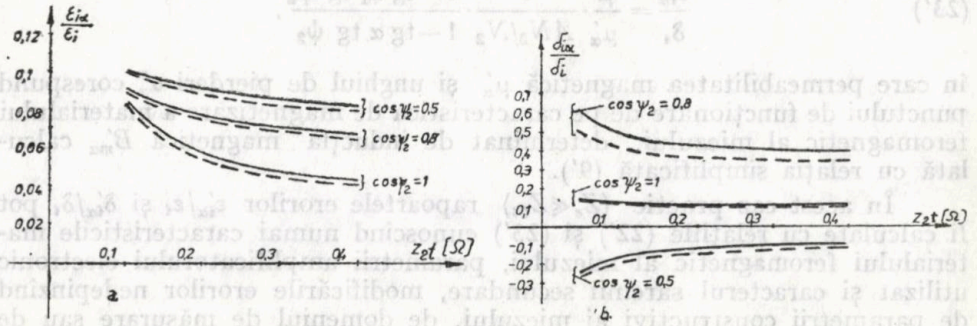

Fig. 3

Curbele trasate în fig. 2 și 3 evidențiază faptul că eficacitatea compensării electronice a erorilor transformatorului de curent este cu atît mai ridicată cu cît impedanța de ieșire, Z_e, a amplificatorului electronic utilizat este mai mică în raport cu Z_{2t} ; aceleași curbe arată că eficacitatea compensării crește semnificativ cu creșterea amplificării A numai pînă la o anumită valoare a acesteia după care rapoartele $\varepsilon'_{i\alpha}/\varepsilon_i$ și $\delta'_{i\alpha}/\delta_i$ rămîn practic constante.

De asemenea, efectul de compensare depinde în mare măsură de caracterul sarcinii secundare (valoarea lui $\cos\psi_2$) și mai puțin de valoarea inducției maxime B_m a punctului de funcționare.

De notat că existența diferenței $1 - \operatorname{tg}\alpha \operatorname{tg}\psi_2$ la numitorul expresiei (23) a raportului erorilor de unghi $\delta_{i\alpha}/\delta_i$ arată că, în situația în care produsul $\operatorname{tg}\alpha \operatorname{tg}\psi_2$ se apropie de valoarea 1, efectul de compensare a erorii de unghi. $\delta_{i\alpha}$, încetează, existînd posibilitatea ca această eroare să depășească chiar valoarea erorii de unghi δ_i a transformatorului de curent necompensat, deși eroarea de raport este mult micșorată.

3. Concluzii

1. S-au determinat expresiile analitice ale erorilor $\varepsilon_{i\alpha}$ — de raport și $\delta_{i\alpha}$ — de unghi, ale transformatorului de curent compensat electronic.

2. Pe baza expresiilor analitice obținute s-au trasat teoretic și verificat experimental curbele rapoartelor $\varepsilon_{i\alpha}/\varepsilon_i$ și $\delta_{i\alpha}/\delta_i$ dintre erorile transfor-

matorului compensat electronic și necompensat. Din analiza expresiilor analitice ale erorilor și a curbelor trasate pe baza lor, au rezultat următoarele concluzii practice :

a) eficacitatea dispozitivului electronic de compensare crește sensibil cu creșterea amplificării sale în tensiune numai pînă la o anumită valoare a acesteia ; creșterea amplificării A peste această valoare este nejustificată practic ;

b) de asemenea, această eficacitate crește dacă impedanța de ieșire Z_e a amplificatorului electronic utilizat devine neglijabilă comparativ cu impedanța totală Z_{2t} a circuitului secundar al transformatorului de compensat ;

c) eficiența compensării electronice depinde sensibil de caracterul sarcinii (v. fig. 2 *a* și *b*) și este practic independentă de modulul acesteia dacă $Z_e \ll Z_{2t}$ (fig. 3 *a* și *b* și relațiile (22') și (23')). În cazul erorilor de raport $\varepsilon_{i\alpha}$ (fig. 2 *a*), eficacitatea compensării crește odată cu creșterea lui $\cos \psi_2$; de exemplu la o amplificare $A=50$, în cazul $\cos \psi_2=1$, eficacitatea compensării este de aproximativ $0{,}074/0{,}027=2{,}7$ ori mai mare decît la $\cos \psi_2=0{,}5$.

Și în cazul erorilor de unghi, așa cum rezultă din fig. 2 *b* și 3 *b*, eficacitatea compensării crește, în general, cu creșterea lui $\cos \psi_2$. Totuși, în acest caz, așa cum rezultă din expresia (23), în funcție de caracterul sarcinii și a calității miezului transformatorului, există anumite situații particulare în care conectarea dispozitivului electronic poate conduce la creșterea acestor erori .

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LE CALCUL ET L'ANALYSE DES ERREURS DU TRANSFORMATEUR DE COURANT COMPENSÉ PAR DES AMPLIFICATEURS DE CORRECTION

(Résumé)

On a déterminé les expressions analytiques des erreurs de rapport et de phase des transformateurs de courant compensés à l'aide des amplificateurs de correction. L'analyse de ces expressions aboutit à des conclusions ayant un remarquable intérêt pratique, concernant la dépendence de l'efficacité des dispositifs de compensation électronique, de leur facteur d'amplification en tension, de leur impédance de sortie et de l'impédance secondaire totale du transformateur compensé.

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BASIC PARAMETERS FOR THE STABILITY OF ATTRACTIVE ELECTROMAGNETIC LEVITATION SYSTEMS

BY

SAVEL MITREA, LORIN CANTEMIR, ILIE POPESCU and M. JUNCU

1. Introduction

In a previous paper [1] the authors investigated the stability of a second-order electromagnetic levitation sistem by using the phase-plane method. The second-order system is obtained if a controlled current-source is used to supply the electromagnet. This simplification enables one to understand the importance of the shape of force-gap characteristic in order to achieve stability. Also, the phase-portrait shows clearly the way the velocity control-signal ensures stability for large perturbations. Yet, current-source-driven electromagnets are almost impracticable especially in the case of large units. The use of controlled voltage-sources lead to third-order levitation systems which are difficult to investigate by phase-space methods. Nevertheless the investigation of stability of such a system by the small-perturbation method gives a good insight into its behaviour. Also, as it will be shown later, this method enables one to select appropriate magnitude of control signals in order to obtain prescribed damping coefficients.

2. Stability Investigation by the Method of Small Perturbations

The schematic diagram of the system under investigation is shown in Fig. 1.

In the following, eddy currents, fringing and hysteresis are ignored. The equations of the system are the following :

$$(1) \qquad Ri + \frac{\mathrm{d}(Li)}{\mathrm{d}t} = u, \qquad m\frac{\mathrm{d}^2 z}{\mathrm{d}t^2} = F - F_z ,$$

where :

$$\frac{\mathrm{d}(Li)}{\mathrm{d}t} = \frac{k_L}{z}\cdot\frac{\mathrm{d}i}{\mathrm{d}t} - k_L\frac{i}{z^2}\cdot\frac{\mathrm{d}z}{\mathrm{d}t}, \qquad u = k_z z^{\gamma} + k_v\frac{\mathrm{d}z}{\mathrm{d}t} + k_a\frac{\mathrm{d}^2 z}{\mathrm{d}t^2},$$

$$F=mg, \qquad F_z=k_F\frac{i^2}{z^2}.$$

Equations (1) can be written in normal form as follows:

$$(2)\quad \begin{cases} \dfrac{di}{dt}=\dfrac{1}{k_L}\left(k_z z^{\gamma+1}-Riz+\dfrac{k_L iv}{z}+k_v vz+k_a gz-\dfrac{k_a k_F}{m}\cdot\dfrac{i^2}{z^2}\right)\overset{\Delta}{=}f_1, \\ \dfrac{dz}{dt}=v\overset{\Delta}{=}f_2, \\ \dfrac{dv}{dt}=g-\dfrac{k_F}{m}\cdot\dfrac{i^2}{z^2}\overset{\Delta}{=}f_3. \end{cases}$$

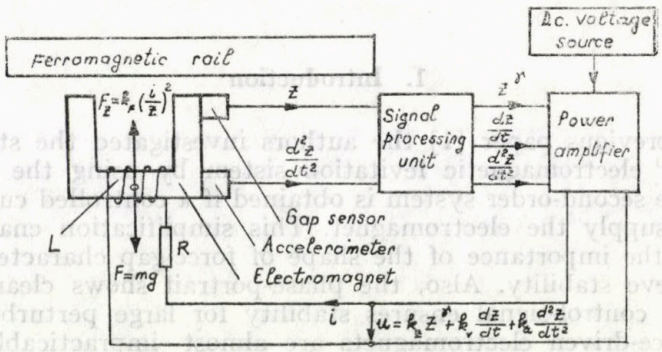

Fig. 1

As it is well known from the nonlinear systems theory, the equilibrium points of the system are found by making $f_1=f_2=f_3=0$. We obtain the equations

$$(3)\qquad Ri_0=k_z z_0^{\gamma}, \qquad g=\frac{k_F}{m}\cdot\frac{i_0^2}{z_0^2},$$

from where one gets the currents i_0 and the gaps z_0, corresponding to the equilibrium points. If the exponent γ is a rational number, Eqs. (3) give four real values for the gap z_0, out of which, two are in the origin and the other two are equal and opposite in sign. The equilibrium point that presents practical interest, has the coordinates:

$$(4)\qquad z_0=\left(\frac{mgR^2}{k_F k_z^2}\right)^{1/[2(\gamma-1)]}, \qquad i_0=z_0\sqrt{\frac{mg}{k_F}}.$$

It should be noted that in order to get a prescribed gap z_0 one has to adjust the coefficient k_z, the other parameters being left unchanged.

The characteristic equation $f(\lambda)=0$ of the system of differential Eqs. (2) is obtained as follows:

$$(5)\qquad \begin{vmatrix} \left.\frac{\partial f_1}{\partial i}\right|_0 - \lambda & \left.\frac{\partial f_1}{\partial z}\right|_0 & \left.\frac{\partial f_1}{\partial v}\right|_0 \\ 0 & -\lambda & 1 \\ \left.\frac{\partial f_3}{\partial i}\right|_0 & \left.\frac{\partial f_3}{\partial z}\right|_0 & -\lambda \end{vmatrix} = 0,$$

where the partial derivatives $\partial fi/\partial x_i|_0$ take values corresponding to the chosen equilibrium point. By expanding the above determinant one gets the third-order algebraic equation

$$(6)\qquad a_0\lambda^3+a_1\lambda^2+a_2\lambda+a_3=0,$$

where the coefficients have the form :

$$(7)\qquad a_0=1,\quad a_1=\frac{Rz_0}{k_L}+\frac{2k_a}{k_L}\sqrt{\frac{gk_F}{m}},\quad a_2=\frac{2k_v}{k_L}\sqrt{\frac{gk_F}{m}},$$
$$a_3=\frac{2gR}{k_L}(\gamma-1).$$

According to the H u r w i t z criterion the system is stable in the vicinity of equilibrium point if the following conditions are fulfilled :

$$(8)\qquad \begin{cases} a_0,\ a_1,\ a_2,\ a_3>0, \\ \Delta=\begin{vmatrix} a_1 & a_0 \\ a_3 & a_2 \end{vmatrix}=a_1a_2-a_0a_3>0. \end{cases}$$

The coefficient a_1 is greater than zero even if acceleration signal is absent, i.e. $k_a=0$. The coefficient a_2 is greater than zero only if the velocity signal is present. The coefficient a_3 is greater than zero only if the exponent γ is greater than unity, a condition that have been stressed upon in the paper [1] as well.

By taking account of (7) the above determinant becomes

$$(9)\qquad \Delta=\frac{2k_v}{k_L^2}\sqrt{\frac{gk_F}{m}}\left(Rz_0+2k_a\sqrt{\frac{gk_F}{m}}\right)-\frac{2gR}{k_L}(\gamma-1).$$

It is seen from (9) that condition (8_2) is fulfilled if k_v exceeds a certain value. Also, stability can be obtained even if acceleration signal is absent so as at the first glance the coefficient k_a seems to be superfluous. Nevertheless this coefficient must be introduced if quality of stabilization is pursued. In order to show this the characteristic equation will be written in terms of its roots $\lambda_1, \lambda_2, \lambda_3$ as follows :

$$(10)\qquad (\lambda-\lambda_1)(\lambda-\lambda_2)(\lambda-\lambda_3)=0.$$

In order that the system be stable and at the same time the damping be smooth the roots must be real and negative. Under these conditions the coefficients of characteristic equation become

$$(11)\qquad a_0=1,\ a_1=\lambda_1+\lambda_2+\lambda_3,\ a_2=\lambda_1\lambda_2+\lambda_1\lambda_3+\lambda_2\lambda_3,\ a_3=\lambda_1\lambda_2\lambda_3.$$

If construction parameters as m, z_0, R, k_L, k_F are to be left unchanged then one sees from (7) that only having at hand three adjustable paramete rs. as γ, k_v, k_a one can obtain the desired values for the roots $\lambda_1, \lambda_2, \lambda_3$. The authors consider that the above parameters are basic to the obtaining of a good stability since the coefficients of characteristic equation, hence the roots, can be easily adjusted by their aid. Nonetheless one may find it more appropriate to use other quantities as control signals that have the same effect upon the coefficients of characteristic equation. The authors propose themselves to deal with this problem in a future paper.

Finally, a few words should be said about the equilibrium point in the origin. Because the corresponding root is multiple this represents a nonsimple singular point and its nature, as nonlinear systems theory shows, can be studied by resorting to a higher order approximation which makes the problem more difficult. Physically, one may conjecture that the electromagnet cannot stay in that point once the current through is zero, provided that the hysteresis is absent. From the phase trajectories shown in [1] for the second order system it appears to be a saddle point, consequently it is unstable.

3. Conclusions

The third-order electromagnetic levitation system can be stabilised only if the position control-signal increases as z^γ, where $\gamma > 1$. The above condition being fulfilled, the system is stable in the vicinity of equilibrium point if a velocity control signal of an appropriate magnitude is applied. In order that the three damping coefficients assume prescribed real values an acceleration control signal must be applied if construction parameters are to be left unchanged.

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PARAMETRII DE BAZĂ PENTRU STABILITATEA SISTEMELOR DE LEVITAȚIE ELECTROMAGNETICE ATRACTIVE

(Rezumat)

Dacă electromagnetul de suspensie este alimentat de la o sursă comandată de tensiune sistemul este descris de un sistem de ecuații diferențiale neliniare de ordinul trei. Stabilitatea în jurul punctului de echilibru se studiază prin metoda perturbațiilor mici. Rezultă că sistemul poate fi stabilizat numai dacă semnalul de comandă după întrefier variază ca z^γ, unde $\gamma > 1$. În plus sistemul este stabil dacă se aplică și un semnal de comandă dependent de viteză. Pentru ca rădăcinile ecuației caracteristice să aibă valori dorite trebuie aplicat și un semnal de comandă dependent de accelerație. Se consideră că cei trei parametri menționați mai sus reprezintă condițiile de bază ale unei stabilități bune a sistemului.

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D.C. 621.318.3

THE DRIVING CONDITION FOR THE DIRECT CURRENT ELECTROMAGNETS

BY

EUGEN HNATIUC

1. Introduction

It is considered a direct current electromagnet of the type $U-I$ with translation mobile armature, (Fig. 1), as an electromechanical system of conversion of electric energy with two degrees of liberty — one electric, the electric current $i(t)$ or the electric charge $q(t)$, and the other mechanic, the displacement $x(t)$ of its mobile armature, [2]...,[4].

With help of the analytical mechanics principles, the system of L a g r a n g e's equations which describes the ensemble, can be written as follows:

$$(1)\quad \begin{cases} M\ddot{x}(t)+D\dot{x}(t)+k[x(t)-b]+\dfrac{1}{2}\cdot\dfrac{L'}{[x(t)+a]^2}\,i^2(t)=F_R(x), \\ \dfrac{L'}{x(t)+a}\cdot\dfrac{\mathrm{d}i(t)}{\mathrm{d}t}+\left\{R-\dfrac{L'\dot{x}(t)}{[x(t)+a]^2}\right\}i(t)=U, \end{cases}$$

in which: M is the mass of mobile armature of the electromagnet, $D-$the ratio which takes into consideration the damping of the movement of armature due to the viscousness of the medium (in the case of the air it can

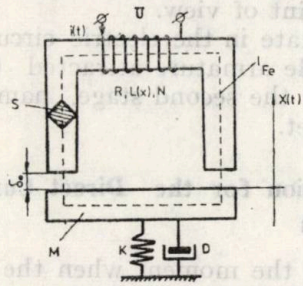

Fig. 1

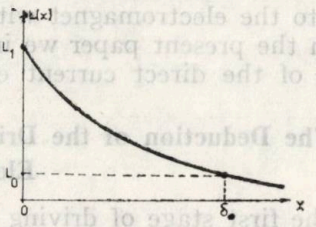

Fig. 2

be neglected), $k-$the elastic constant of the antagonist spring fixed on the mobile armature of the electromagnet which drives with a null force against it for $x=b$, $L(x)-$the inductance of the coil of the electromagnet (Fig. 2) :

$$(2) \qquad L(x)=\frac{L'}{x(t)+a}, \qquad L'=\frac{N^2\mu_0 S}{2}, \qquad a=\frac{1}{2}\cdot\frac{\mu_0}{\mu_{Fe}}\,l_{Fe},$$

N — the number of widings of the electromagnet's coil, μ_0, μ_{Fe} — the magnetic permeabilities of the air, respective of the magnetic core with the cross section S and the total length with iron's core l_{Fe}, $F_R(x)$ — the resisting force that drives on mobile armature and that opposes to the displacement, supposed as being a constant, R — the electrical resistance of the coil of the electromagnet, U — the voltage of the source, $x(t)$ — the mechanical displacement of mobile armature, $i(t)$ — the current through the circuit of the coil.

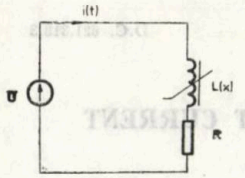

Fig. 3

The system (1) can be written as follows:

$$(3) \qquad \begin{cases} M\ddot{x}(t)+\Delta\dot{x}(t)+k[x(t)-b]+\dfrac{1}{2L'}\psi^2(t)=F_R(x), \\ \dfrac{d\psi(t)}{dt}+\dfrac{R}{L'}[x(t)+a]\psi(t)=U, \end{cases}$$

where

$$(4) \qquad \psi(t)=L(x)\,i(t)$$

represents the total magnetic flux which passes through the magnetic circuit of the electromagnet.

For the electromagnets can be accepted that their „driving" is achieved in three stages, namely:

a) The transient regime of the growth of the current value (of the magnetic flux), its mobile armature remaining fixed in the initial position, ($x=\delta_0$) and the inductance of the coil remaining constant (L_0 — Fig. 2).

b) The transient regime corresponding to the displacement of the mobile armature of the electromagnet, from the initial in to the final position, corresponding to a minimum value (practically null) of the airgap of the magnetic circuit. At the end of this stage the „driving" of the electromagnet is finished from the mechanical point of view.

c) The etablishment of the steady state in the electric circuit equivalent to the electromagnet with the mobile armature attracted (Fig. 3).

In the present paper we interest only the second stage, namely the driving of the direct current electromagnet.

2. The Deduction of the Driving Condition for the Direct Current Electromagnets

The first stage of driving lasts till in the moment when the current from circuit (magnetic flux ψ) reaches the value which correspond to the starting of the mobile armature, ψ_p, that verifies the equation (3_1):

$$(5) \qquad k(\delta_0-b)+\frac{1}{2L'}\psi_p^2=F_R, \qquad \psi_p=\sqrt{2L'[F_R-k(\delta_0-b)]}.$$

Taking into account the equation (3_2), which corresponds to this first stage;

$$\frac{d\psi(t)}{dt}+\frac{R}{L'}(\delta_0+a)\psi(t)=U, \tag{6}$$

the variation in time of the magnetic flux is illustrated by the equation :

$$\psi(t)=\psi_0(1-e^{-\frac{t}{T_0}}), \qquad \psi_0=UT_0, \qquad T_0=\frac{L_0}{R}=\frac{L'}{R(\delta_0+a)}, \tag{7}$$

thus the value of starting magnetic flux will be obtained after a „starting time" :

$$t_p=T_0\ln\frac{\psi_0}{\psi_0-\psi_p}. \tag{8}$$

We mention that this „condition of starting" for the mobile armature of the direct current electromagnet is usually improperly accepted as a condition of driving.

It can be noticed that the value of the starting magnetic flux is, in fact, higher, which has to be „corrected" taking into account the dispersal factor $\sigma(x)$:

$$\psi'_p=\sigma(x)\psi_p>\psi_p. \tag{9}$$

The second stage in the driving of the direct current electromagnet begins in the moment when the magnetic flux gets the value ψ_p and lasts as long as its mobile armature move from its initial position $x(t)=\delta_0$, $L(x)=L_0$, to the final one $x(t)\approx 0$, $L(x)=L_1$.

In order to estimate the behaviour of the system in the second stage, is usefull to change the variables as follows :

$$x(t)=\delta_0-y(t), \qquad \psi(t)=\psi_p+\varphi(t)\ ; \tag{10}$$

thus the system (3) can be written under the form :

$$\begin{cases} M\ddot{y}(t)+D\dot{y}(t)+ky(t)=\dfrac{\psi_p}{L'}\varphi(t)+\dfrac{1}{2L'}\varphi^2(t), \\ \dot{\varphi}(t)+\dfrac{R}{L'}[\delta_0+a-y(t)]\varphi(t)=\left.\dfrac{d\psi}{dt}\right|_{t=t_p}+\dfrac{R}{L'}\psi_p y(t), \\ \left.\dfrac{d\psi}{dt}\right|_{t=t_p}=U-\dfrac{R}{L'}(\delta_0+a)\psi_p. \end{cases} \tag{11}$$

Taking into consideration that the damping viscousity owed to the abrasion of the mobile armature with the air is small enough ($D\approx 0$), the system (11) becomes :

$$\ddot{y}(t)+\omega_0^2 y(t)=A\varphi(t)[B+\varphi(t)]\ ,\ \dot{\varphi}(t)+c[D-y(t)]\varphi(t)=H+Gy(t), \tag{12}$$

in which the following notations have been utilized :

$$(13)\quad \begin{cases} \omega_0^2=\dfrac{k}{M}, \quad A=\dfrac{1}{2ML'}, \quad B=2\psi_p, \quad c=\dfrac{R}{L'}, \quad D=\delta_0+a, \\ H=\left.\dfrac{d\psi}{dt}\right|_{t=t_p}=Ue^{-\frac{t_p}{T_0}}, \quad G=\dfrac{R}{L'}\psi_p. \end{cases}$$

Now, taking into consideration the linear system which corresponds to the Eqs. (12) [1], that describes the behaviour of the electromagnet for the small perturbations, $y(t)$ and $\varphi(t)$, it results:

$$(14)\quad \ddot{y}(t)+\omega_0^2 y(t)-AB\varphi(t)=0, \quad \dot{\varphi}(t)+CD\varphi(t)-Gy(t)=H.$$

The characteristic equation which corresponds to this system of equations is:

$$(15)\quad \begin{vmatrix} p^2+\omega_0^2 & -AB \\ -G & p+CD \end{vmatrix}=0,$$

or:

$$(16)\quad p^3+CDp^2+\omega_0^2 p+\omega_0^2 CD-ABG=0,$$

which permits the estimation, in the first approximation, of the electromagnet behaviour which is considered in the second stage of its driving.

Thus, the stable behaviour of the system, which corresponds to the solutions of the characteristic equation situated in the left complex half-plane, impose positive values of their coefficients [5]:

$$(17)\quad CD>0, \quad \omega_0^2>0, \quad \omega_0^2 CD-ABG>0,$$

and the fulfilling of the condition $a_1a_2-a_0a_3>0$, namely:

$$(18)\quad \omega_0^2 CD-(\omega_0^2 CD-ABG)=ABG>0.$$

It can be observed that the electromagnet has a stable behaviour if the relation (17_3) is fulfilled.

From the electromagnet driving point of view we are interested in its unstable behaviour which is obtained when the conditions (17) and (18) are not fulfilled, therefore for:

$$(19)\quad \omega_0^2 CD-ABG<0,$$

so that the existence of a positive solution of the characteristic equation (16) is necessary.

Refering to the notations (13) the condition of sure driving of the electromagnet can be written in one of the equivalent forms:

$$(20)\quad \psi_p>\sqrt{KL'(\delta_0+a)}, \quad \delta_0<\frac{\psi_p^2}{KL'}-a,$$

or considering the expression (5) of the starting magnetic flux it results:

$$(21)\quad \delta_0<\frac{2F_R+k(2b-a)}{3k}.$$

Taking into account the fact that the maximum value of the starting magnetic flux is:

$$\psi_{p\max}=\psi_0=UT_0, \tag{22}$$

the condition of sure driving can be also written as:

$$U\geqslant R(\delta_0+a)\sqrt{\frac{k}{L_0}} \quad \text{or} \quad \delta_0\leqslant \frac{U}{R}\sqrt{\frac{L_0}{k}}-a. \tag{23}$$

If we consider the expression (2) of the inductance of the electromagnet coil $L(x)$ and the conditions:

$$L(\delta_0)=L_0, \quad L(0)=L_1, \tag{24}$$

we obtain:

$$L'=\frac{L_1L_0}{L_1-L_0}\,\delta_0, \qquad a=\frac{L_0}{L_1-L_0}\delta_0. \tag{25}$$

By replacing a with its expression (25_2) in the relations (23) and imposing conditions of extreme of the form:

$$\frac{dU}{dL_0}=0, \qquad \frac{d\delta_0}{dL_0}=0, \tag{26}$$

it result the minimum values of the supply voltage, respectively of the air-gap for the sure driving of the electromagnet:

$$U_{\min}=\frac{3R\delta_0}{2}\sqrt{\frac{k}{L_0}}, \qquad \delta_0=\frac{2}{3}\cdot\frac{U}{R}\sqrt{\frac{L_0}{k}}, \tag{27}$$

satisfied only if the optimum relation

$$L_1=3L_{0\min}. \tag{28}$$

is verified.

In the driving of the electromagnet after the displacement of its mobile armature from the initial position to the final one, the third stage follows (there is not a movement of its mobile armature)—the electric transient regime—in which the values $i(t)$ and $\psi(t)$ tend to the values of the steady state.

3. Experimental

It was determined experimentally the values of the voltage of the sure driving for the usual direct current electromagnets and it was compared with those resulted from the direct measurement, obtaining a good concordance between calculation and experiment, although in calculation the magnetic dispersion and nonlinearity of the magnetization characteristic of the iron core was neglected (Table 1).

Table 1

Type of relay	PB -45	UA 3 KUHNKE	WHBH Siemens
$U_{calc.}$, [V]	14,289	134,89	15,05
$U_{exp.}$, [V]	14,5	122	15,75
ε%	1,45	9,8	4,4

4. Conclusions

Considering that the direct current electromagnet is an electromechanical system of energy conversion with two degrees of liberty, which is described by L a g r a n g e's equations and taking into account the three stages of its driving, with help of the first approximation of the stability of the ensemble it is obtained a condition of sure driving, which was, then, experimentally confirmed.

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CONDIȚIA DE ACȚIONARE PENTRU ELECTROMAGNEȚII DE CURENT CONTINUU

(Rezumat)

Se consideră electromagneții de curent continuu ca sisteme electromecanice de conversie a energiei descrise de sisteme de ecuații diferențiale de tip L a g r a n g e. Analizînd, în primă aproximație, stabilitatea sistemului de ecuații stabilit se obține o condiție de acționare sigură pentru electromagneții considerați. Rezultatele obținute sînt pe deplin confirmate experimental, diferențele rezultate între valorile tensiunii minime de acționare, obținute prin calcul și prin măsurare directă, fiind sub 10%.

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C.D. 621. 318.5

ÉLÉMENT SENSIBLE POUR LA CONSTRUCTION DES RELAIS

PAR

ADRIAN BARABOI

1. Les relais sont des appareils électriques caractérisés par des phénomènes de saut du signal de sortie, pour des signaux d'entrée représentant des fonctions continues, la caractéristique de transfert d'un tel appareil ayant l'alure représentée dans la fig. 1, où par x on a noté le signal d'entrée et par y—le signal de sortie. On constate que le signal de sortie y, d'un relais, présente deux états stables, notées par 0 et 1.

Le schéma-bloc (fig. 2), de n'importe quel type de relais contient [1] : a) l'élément sensible (*1*), qui transforme le signal d'entrée, x, dans une grandeur physique nécessaire au fonctionnement du relais ; b) l'élément comparateur (*2*), qui compare le niveau du signal de sortie de l'élément sensible (*1*) avec sa valeur préréglée et transmet l'action, si le résultat de la comparaison l'impose ; c) l'élément final, qui réalise pratiquement l'un des états 0 ou 1 du signal de sortie, y.

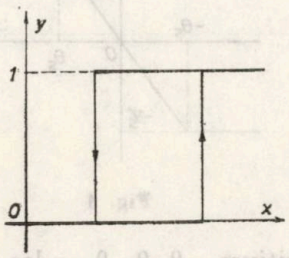


Fig. 1

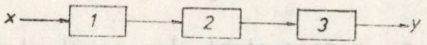


Fig. 2

Dans cette Note on analyse les possibilités de l'utilisation, comme élément sensible pour la construction des relais, d'un dispositif electromagnétique qui contient des bobines non-linéaires ayant les noyaux magnétisés orthogonalement (BNNMO).

2. L'élément sensible, ayant le schéma électrique représenté graphiquement dans la fig. 3, est un dispositif électromagnétique réalisé à l'aide de deux bobines identiques, $BNNMO_1$ et $BNNMO_2$, ayant comme signaux d'entrée les courants sinusoïdaux de même fréquence $i_{ex}(t)$ et $i_c(t)$, le signal de sortie étant considéré la t.e.m. $e(t)$, obtenue entre les bornes *3-4*. Le circuit *7-8* est parcouru par le courant continu I_0, de prémagnétisation.

Compte tenu de la nature des signaux d'entrée et de prémagnétisation, pour le schéma électrique du dispositif, donné dans la fig. 3, on con-

sidère la t.m.m. $\theta(t)$, produite par chaque enroulement parcouru par le courant $i_{ex}(t)$, de la forme :

$$\theta(t) = \theta_m \cos(\omega t + \varphi), \tag{1}$$

et les t.m.m. de commande, produites par les enroulements parcourus par les courants $i_c(t)$ et I_0, ayant les expressions :

$$\theta_c(t) = \begin{cases} \theta_0 + \theta_{cm} \cos \omega t, & \text{pour la BNNMO}_1, \\ \theta_0 - \theta_{cm} \cos \omega t, & \text{pour la BNNMO}_2. \end{cases} \tag{2}$$

Si l'on peut linéariser la courbe d'aimantation du matériel ferromagnétique utilisé pour les noyaux des BNNMO, sous la forme représentée dans la fig. 4, la fonction qui permet l'approximation analytique de la caractéristique $\psi(\theta, \theta_c)$ d'une BNNMO, [4], prend la forme particulière :

$$\psi(\theta, \theta_c) = \begin{cases} (a_{10} - a_{12}\theta_c^2)\theta, & \theta \in [-\theta_s, \theta_s], \\ \pm\psi_s, & \theta \notin [-\theta_s, \theta_s], \end{cases} \tag{3}$$

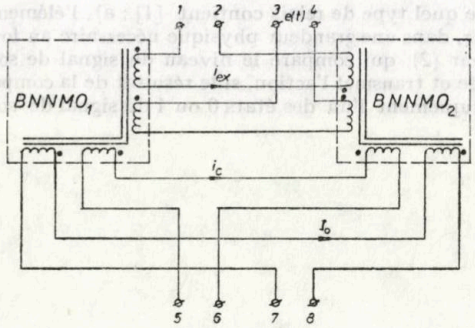


Fig. 3

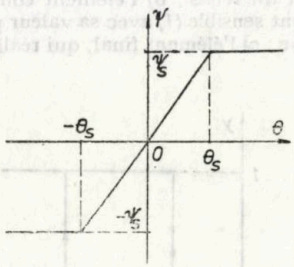


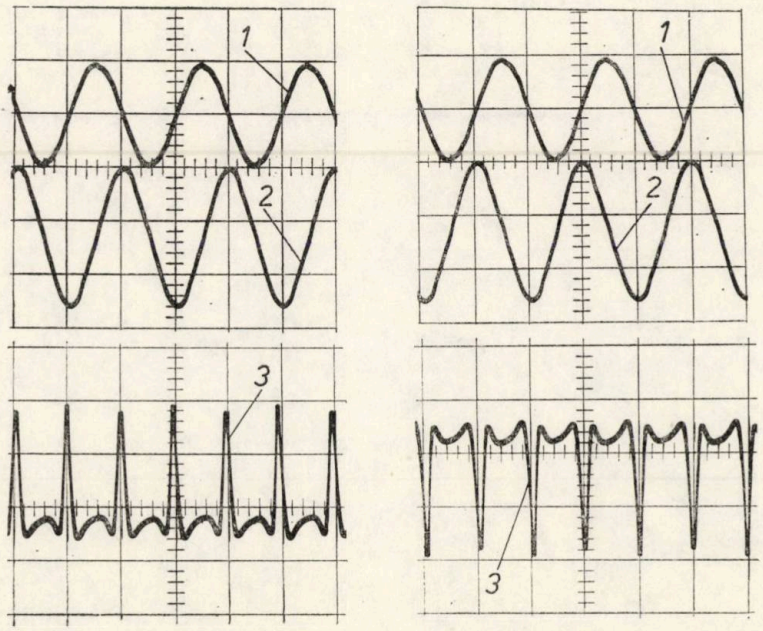
Fig. 4

où a_{10}, a_{12} représentent des constantes positives, θ, θ_c, θ_s — les t.m.m. d'excitation, de commande et de saturation, respectivement, ψ, ψ_s — les flux magnétiques totaux d'excitation et de saturation, respectivement. Les fonctions qui permettent l'approximation analytique des caractéristiques $\psi_1(\theta, \theta_{c1})$, $\psi_2(\theta, \theta_{c2})$, écrites pour les circuits d'excitation des bobines BNNMO$_1$, respectivement BNNMO$_2$, s'obtiennent par la substitution des t.m.m. $\theta(t)$, $\theta_c(t)$, dans la relation (3), avec leurs expressions (1) et (2). Par cette voie la t.e.m. $e(t)$, qui constitue le signal de sortie de l'élément sensible analysé, produite par la différence $\Delta\psi(t)$ des flux magnétiques $\psi_1(t)$ et $\psi_2(t)$, résulte de la forme :

$$e(t) = \begin{cases} E_m \sin(2\omega t + \varphi), & \theta \in [-\theta_s, \theta_s], \\ 0 & \theta \notin [-\theta_s, \theta_s], \end{cases} \tag{4}$$

où :

$$E_m = 4qa_{12}\theta_0\theta_m\theta_{cm}\omega, \tag{5}$$



a b

Fig. 5

la constante q représentant le rapport du nombre de spires du circuit *3-4*, par le nombre de spires du circuit *1-2*.

3. Si pour les noyaux ferromagnétiques des BNNMO du schéma de l'élément sensible (fig. 3) on utilise des matériaux d'haute perméabilité, caractérisés par des intensités faibles du champ coercitif, en acceptant l'hypothèse :

$$\theta_m \gg \sqrt{2}\theta_s , \tag{6}$$

le signal de sortie $e(t)$, donné par la relation (4), représente une suite d'impulsions, ayant la fréquence ω/π ; les deux fronts de chaque impulsion se produisent dans les instants :

$$t_{1,2} = \frac{1}{\omega}\left(k\pi \pm \arccos\frac{\theta_s}{\theta_m} - \varphi\right), \quad (k=1, 2, 3,...). \tag{7}$$

Vu que les noyaux ferromagnétiques des BNNMO utilisées fonctionnent fortement saturés, la relation (7), l'hypothèse (6) en étant acceptée, peut s'écrire sous la forme :

$$t_{1,2} \approx t_0 = \frac{1}{\omega}\left[(2k+1)\frac{\pi}{2} - \varphi\right], \quad (k=0, 1, 2, 3,...), \tag{8}$$

de sorte que pour l'amplitude des impulsions du signal de sortie, $e(t)$, compte tenu de (4), on obtient :

$$e(t_0) = e_0 = E_m \sin\varphi, \tag{9}$$

où l'amplitude E_m est donnée par (5).

L'expression (9) met en évidence que l'amplitude des impulsions de tension obtenues dans le circuit de sortie de l'élément sensible (fig. 3), respecte une relation de proportionalité en $\sin\varphi$, où φ est le déphasage entre les courants sinusoïdaux d'entrée, $i_{ex}(t)$ et $i_c(t)$. Dans les conditions ci-dessus ce dispositif peut servir comme élément sensible dans la construction des relais à retour de puissance, le signal de sortie en impulsions étant avantageusement utilisé pour la commande des thyristors de l'élément final *3* (fig. 2). Dans la fig. 5 on présente les oscillogrammes des signaux d'entrée $i_{ex}(t)$, $i_c(t)$ et du signal de sortie, $e(t)$, pour le déphasage φ ayant les valeurs $\pm\pi/2$, en confirmant ainsi, par voie expérimentale, les résultats théoriques obtenus. On a représenté par: *1* — le courant $i_{ex}(t)$, *2* — le courant $i_c(t)$ et *3* — la t.é.m. $e(t)$ pour $\varphi=\pi/2$ (fig. 5 *a*) et $\varphi=-\pi/2$ (fig. 5 *b*).

Conclusions

1. On étudie les possibilités de l'utilisation d'un dispositif électromagnétique contenant deux bobines BNNMO identiques (fig. 3), comme élément sensible pour la construction des relais ; on établit théoriquement la dépendence, par une relation de proportionalité, entre l'amplitude des impulsions du signal de sortie, $e(t)$ et $\sin\varphi$, où φ est l'angle de déphasage entre les courants $i_{ex}(t)$ et $i_c(t)$, qui circulent par les circuits d'entrée.

2. On présente quelques résultats expérimentaux qui confirment, qualitativement, les résultats obtenus par voie théorique.

3. Le dispositif analysé peut fonctionner comme élément sensible des relais à retour de puissance, l'utilisation des BNNMO présentant des avantages par rapport aux autres dispositifs de même type, [3], à savoir la réduction à la moitié du nombre des noyaux ferromagnétiques et une rapidité de fonctionnement augmentée, grâce à l'absence des couplages magnétiques entre les enroulements d'excitation et de commande.

Reçue le 10 novembre 1980

Institut Polytechnique, Jassy,
Chaire d'Électroénergétique

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ELEMENT SENSIBIL PENTRU CONSTRUCȚIA RELEELOR

(Rezumat)

Se studiază, pe cale analitică și experimentală, posibilitățile de utilizare a unei scheme electrice conținînd bobine neliniare avînd miezurile magnetizate pe direcții ortogonale (BNMMO), ca element sensibil în construcția releelor.

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D.K. 621.316.543 ; 621 - 52

SYNTHESEVERFAHREN DER LINEAREN SEQUENTIELLEN KREISEN DIE ZUR WECHSELRICHTERSETEUERUNG VERWENDET SIND

VON

VALERIU MUNTEANU, M. LUCANU und M. ANTONESCU

1. Einleitung

Die Haupt- und Hilfsthyristoren aus der Starkstromkreisen der Wechselrichtern leiten bzw. sind gesperrt gewisse Zeitintervallen und der Zyklus wiederholt sich nach einer bestimmten Periode. Unter diesen Bedingungen lassen sich in den Steuerungskreisen der Wechselrichtern linearen, sequentiellen Kreisen verwenden und zwar Kreisen deren unterschiedlichen Zustände gleich mit dem Zyklus sind nach dem sich die Funktionierung des Schemas sich wiederholt.

Es muß noch erwähnt werden, daß die Anwendung von linearen, sequentiellen Kreisen, mit passenden gewählten Rückwirkungen, ermöglicht die Generierung der Leit- und Sperreintervallen für jeden Haupthyristor und das beseitigt die Dekodierschaltungen die in den klassischen Schemen erscheinen [1]...[3].

Die Verwendung in den Steuerunskreisen der linearen, sequentiellen Kreisen bietet zusätzlich den Vorteil einer leichten Umkehrung der Drehrichtung der vom Wechselrichter gespeisten Motoren und zwar durch Verschiebung des Registerinhalts nach links oder nach rechts [4].

Im folgenden wird einen Steuerungskreis eines Wechselrichters vorgeschlagen, der zur umkehrbaren Betätigung der Motoren in einem breiten Frequenzbereich verwendbar ist.

2. Die Synthese des linearen, sequentiellen Kreises

In der Abb. 1 ist der Starkstromkreis des Wechselrichters dargestellt. Die Spannungen u_R, u_S, u_T haben die Wellenformen aus der Abb. 2 falls die Hauptthyristoren $T_1 \dots T_6$ und die Hilfsthyristoren $T_{11} \dots T_{16}$ leiten in den in der Tabelle 1 angegebenen Zeitintervallen [2].

Es folgt also daß nach sechs Takten die Funktionierung des Schemas wiederholt sich.

Um die leitenden Zustände der Hauptthyristoren zu realisieren wird ein linearer, sequentieller Kreis nötig sein, der einem Polynomen $g(D)$ mit Koeffizienten in $GF(2)$, mit einer Periode gleich 6 entspricht.

Die Bedingung, daß das Polynom $g(D)$ eine Periode gleich 6 hat ist, da $D^6 \oplus 1$ teilbar durch $g(D)$ ist [5].

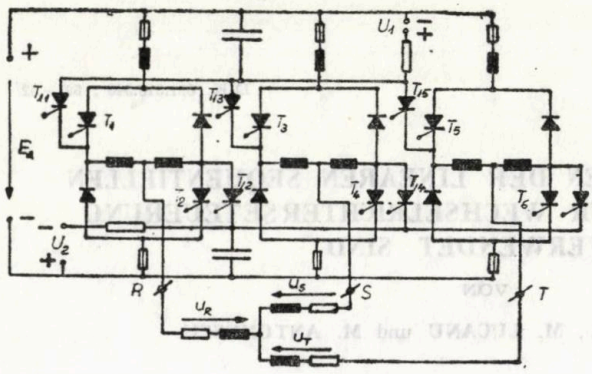

Abb. 1

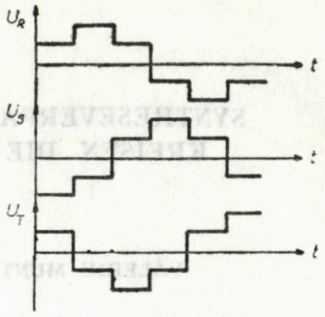

Abb. 2

Tabelle 1

T_5	1	0	0	0	1	1
T_4	1	1	0	0	0	1
T_1	1	1	1	0	0	0
T_6	0	1	1	1	0	0
T_3	0	0	1	1	1	0
T_2	0	0	0	1	1	1
T_{15}	1	0	0	0	0	0
T_{14}	0	1	0	0	0	0
T_{11}	0	0	1	0	0	0
T_{16}	0	0	0	1	0	0
T_{13}	0	0	0	0	1	0
T_{12}	0	0	0	0	0	1

Wählt man ein Polynom $g(D)$ der Form:

$$g(D) = (D^p \oplus 1) \cdot g_1(D) \tag{1}$$

dann ist es nötig, daß D^6+1 sowohl durch das Polynom D^p+1, wobei p eine Ganzzahl ist, als auch durch das Polynom $g_1(D)$ mit Koeffizienten in $GF(2)$, teilbar sein soll.

Das Polynom D^6+1 ist teilbar durch D^p+1 falls 6 das Vielfache von p ist [5].

Wählt man das Polynom $g_1(D)$ primitiv, m-Grades, mit Koeffizienten in $GF(2)$, gewinnt man seine Periode aus der Gleichung [5]:

$$q = 2^m - 1. \tag{2}$$

Es folgt also, daß das Polynom $g(D)$ ein Vielfache des Polynomen $g_1(D)$ ist falls die Zahl 6 teilbar durch q ist.

Die Vereinigung der zwei Bedingungen hat als Folge, daß:

(3) $6 =$ das kleinste kommune Vielfach $[p, q]$

ist.

Die einzigen Divisoren der 6 die verschieden von 1 sind sind 2 und 3. Es folgt also nach der Gl. (2), daß $q=3$, also $m=2$ und $p=2$.

Das einzige primitive Polynom zweiten Grades über $GF(2)$ ist:

(4) $$g_1(D)=D^2\oplus D\oplus 1.$$

Unter Einbeziehung der Gl. (4) und der Tatsache, daß $p=2$ wird Gl. (1):

(5) $$g(D)=(D^2\oplus 1)(D^2\oplus D\oplus 1)=D^4\oplus D^3\oplus D\oplus 1.$$

Der lineare, sequentielle Kreis der dem Polynom $g(D)$ aus der Gl. (5) entspricht ist in der Abb. 3 dargestellt, wobei $C_1 \ldots C_4$ bistabilen Kippschaltungen sind und die Rückwirkungen mittels Addierern modulo 2 realisiert sind.

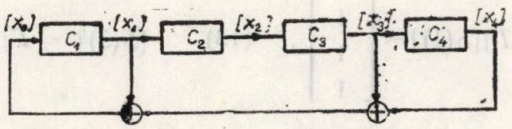

Abb. 3

Es seien s_1, s_2, s_3, s_4 die Zustände der bistabilen Kippschaltungen bei einem bestimmten Zeitpunkt (nicht alle gleich Null) und s_1', s_2', s_3', s_4' die Zustände derselben Kreisen nach der Anbringung des ersten Taktes (der in der Abb. 3 nicht indiziert ist).

Entsprechend der Abb. 3 läßt sich folgendes System schreiben:

(6) $$s_2'=s_1,\quad s_3'=s_2,\quad s_4'=s_3,\quad s_1'=s_1\oplus s_3\oplus s_4.$$

Falls man den anfänglichen Zuständ mit der Matrix:

(7) $$[S(1)]=\begin{bmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \end{bmatrix}$$

und den Zustand nach der Anbringung des ersten Taktes mit der Matrix:

(8) $$[S(2)]=\begin{bmatrix} s_1' \\ s_2' \\ s_3' \\ s_4' \end{bmatrix}$$

aufzeichnet, das Gleichungssystem (6) läßt sich in der Form:

$$[S(2)]=[T][S(1)] \tag{9}$$

kompakt schreiben, wobei:

$$[T]=\begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}. \tag{10}$$

Zum Beispiel, falls man annimt, daß:

$$[S(1)]=\begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix} \tag{11}$$

die anderen verschiedenen fünf Zustände folgen aus den Beziehungen:

$$[S(2)]=[T][S(1)]=\begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \tag{12}$$

$$[S(3)]=[T][S(2)]=\begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \tag{13}$$

$$[S(4)]=[T][S(3)]=\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \tag{14}$$

$$[S(5)]=[T][S(4)]=\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \tag{15}$$

$$[S(6)]=[T][S(5)]=\begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}. \tag{16}$$

Man kann prüfen, daß die Anbringung des nächsten Taktes den linearen, sequentiellen Kreis in dem anfänglichen Zustand bringt.

Falls man als Anfangszustand jeder der Zustände (12)...(16) betrachtet, ergeben sich dieselben Zustände.

Aus den Beziehungen (11)...(16) geht hervor, daß bei den Ausgängen der bistabilen Kippschaltungen werden die Folgen:

$$\begin{cases} [x_1]=[1\ 0\ 0\ 0\ 1\ 1], & [x_2]=[1\ 1\ 0\ 0\ 0\ 1], \\ [x_3]=[1\ 1\ 1\ 0\ 0\ 0], \text{ bzw.} & [x_4]=[0\ 1\ 1\ 1\ 0\ 0] \end{cases} \tag{17}$$

generiert.

Vergleicht man diese Folgen mit der Tabelle 1, so stellt man fest, daß die Ausgänge der bistabilen Kippschaltungen die Leitintervallen der Thyristoren T_5, T_4, T_1, bzw. T_6 bestimmen.

Wiederum, aus der Abb. 3, leitet man her, daß man die Folge $[x_0]$ mit der Beziehung:

$$[x_0]=[x_1]\oplus[x_3]\oplus[x_4]=[0\ \ 0\ \ 0\ \ 1\ \ 1\ \ 1] \tag{18}$$

bestimmen kann, was den Leitintervallen des Hauptthyristors entspricht.

Falls man die Folge $[x_0]$ als Ursprung betrachtet, die Folge $[x_1]$ stellt die ursprüngliche, mit einem Takt verzögerte Folge dar, die Folge $[x_2]$ stellt eine Verzögerung mit zwei Takten dar u.s.w. die Folge $[x_4]$ eine Verzögerung mit vier Takten.

Zeichnet man mit D einen linearen Operator auf, der eine Verzögerung mit einem Takt realisiert [5], folgt:

$$[x_1]=D[x_0],\quad [x_2]=D^2[x_0],\quad [x_3]=D^3[x_0],\quad [x_4]=D^4[x_0]. \tag{19}$$

Mit dem Operator D wird die Beziehung (18):

$$[x_0]=D[x_0]\oplus D^3[x_0]\oplus D^4[x_0], \tag{20}$$

oder:

$$(D^4\oplus D^3\oplus D\oplus 1)\ [x_0]=0. \tag{21}$$

Die Folge $[x_0]$ ist verschieden von Null, also folgt als nötig, daß:

$$D^4\oplus D^3\oplus D\oplus 1=0. \tag{22}$$

Aus der Inspizierung der Tabelle 1 bemerkt man, daß die Leitintervallen für den Thyristor T_3 lassen sich mit einer mit fünf Takten verzögerte Folge im Vergleich mit der Ursprungfolge, $[x_0]$, beschreiben. Diese Verzögerung erhält man entweder unter zusätzlichen Schaltung einer bistabilen Kippschaltung nach C_4, oder als lineare Kombination der vorher generierten Folgen.

Tatsächlich, unter Einbeziehung der Gl. (22), folgt:

$$D^5=D\,.\,D^4=D(D^3\oplus D\oplus 1)=D^4\oplus D^2\oplus D=D^3+D+1. \tag{23}$$

Aus der Beziehung (23) ergibt sich, daß die mit fünf Takten verzögerte Folge, die den Leitintervallen des Hauptthyristors T_3 entspricht, erhält man unter modulo 2 Addierung der Folgen $[x_0]$, $[x_1]$, $[x_3]$.

Der lineare, sequentielle Kreis der die Leitintervallen der Hauptthyristoren $T_1 \ldots T_6$ generiert, ist in der Abb. 4 dargestellt.

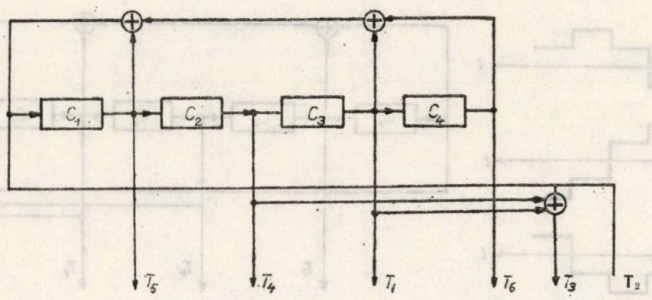

Abb. 4

3. Die Umkehrung der Drehrichtung

Um die Drehrichtung der vom Wechselrichter gespeisten Motoren umzukehren müßen die Leitintervallen der Haupt- und Hilfsthyristoren denen aus der Tabelle 2 entsprechen.

Tabelle 2

T_5	1	1	0	0	0	1
T_4	1	0	0	0	1	1
T_1	0	0	0	1	1	1
T_6	0	0	1	1	1	0
T_3	0	1	1	1	0	0
T_2	1	1	1	0	0	0
T_{15}	0	1	0	0	0	0
T_{14}	1	0	0	0	0	0
T_{11}	0	0	0	0	0	1
T_{16}	0	0	0	0	1	0
T_{13}	0	0	0	1	0	0
T_{12}	0	0	1	0	0	0

Die Wellenformen der Spannungen u_R, u_S, u_T in dieser Situation sind in der Abb. 5 dargestellt.

Die Vergleichung der Wellenformen aus der Abb. 2 und 5 führt zu der Schlußfolgerung, daß die Phasen S und T umkehrt worden sind.

Anderseits, die Inspizierung der Tabellen 1 und 2 zeigt, daß für eine Drehrichtung, die in den bistabilen Kippschaltungen gespeicherte Information nach rechts, für die andere Drehrichtung nach links verschieben werden muß.

Die Verwendung desselben Polynomes $g(D)$ (s. Gl. (5)) führt für die Umkehrung der Drehrichtung zu einen linearen, sequentiellen, in der Abb. 6 dargestellten Kreis.

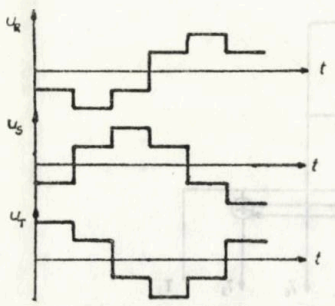

Abb. 5

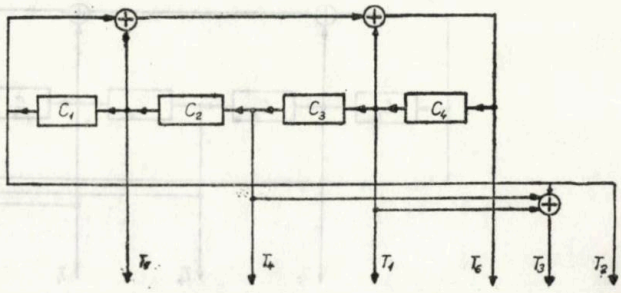

Abb 6.

Eine logische Schaltung die zum Anklemmen der vier bistabilen Kippschaltungen und der 3 modulo 2 Addierern dient, die in den linearen, sequentiellen Kreisen (Abb. 4 und 6) die Umkehrung der Drehrichtung ermöglichen, ist nicht schwierig zu entwerfen [4].

4. Schlußfolgerungen

Die vorgeschlagene Steuerungsschaltung hat folgende Vorteile im Vergleich mit den klassischen Schemen :

a) Die Verwendung von linearen, sequentiellen Kreisen in den Steuerungsschaltungen der Wechselrichtern beseitigt die Dekodierschaltungen.

b) Die Steuerungsschaltung läßt sich in einem breiten Frequenzbereich verwenden und zwar von Frequenzen der Hz-Größenordnung bis zu der maximalen Kommutierungsfrequenz der Thyristoren vom Starkstromkreis. Zu diesem Zweck muß man nur die Taktfrequenz des Impulses der den linearen, sequentiellen Kreis steuert ändern.

c) Die Möglichkeit der Umkehrung der Drehrichtung unter Belastung.

d) Materialwirtschaftlichkeit und erhöhte Zuverlässigkeit.

Das Syntheseverfahren für die linearen, sequentiellen, in den Steuerungsschaltungen der Wechselrichtern verwendeten Kreisen läßt sich auch für anderen Starkstromkreisen verallgemeinern unter Aufbewahrung der Vorteile im Bezug auf den klassischen Schemen.

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Rechenanlagen

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METODĂ DE SINTEZĂ A CIRCUITELOR LINIARE SECVENȚIALE FOLOSITE LA COMANDA INVERTOARELOR

(Rezumat)

Se sintetizează circuitul secvențial liniar folosit pentru generarea intervalelor de conducție ale tiristoarelor principale. Schema de comandă completă rezultă imediat ținînd cont de prescripțiile și soluțiile propuse în [3], [4].

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D.K. 621.3.018.14 ; 621.314.6

DIE ERHÖHUNG DES TOTALEN LEISTUNGSFAKTORS EINES SELBSTGEFÜHRTEN STROMRICHTERS

VON

D. ALEXA, M. ANTONESCU und LUCIUS DUMITRESCU

1. Einleitung

Stromrichteranlage zur Gleich-, Wechsel- und Umrichtung des Netzstromes erzeugen Oberschwingungsströme und benötigen die Blindleistung des Speisenetzes.

Zur Vergrößerung des totalen Leistungsfaktors wurden Schaltpläne sechspulsiger Stromrichter mit Nullanode, mit unsymmetrischer Steuerung, bestehend aus zwei Stromrichterbrücken in Reihenschaltung entworfen. Die Blindleistung verringert sich dabei um 40 ... 50% gegenüber der Drehstrombrückenschaltung. Solche Schaltungen führen aber nicht zur Herabsetzung der Stromoberschwingungen.

Es werden auch Mittel verwendet, ausserhalb der Stromrichteranlage für den Ausgleich der Blindleistung und der Verzerrungsleistung zu sorgen, wie z. B. spezielle synchrone Ausgleicher, Kondensatorbatterien und Filteranlagen für Oberschwingungen.

Den Netzen mit sehr schneller Änderung des Blindleistungbedarfes werden zusätlich Kompensationsverfahren mit sättigbaren Drosselspulen und mit Thyristorsteuerung eingesetzt.

Diese Kompensationsverfahren benötigen größere Investitionen, können aber die Blindleistung auf Grund schneller Schwankungen nicht gänzlich ausgleichen.

Die Zwangskommutierung der Thyristoren ermöglicht die Herstellung von Stromrichter mit Impulsfunktionierung, während die aus dem Speisenetz geforderte oder abgegebene Blindleistung nahezu völlig herabgesetzt wird.

Das Funktionierungsprinzip des selbstgeführten Stromrichters die weiter Diskutiert wird ist in der Arbeit [5] angegeben.

2. Der Strom aus dem Speisenetz

Im prinzipiellen Schema des selbstgeführten Stromrichters wird angenommen daß die Phasenspannungen u_{10}, u_{20} und u_{30} des Speisenetzes ein symmetrisches Drehphasensystem bilden.

Die Stromform in den einzelner Phasen des Speisenetzes ist nicht sinusförmig und für Phase R ist sie im Abb. 1 wiedergegeben.

Die Gleichung dieses periodischen Stromes ist eine F o u r i e r sche Reihe:

$$i_R(t) = \sum_{\nu=1}^{\infty} \sqrt{2}\, I_{\nu k} \sin \nu\omega t, \tag{1}$$

worin ω die Netzwinkelfrequenz ist.

Der Effektivwert der Harmonischen ν-ter Ordnung des Speisestromes ist:

$$|I_{\nu k}| = \left| \frac{8 \sin^2 \nu \frac{\pi}{2} \cos \nu \frac{\pi}{6}}{\sqrt{2}\, \nu\pi} \left[\sin \nu \frac{\pi}{6} - \sin \nu(1-k) \frac{\pi}{6} \right] I_d \right|, \tag{2}$$

wobei k die relative Einschaltdauer der Stromrichters ist.

Aus dem Ansatz der Harmonischen $I_{\nu k}$ kan man schliessen daß für den selbstgeführten Stromrichter keine geradzahligen Harmonischen und Vielfache von drei im Netzstrom vorkommen.

Der Effektivwert des Grundschwingungsstromes bei voller Steuerung ist $(k=1)$:

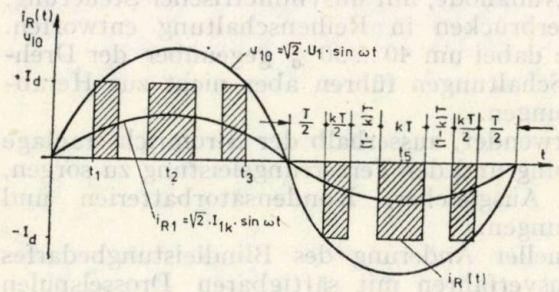

Abb. 1.—Stromform des Speisenetzes.

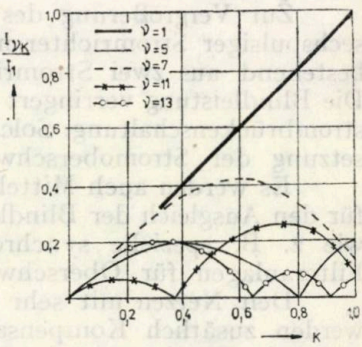

Abb. 2.—Abhängigkeit der Effektivwerte der Oberschwingungen des Speisestromes vom der relativen Einschaltdauer, k.

$$I_1 = \frac{\sqrt{6}}{\pi} I_d. \tag{3}$$

Im Abb. 2 sind die Effektivwerte der Oberschwingungen gezeigt, gleichzeitig mit Effektivwert des Grundschwingungsstromes I_1 für $k=1$, als Funktion der relativen Einschaltdauer des Stromrichters.

3. Ungeglättete Gleichspannung

Die zeitliche Änderung der Ausgangsspannung des Stromrichters ist im Abb. 3 für den Fall eines stationären Arbeitsregime des Stromrich-

ters wiedergegeben. Aus diesem Abbildung ersichtlich daß am System Motor—Filterinduktivität Spannungsimpulse variabler Amplitude und Zeitdauer kT anliegen und die unfiltrierte Gleichspannungs - Mittelwert und den Wechselspannungen mit höheren Frequenzen, die Vielfache 6.ter Ordnung der Frequenz des Speisentzes sind, zusammen.

Der Gleichspannungs-Mittelwert ist :

$$(4) \quad U_{dkm} = \frac{3\sqrt{6}}{\pi}\left[1-2\sin(1-k)\frac{\pi}{6}\right]U_1 .$$

Die Änderung der mittleren Gleichspannung U_{dkm} im Abhängigkeit von k ist nahezu linear.

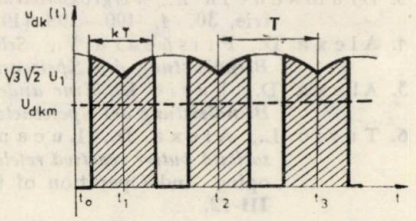

Abb. 3.—Ungeglättete Gleichspannung.

4. Verzerrungsleistung

Die Wirkleistung die der Stromrichter dem Netze abnimmt ist :

$$P = 3\,U_1\,I_{1k}\cos\varphi_1 = 3U_1 I_{1k}.$$

Berücksichtigend (2) erzielt man :

$$(5) \quad P = \frac{3\sqrt{6}}{\pi}\,U_1\,I_d\left[1-2\sin(1-k)\frac{\pi}{6}\right],$$

worin $\cos\varphi_1 = 1$ den Verschiebungsfaktor bedeutet.

Aus Abb. 1 ist der Effektivwert des Speisenetzstromes zu folgern :

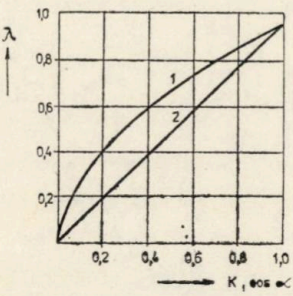

Abb. 4. — Totale Leistungsfaktor in Abhängigkeit von k oder cos α.

$$(6) \quad I_{\text{eff}} = \sqrt{\frac{1}{6T}\left(4\frac{kT}{2}I_d^2 + 2kTI_d^2\right)} = \sqrt{\frac{2k}{3}}\,I_d .$$

Der totale Leistungsfaktor des Stromrichters ist :

$$(7) \quad \lambda = \frac{P}{\sqrt{P^2+D^2}} = \frac{3\,U_1\,I_{1k}}{3\,U_1\,I_{\text{eff}}} = \frac{3}{\pi\sqrt{k}}\left[1-2\sin(1-k)\frac{\pi}{6}\right],$$

worin D im Speisenetz bezeugte die Verzerrungsleistung darstellt.

Die Änderung von λ nach Gleichung (7) im Abhängigkeit von k, ist im Abb. 4 durch Kurve *1* wiedergegeben. Zum Vergleich wurde im selben Abbildung durch Kurve *2* die Abhängigkeit des totalen Leistungsfaktors, für einem ideal netzgeführten Stromrichter in Drehstrombrückenschaltung eingezeichnet.

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Lehrstuhl für Elektronik und Rechenanlage

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ÎMBUNĂTĂŢIREA FACTORULUI DE PUTERE TOTAL LA UN REDRESOR CU COMUTAŢIE FORŢATĂ

(Rezumat)

Se prezintă formele de undă ale tensiunii redresate nefiltrate şi ale curentului de reţea şi se efectuează analiza spectrului de armonici superioare ale acestui curent pentru redresorul comandat descris în lucrarea [5]. Se deduce apoi expresia factorului de putere total al acestui redresor.

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D.K. 621.314.5

EINE VARIANTE VON UMRICHTER SEMIVERTER MIT DEN PHASENAUSGANGSSPANNUNGEN IN TREPPENFORM MIT 12 PULSEN

VON

D. ALEXA, ROMAN SIREŢCHI, M. ANTONESCU und I. BOGDAN

1. Einleitung

Umrichter für steuerbare Drehstromantriebe werden vornehmlich dann angewendet, wenn Aufgaben mit besonderen, technologischen Bedingungen zu lösen sind. Einige dieser Voraussetzungen sind: wartungsfreier Dauerbetrieb, Verzicht auf Maschinenübertragungsglieder, extrem hohe Drehzahl, niedertourige Drehbewegung, robuste Antriebe für aussetzenden Betrieb, aggressive Umweltbedingungen.

Einer von den am meisten verbreiteten statischen Frequenzumrichtern ist auch der Umrichter Semiverter. Dieser Umrichter wird besonders bei der Speisung von Mehrmotorenantrieben angewendet und hat eine Ausgangsspannung in Treppenform mit 6 Pulsen mit einem Grundschwingungsgehalt von 0,955.

Mit der im folgenden beschriebenen Variante des Umrichters Semiverter erhält man regelbare Ausgangsfrequenzen von 1...300 Hz und Ausgangsspannungen mit einem Grundschwingungsgehalt von 0,988.

2. Funktioniorungsprinzip

Entsprechend Abb. 1 ist der statische Frequenzumrichter aus folgenden Komponenten zusammengesetzt: Netzstromrichter, Zwischenkreisschaltung, Wechselrichter und 6 Nebenthyristoren (T'_a, T''_a, T'_b, T''_b, T'_c, T''_c). Der Netzstromrichter ist bei nur motorischer Betriebsweise der Belastungsmaschinen als einfache Drehstrombrücke ausgeführt, bei motorischer und generatorischer Betriebsweise kann er jedoch durch eine antiparallele Drehstrombrücke ergänzt werden. Der Zwischenkreis besteht aus zwei gleichen Glättungsdrosselspulen L und zwei gleichen Glättungskondensatoren C zur Entkopplung des Netzstromrichters vom Wechselrichter.

Die Kommutierungseinrichtung des Wechselrichters enthält zwei zentrale Kondensatoren, C_1 und C_2. Kondensator C_1 erlaubt nur die Löschung der Hauptthyristoren $T_1 ... T_6$, hingegen Kondensator C_2 nur die der Nebenthyristoren. Um ein großes Regulierungsverhältnis der Ausgangsspannung zu erreichen, werden die Löschkondensatoren C_1 und C_2 von zwei Gleichspannungsquellen E' und E'' geladen [1], [2].

Der Netzstromrichter SR ermöglicht die Variation der Mittelgleichspannung U_d, beim Eingang des Wechselrichters und dieser modifiziert die Ausgangsfrequenzen f_1.

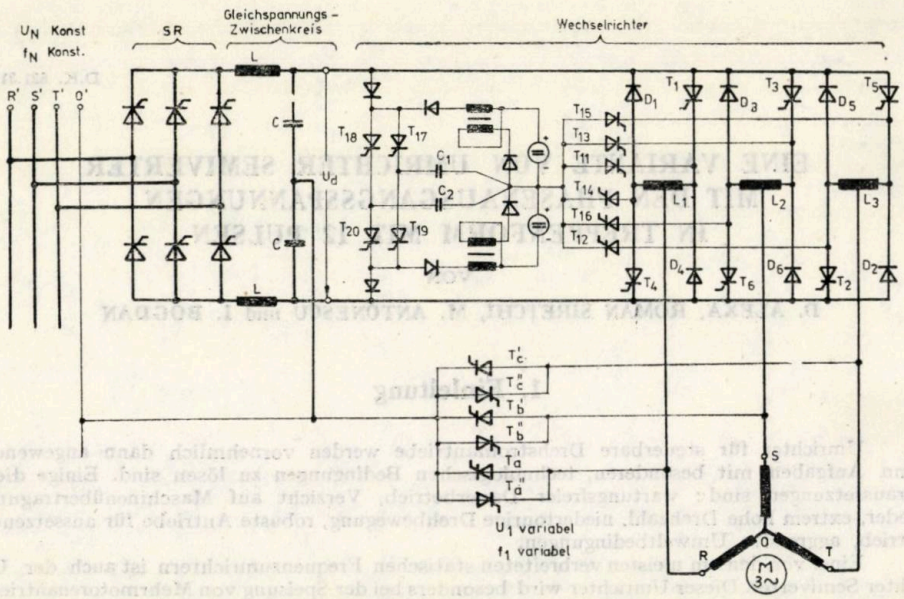

Abb. 1. — Prinzipielles Schema des statischen Frequenzumrichter.

Weiter wird die Funktion des Wechselrichters zwischen den Zeitpunkten t_0 und t_4 laut Abb. 1 und 2 beschrieben. Man nimmt den Fall, wenn der Wechselstrommotor für den Wechselrichter eine Belastung mit induktiv-resistiven Charakter hat, also die Statorströme eilen den Spannungen nach.

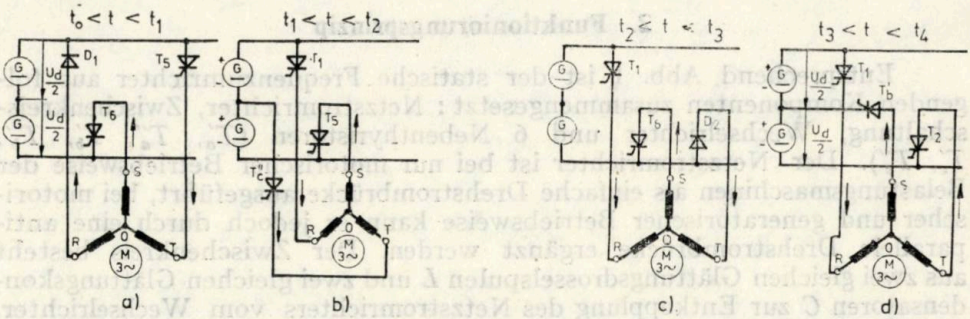

Abb. 2. — Stromlaufschema im Wechselrichter für $t_0<t<t_4$: a)...d) für verschiedene Zeitpunkte.

Vor dem Zeitpunkt t_0 sind die Hauptthyristoren T_5 und T_6 und der Nebenthyristor T'_a leitend.

Zum Zeitpunkt t_0 kommt von der Steuerung der Löschung für den Nebenthyristor T'_a durch Zündung der Löschthyristoren T_{14} und T_{20}.

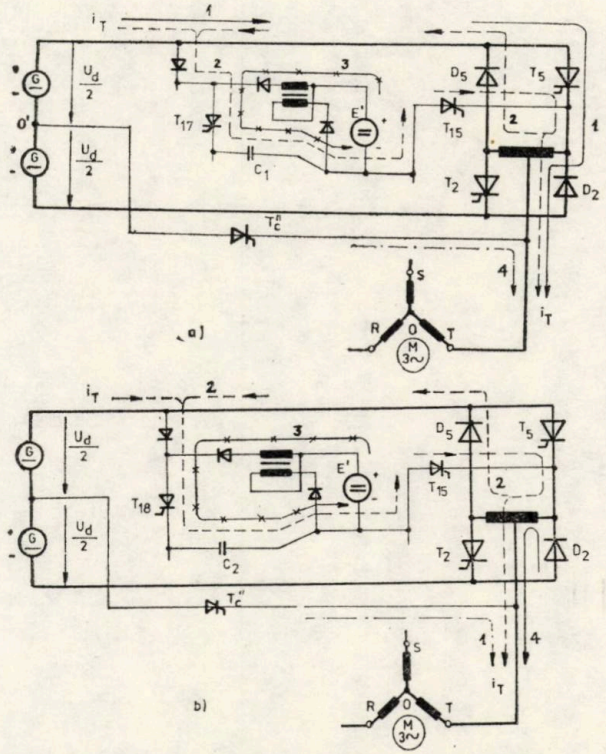

Abb. 3. — Die Ströme für verschiedene Zeitintervalle:
a) 1 — im Zeitintervall $t_0+\Delta t'<t<t_1$; 2 — im Löschprozeß; 3 — Ladestrom des Kondensators C_1; 4 — im Zeitintervall $t_1+\Delta t''<t<t_2$.
b) 1 — im Zeitintervall $t_1+\Delta t''<t<t_2$; 2 — im Löschprozeß; 3 — Ladestrom des Kondensators C_2; 4 — im Zeitintervall $t_2+\Delta t'<t<t_3$.

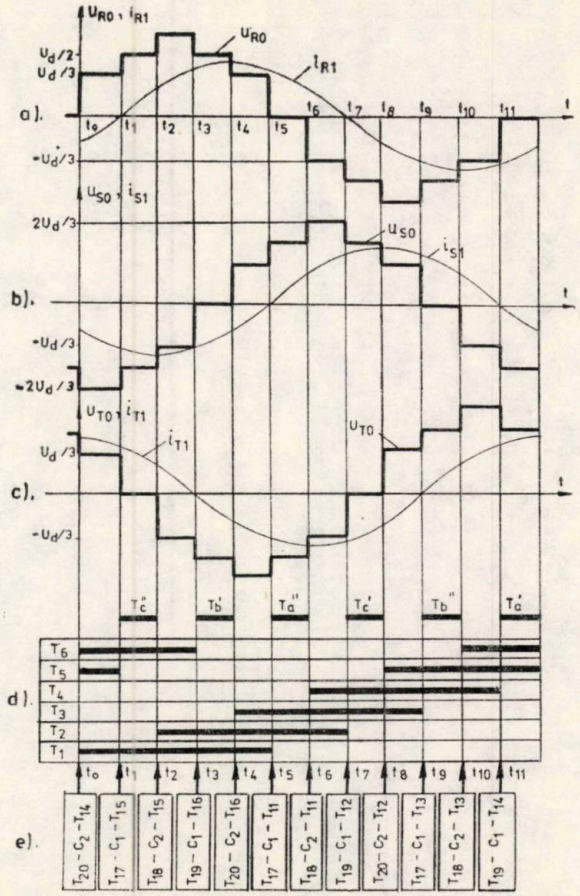

Abb. 4. — a), b), c) Ausgangsspannungen u_{R0}, u_{S0} und u_{T0}; d) Programm der Einschaltdauer der Haupt- und Nebenthyristoren; e) Löschthyristoren und Kondensatoren, die in die Löschprozeße der Haupt- und Nebenthyristoren eingreifen.

Abb. 4. — a) b) c) Anlegungsmöglichkeiten von n- und p-Dioden; d) Diagramm der Einschaltdauer der Haupt- und Zusatzventile; e) Lösungsprinzip der Haupt- und Nebenventile.

Abb. 5. — Die Stufen der gesteuerten Zeitumkehr.

a) 1 — im Zeitintervall $t_0 \div \Delta t < t < t_1$; 2 — im Intervall $t_1 \div \Delta t < t < t_2$; 3 — Ladekondensatoren C_1; 4 — im Zeitintervall $t_1 \div \Delta t < t < t_2$.

b) 1 — im Zeitintervall $t_1 + \Delta t < t < t_2$; 2 — im Intervall $t_2 \div \Delta t < t < t_3$; 3 — Ladekondensatoren C_2; 4 — im Zeitintervall $t_2 + \Delta t < t < t_3$.

Der Kondensator C_2 aus dem Kreis $T_{14}-C_2-T_{20}$ wurde bei einer Polarität geladen, die dieses Löschung zuläßt. Der Strom, der durch Phase R fließt, wird am Ende des Kommutierungsprozeßes auf Diode D_1 umgeschaltet, nach Ablauf einer Zeitdauer von $\Delta t'$.

Im Zeitintervall zwischen $t_0+\Delta t'$ und t_1 werden Steuerimpulse in Form von Impulsreihen den Hauptthyristoren T_1, T_5 und T_6 gegeben. Folglich sind die Spannungen an den Klemmen R, S und T: $+U_d/3, -2U_d/3$ und $+U_d/3$, laut Abb. 2 *a*. So lange der Strom, der durch die Phase R fließt, die im Abb. 2 *a* gezeigte Richtung hat, wird die Diode D_1 offen sein. Nachdem dieser Strom den Nullpunkt durchläuft, zündet der Thyristor T_1 und der Strom wechselt die Richtung.

Zum Zeitpunkt t_1 werden die Löschthyristoren T_{15} und T_{17} gezündet, um den Hauptthyristor T_5 zu löschen. Der Strom aus Phase T wird zuerst auf den Kreis $T_{17}-C_1-T_{15}$ umgeschaltet, dann auf Diode D_2, entsprechend Abb. 3 *a*. Zum Zeitpunkt $t_1+\Delta t''$ (wobei $\Delta t''$ ungefähr gleich der Dauer des Kommutierungsprozeßes zwischen T_5 und D_2 ist) wird ein Zündungsimpuls für T_c'' gegeben. Man kann feststellen, daß $\Delta t'$ ungefähr gleich $\Delta t''$ ist. Durch Zündung von T_c'' wird die Diode D_2 blockiert. Der Steuerbefehl für T_c'' kann auch früher durch einen Impuls von längerer Dauer gegeben werden, derart, daß die Diode D_2 nicht leitfähig wird. Am Ende des Kommutierungsprozeßes zwischen T_5 und T_c'' ist der Kondensator C_1 von Quelle E' mit einer Ladung geladen, die eine entgegengesetzte Polarität gegenüber der vorherigen dieses Prozeßes hat.

Im Zeitintervall zwischen $t_1+\Delta t''$ und t_2 werden Steuerimpulse in Form von Impulsreihen nur den Hauptthyristoren T_1 und T_6 gegeben. Also werden die Spannungen an den Klemmen R, S und T : $+U_d/2, -U_d/2$ und Null sein, laut Abb. 2 *b*.

Zum Zeitpunkt t_2 wird das Löschung des Thyristors T_c'' durch die Zündung der Thyristoren T_{15} und T_{18} befohlen. Der Strom, der durch die Phase T fließt, wird zuerst auf den Kreis $T_{18}-C_2-T_{15}$ umgeschaltet, dann, am Ende des Kommutierungsprozeßes, auf Diode D_2, entsprechend Abb. 3 *b*. Im Zeitintervall zwischen $t_2+\Delta t'$ und t_3 werden Steuerimpulse in Form von Impulsreihen den Hauptthyristoren T_1, T_2 und T_6 gegeben, und die Spannungen an den Klemmen R, S und T sind : $+2U_d/3, -U_d/3$ und $-U_d/3$ laut Abb. 2 *c*.

Schließlich wird zum Zeitpunkt t_3 die Löschung des Thyristors T_6 befohlen und zwar durch Zündung des Thyristoren T_{16} und T_{19}, zwischen denen sich der entsprechend geladene Kondensator C_1 befindet. Zum Zeitpunkt $t_3+\Delta t''$ wird der Nebenthyristor T_b' gezündet. Also sind die Spannungen an den Klemmen R, S und T im Zeitintervall zwischen $t_3+\Delta t''$ und t_4 : $+U_d/2$, Null und $-U_d/2$ laut Abb. 2 *d*. Im Zeitpunkt t_4 wird der Nebenthyristor T_b' durch Zündung der Löschthyristoren T_{16} und T_{20} gelöscht.

Das Steuerprogramm mit den Impulsreihen für die Hauptthyristoren des Wechselrichters, für eine ganze Periode der Ausgangsspannung ist in Abb. 4 *d* dargestellt. Die Nebenthyristoren T_a', T_a'', T_b', T_b'', T_c' und T_c'' können nur mit einem Impuls am Anfang ihres Leitintervalls gesteuert werden, laut dem Diagramm aus Abb. 4 *d*. Im Falle von nur ohmischer Belastung sind diese Thyristoren nicht leitend.

Die Schwingungsform der Phasenausgangsspannungen u_{R0} u_{S0}, u_{T0} und der Grundschwingungsströme i_{R1}, i_{S1}, i_{T1} sind in den Abb. 4 *a*, *b* und *c* dargestellt. Durch das Steuerprogramm, entsprechend Abb. 4 *d*, ist es möglich Ausgangsspannungen in Treppenform mit 12 Pulsen zu erhalten. Der Grundschwingungsgehalt der Ausgangsspannungen, wiedergegeben in den Abb. 4 *a*, *b* und *c*, beträgt 0,986. In Abb. 4 *e* wird die Löschthyristoren und Kondensatoren angegeben, die in die Löschprozeße der Haupt- und Nebenthyristoren eingreifen.

Eine Besonderheit des Wechselrichters aus Abb. 1 ist, daß die Hauptthyristoren eine relative Einschaltdauer von 150° haben, laut Abb. 4 *d*. Aus diesem Grund folgt eine Verkleinerung der Mittelwerte der Ströme, die durch diese Thyristoren fließen.

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Technische Hochschule, Jassy,
Lehrstuhl für Elektronik und Rechenanlage

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VARIANTĂ DE CONVERTIZOR STATIC DE FRECVENŢĂ SEMIVERTER CU TENSIUNI DE IEŞIRE PE FAZE SUB FORMĂ DE TREPTE CU 12 PULSURI

(Rezumat)

Se analizează posibilitatea reducerii armonicilor curentului de alimentare al motoarelor asincrone cu turaţie reglabilă prin alimentarea acestora de la o variantă de convertizor de frecvenţă tip Semiverter, a cărui formă de undă a tensiunii de ieşire este necesar să fie cît mai apropiată de o lege sinusoidală. Se descrie principiul de funcţionare şi se efectuează analiza armonică a tensiunii de ieşire al cărei conţinut în fundamentală este de 0,986.

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D.C. 621-52

MICROSYSTEM WITH MICROPROCESSOR FOR DIGITAL CONTROL

BY

CORNELIU HUŢANU and DORU PĂNESCU

1. Introduction

The apparition of the microprocessors means the reappraisal of all application areas of semiconductors. The digital control systems require more the (V) LSI technology than any other area of application.

This revolution performed by the (V)LSI technology makes available the up-to-date information systems more saving and smaller than the former huge information systems. The advantage of these small information systems is that they are available for the users.

The materialization of some powerful and economical information automatons is made possible when we use a microsystem built around a microprocessor. The standard structure of a microsystem with a microprocessor is shown synthetically in Fig. 1.

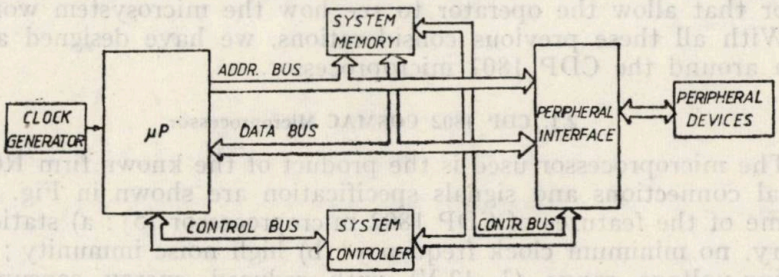


Fig. 1

Concretely the elements of a microsystem are determined both by the microprocessor (that requires the use of some certain additional circuits) and by the destination of the microsystem (which determines the use of some certain interface circuits and peripherals).

In the following it is described a microsystem with a microprocessor that permits the achievement of two important tasks: the study of the performances of a microsystem and the possibility of using it for digital control with flexible programs.

2. Microsystem Design with Microprocessor

The design of a microsystem differs from the traditional design of the digital systems. In this case we begin with the goal too, but the design includes two different aspects : the hardware and the software. Generally, the programmer is not interested in the hard when operating a digital computer ; but in the case of microsystems the design of the hard and the soft are interdependent.

When we design the hard of a microsystem we must take into account the following elements :

a) The memory is determined by the dimensions of the program, which must be defined approximatively at the begining.

b) The interface circuits are determined by the controlled elements (peripherals).

c) There must synchronize the microprocessor with the other elements of the microsystem because they have different speeds.

d) As the microsystem is programmable with many possibilities of application at a certain moment, these possibilities are used only partially. For this reason an expandition area must be alocated.

e) The operator must easily communicate with the microsystem (the introduction and control of the program, the possibility of obtaining some information from microprocessor towards exterior).

When we design the soft of a microsystem with a microprocessor, excepting the usual and known elements (the suitable algorithm, the flow chart and the program in machine-language or in superior language), we must take into account the following elements too : a) there must exist a monitor program to prevent the accidental state of the microsystem when it is powered on ; b) the program must include some communications to exterior that allow the operator to see how the microsystem works.

With all these previous considerations, we have designed a microsystem around the CDP 1802 microprocessor.

2.1. CDP 1802 COSMAC Microprocessor

The microprocessor used is the product of the known firm RCA. The external connections and signals specification are shown in Fig. 2. Here are some of the features of CDP 1802 microprocessor [5] : a) static CMOS circuitry, no minimum clock frequency ; b) high noise immunity ; c) wide operating-voltage range (3...12 V) with reduced energy consumption ; d) full temperature range (−55°...+125°C) ; e) 91 easy-to-use instructions ; f) TTL compatibility ; g) no monitor program ; h) any combination of standard RAM's and ROM's via common interface; i) direct memory addresing up to 65,536 bytes ; j) hexadecimal code instructions ; k) a 16×16 matrix of registers with multiple possibilities of use. For instance, the program counter can be changed from a register into another ; this constitutes an advantage for the programming (e.g. in using the subroutines).

Compared with the well known Intel 8080, which is usually thought to be a reference element [3], [6], the CDP 1802 has the following advantages :

a) Single-phase internal clock (Intel 8080 requires the additional circuit 8224).

b) DMA facility, very advantageous for programming (8080 doesn't have DMA).

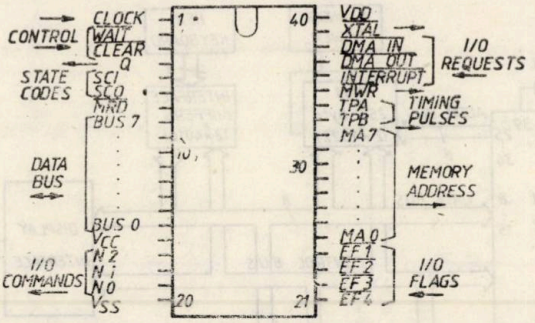


Fig. 2

c) Control bus wider than the Intel 8080 one that requires some more additional circuits to multiplex the control signals.

d) A single voltage-supply.

Because the instructions are executed in two cycles and the microprocessor is made in CMOS technology, the CDP 1802 works at a lower speed that the one of the Intel 8080. Also CDP 1802 operates only in the binary code, which is an incovenient if the BCD code is required (the Intel 8080 works also in BCD).

The CDP 1802 is advantageous too, as it permits the achievement of microsystems with simple peripherals (therefore the microsystem will be cheap). Thus, the existence of DMA facility and four different operating modes on $\overline{\text{CLEAR}}$ and $\overline{\text{WAIT}}$ lines permits the microsystem with CDP 1802 to have an easy communication with the operator (it is very easy to carry out the following tasks: run the program, reset the microsystem, pause, load the program).

2.2. The Hardware of the System

With all this considerations about the specific elements of the CDP 1802 COSMAC microprocessor and taking into account our aim — to materialize a microsystem for digital control, we have done the real design of the microsystem. For the beginning we supposed a 256 byte memory is sufficient. The necessity of experimentation made us use the RAM memories. The RAM's inconvenient of volatility was compensated by the use of the CMOS devices with a proper saving and protecting circuit. In Fig. 3 the structure of the designed microsystem is shown.

For the program introduction we made a two-range hexadecimal keyboard. Also, the conversation with the operator is made possible by a seven segments display device and an information LED. The interface with the process requires the use of adequate I/0 ports.

The microsystem also contains a control schema and four switches by which is possible to control the operation modes of the microsystem (RESET, RUN, LOAD, PAUSE).

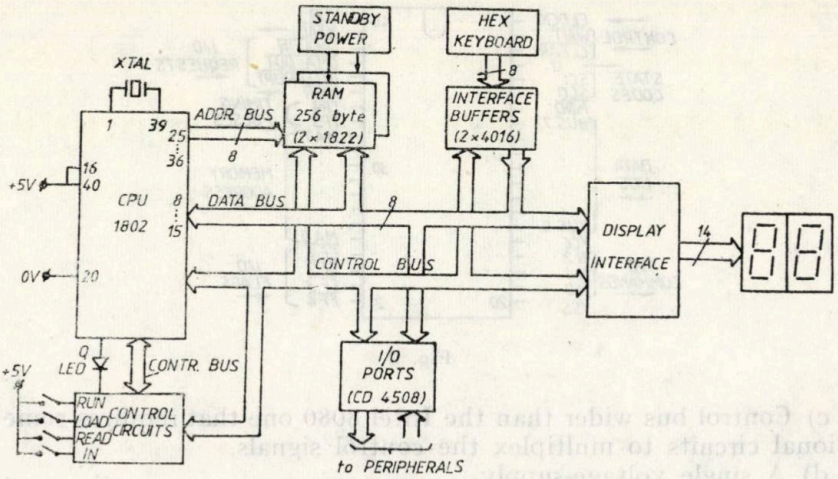
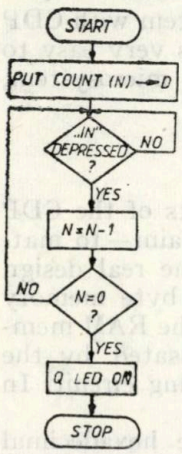


Fig. 3

2.3. The Software of the System

First, the soft of a microsystem with microprocessor must permit to test the main elements of the system. It must be possible that, through the software, to check the microprocessor, the memories and the interface of the microsystem with the operator and the process. Thus, the



a)

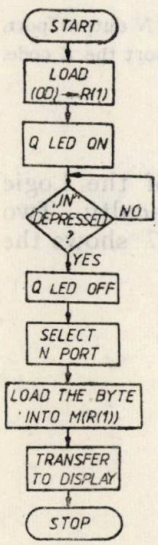
Fig. 4

ADDRESS	INSTRUCTION	COMMENTS
00	F8 N	— N is loaded in D register.
02	3F 02	— If „IN" is not depressed, go to 02 address.
04	37 04	— If it is depressed, is the depression maintained ? If true, go to 04 address.
06	FF 01	— If false, decrement N with 1.
08	3A 02	— If $N \neq 0$, go to 02 address.
OA	7B	— If $N = 0$, set Q on.
OB	30 OA	— Stop at the OA address.

Obs: Instruction of the 04 address was introduced to make sure the depression of IN switch is correctly understand by the microprocessor.

program shown in Fig. 4 is used to check the working state of the microprocessor and the interface with the operator. The flow chart (Fig. 4 *a*) shows a simple algorithm for checking the microsystem (by setting a number N — from 00 to FF — in the memory). If both microprocessor and the interface with the operator are right, when operating the INPUT switch (Fig. 3) for N times, the program is confirmed by setting on the LED Q. The list of the program in machine-language is shown in Fig. 4 *b*. This program permits the checking of the following elements : a) the accepting of the program by the memories ; also the program can be read out to be sure of its correctness ; b) the working state of the microprocessor in RUN mode ; c) the control LED.

The process interface requires the control of the information transfer through the I/O ports. As concerns the communication with the peripherals, the CDP 1802 is one of the most improved microprocessors, it has three different modes of communications: the programmed I/O communication, the interrupt I/O communication and the DMA facility. For check-



ADDRESS	INSTRUCTION	COMMENTS
00	F8 OD	— Load OD address into D register.
02	A1	— Load this address into R(1).
03	7B	— Set Q on.
04	3F 03	— If „IN" is not depressed, go to 03 address.
06	7A	— If „YES", set Q off.
07	62 N	— Select N port.
09	E1	— $1 \rightarrow X$.
OA	69	— Transfer the byte from N port to $M(R(1))$.
OB	EO	— $O \rightarrow X$.
OC*)	64	— The byte brought through the port, present in $M(R(1))$ location, is tansferred to display.
OE	30 OC	— Stop at OC address.

Fig. 5

*) The initial content of the OD location is arbitrary.

ing the ports, we used the programmed communication. The CDP 1802 has 7 different I/0 instructions. In this case the microsystem may use 7 input ports and 7 output ports. Using the decoders, the number of these ports can be increased. Fig. 5 shows the program for checking an input port selected through a decoder ; N is the number of the tested input port. We can easily notice that the program is controlled by the operator by using the INPUT switch and the Q LED. Fig. 6 shows the checking program for an output port. In this case, for more safety, an additional test is made. Thus, at the first depression of the switch IN the port is checked by the code K, and then, at the second depression by the code $\overline{K}$.

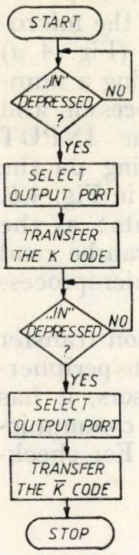


Fig. 6

ADDRESS	INSTRUCTION	COMMENTS
00	3F 00	— If „IN" is not depressed, go to 00 address.
02	37 02	— If „IN" is depressed, is the depression maintained ? If true, go to 02 address.
04	62 *N*	— If false, select the *N* output port.
06	61 *K*	— Transmit in this port the *K* code.
08	3F 08	— If „IN" is not depressed go to 08 address.
OA	37 OA	— If it is depressed, the depression is maintained ? If true, go to OA address.
OC	62 *N*	— If false, select the *N* output port.
OE	61 $\overline{K}$	— Transmit in this port the *K* code.
OF	00	— Stop.

3. Materialization. Checking and Conclusions

The microsystem was materialized in the Laboratory of the Logic Circuits and Digital Control of the Electrical Engineering Faculty. Two CDP 1822 RAM's with protection circuit were used. Fig. 7 shows the

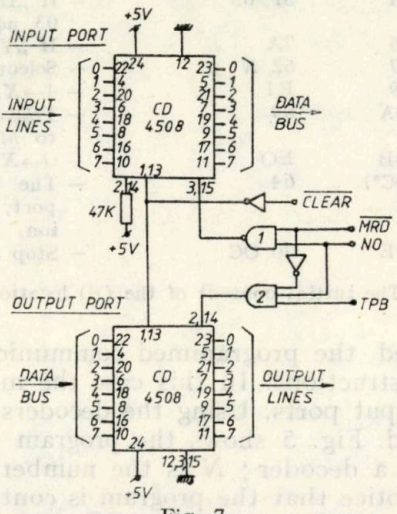


Fig. 7

I/0 interface in which the connections of two I/0 ports are indicated. As the asynchronous data intrance mode is accepted, the synchronized inputs of the IN ports (2,14) are connected to +5 V. In order to write the

data in the memory, the logical condition $\overline{MRD} \cdot NO = 0$ must be satisfied. For this reason the inputs 3 and 15 are drived by the NAND 1 circuit (Fig. 7). In this mode, under the control of the microprocessor it is easy to set the external information into the memory of the microsystem by means of the input ports.

The output port is activated only by the output instructions. As the line NO is used with logic 1 too, by means of the AND 2 circuit (Fig. 7), the condition $MRD \cdot NO \cdot TPB = 1$ is satisfied. For the control of the peripherals, a permanent reading of the output ports is made by connecting the pins 3 and 15 to the ground (activating the outputs of the port).

By means of the INPUT, RUN, LOAD switches and the hex keyboard, the programs shown in Figs. 4...6 were introduced and checked succesively.

The microsystem has given a full satisfaction and has permitted the materialization of a digital control system which will be described later.

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Utilisations and Automation

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MICROSISTEM CU MICROPROCESOR PENTRU COMANDĂ NUMERICĂ

(Rezumat)

Se prezintă un microsistem realizat în jurul microprocesorului CDP 1802 COSMAC (RCA). Sistemul a fost realizat cu dublu scop : de studiere a performanțelor unui microsistem cu microprocesor și de dezvoltare a acestuia în sensul realizării unor comenzi numerice cu program flexibil. Sînt prezentate partea de hardware precum și programele folosite la verificarea microsistemului.

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METODĂ ALGEBRICĂ PENTRU SINTEZA SISTEMELOR AUTOMATE LINIARE CU MODULAȚIE IN AMPLITUDINE

DE

CORNELIU BOȚAN

1. Pornind de la posibilitatea reprezentării mișcărilor modulate prin elemente ale unor algebre peste corpul real [1], în cîteva lucrări anterioare [2], [3] s-au indicat modalități de analiză a sistemelor automate liniare cu modulație în amplitudine pe baza acestei metode. Avînd la bază relațiile stabilite în cadrul analizei, în cele ce urmează se evidențiază unele posibilități de sinteză a acestor sisteme.

2. În cadrul sistemelor automate cu modulație în amplitudine există elemente străbătute de semnale modulate, cel mai des fiind vorba de modulația armonică simplă. Întrucît metoda simbolică se pretează studiului frecvențial se vor considera doar semnale de formele

$$x(t)=A\sin pt, \tag{1}$$

$$x(t)=A_1\sin(pt+\psi)\sin(\omega t+\varphi)+A_2\cos(pt+\psi)\cos(\omega t+\varphi)= \tag{2}$$
$$=a_1\sin pt\cos\omega t+a_2\cos pt\sin\omega t+a_3\sin pt\cos\omega t+a_4\cos pt\cos\omega t,$$

în care ω reprezintă pulsația purtătoarei, iar p — pulsația semnalului util.

Astfel de funcții sînt reprezentabile simbolic [1], [2] prin elemente $\hat{w}$ ale unor algebre peste corpul real, astfel încît derivatei $dx(t)/dt$ să-i corespundă elementul de algebră $d\hat{w}$, în care operatorul de derivare d este element al aceleiași algebre.

Pentru funcții de forma (1), reprezentarea se poate realiza prin elemente de forma:

$$\hat{w}=w_1\varepsilon_1+w_2\varepsilon_2 \tag{3}$$

ale unei algebre de ordinul 2, cu tabela de înmulțire:

$$\varepsilon_1^2=\varepsilon_1,\qquad \varepsilon_1\varepsilon_2=\varepsilon_2\varepsilon_1=\varepsilon_2,\qquad \varepsilon_2^2=-\varepsilon_1, \tag{4}$$

iar operatorul de derivare este

$$\mathrm{d}=p\varepsilon_2. \tag{5}$$

Funcțiile (2) sînt reprezentabile prin elemente de forma:

$$\hat{w} = \sum_{i=1}^{4} w_i \varepsilon_i, \tag{6}$$

ale unei algebre de ordinul 4, produs direct a două algebre de ordin 2 de forma (4). Operatorul de derivare este

$$\mathrm{d} = p\varepsilon_2 + \omega\varepsilon_3, \tag{7}$$

iar tabela de înmulțire este

(8)

	ε_1	ε_2	ε_3	ε_4
ε_1	ε_1	ε_2	ε_3	ε_4
ε_2	ε_2	$-\varepsilon_1$	ε_4	$-\varepsilon_3$
ε_3	ε_3	ε_4	$-\varepsilon_1$	$-\varepsilon_2$
ε_4	ε_4	$-\varepsilon_3$	$-\varepsilon_2$	ε_1

3. În cadrul sistemelor automate cu modulație în amplitudine există cel puțin un ansamblu format dintr-un modulator, element liniar și demodulator (MELD). Particularitatea problemelor de analiză și sinteză a acestor sisteme este legată tocmai de ansamblul MELD. În problemele de sinteză, atunci cînd se folosesc circuite de corecție de curent alternativ, rezolvarea decurge — în prima etapă — ca la sistemele obișnuite, de curent continuu : pornind de la partea fixată a sistemului și avînd în vedere cerințele care se impun, se deduce o funcție de transfer (f.d.t.) $G(s)$ a ansamblului MELD, care asigură rezolvarea problemei. Etapa specifică de sinteză a acestor sisteme o constituie determinarea f.d.t., $W(s)$, a elementului liniar din ansamblul MELD, cunoscînd f.d.t. $G(s)$ a acestui ansamblu.

Calculele se desfășoară în succesiunea inversă celor din cadrul analizei. În analiza efectuată prin metoda simbolică, pornind de la f.d.t. $W(s)$, se stabilește transferanța de ordinul 4, $\hat{w} = \sum_{i=1}^{4} w_i \varepsilon_i$, ca element al algebrei de ordinul 4 definite în §2 ; se stabilește apoi transferanța de ordin 2 (echivalentă cu factorul de amplificare complex) a ansamblului MELD ; în ipoteza utilizării unor modulatoare și demodulatoare ideale, rezultă în final

$$\begin{aligned} G(\mathrm{j}p) = g_1 + \mathrm{j}g_2 = \frac{1}{2}(w_1 + \mathrm{j}w_2)\cos\delta + \frac{1}{2}(w_3 + \mathrm{j}w_4)\sin\delta = \\ = \frac{1}{2}G'(\mathrm{j}p)\cos\delta + \frac{1}{2}G''(\mathrm{j}p)\sin\delta, \quad (\mathrm{j}^2 = -1), \end{aligned} \tag{9}$$

în care $G' = w_1 + \mathrm{j}w_2$, $G'' = w_3 + \mathrm{j}w_4$, iar δ este defazajul între tensiunile de bază ale modulatorului și demodulatorului. De multe ori $\delta = 0$ și atunci

$$G(\mathrm{j}p) = G'(\mathrm{j}p)/2.$$

Aplicarea relației (9) într-un caz general, pentru rezolvarea problemei de sinteză, este dificilă. Din acest motiv este util să se apeleze la descompunerea expresiilor f.d.t. $W(s)$ și $G(s)$ în fracții elementare și să se stabilească relațiile între aceste forme elementare. În adevăr, orice f.d.t. avînd o expresie rațională, cu gradul numitorului n, se poate scrie sub forma

$$W(s)=\sum_{i=1}^{n}\frac{b_i}{s+a_i} \tag{10}$$

unde $-a_i$ sînt polii f.d.t. $W(s)$.

Fiecărei expresii elementare de forma $b_i/(s+a_i)$ îi corespunde o formă tipică $\tilde{G}_i(s)$, iar în final rezultă $G(s)=\sum_{i=1}^{n}\tilde{G}_i(s)$.

Aplicînd regulile de determinare a componentelor w_i, $(i=1, 2, 3, 4)$ [2], rezultă că pentru o transferanță elementară de forma

$$W(s)=\frac{b}{s+a} \tag{11}$$

componentele G' și G'' din (9) au expresiile :

$$\begin{cases} G'(jp)=b\dfrac{a(a^2+p^2+\omega^2)-jp(a^2+p^2-\omega^2)}{(a^2+\omega^2-p^2)^2+4a^2p^2}, \\ G''(jp)=b\dfrac{-\omega(a^2-p^2+\omega^2)+j2a\omega p}{(a^2+\omega^2-p^2)^2+4a^2p^2}, \end{cases} \tag{12}$$

respectiv

$$G'(s)=b\frac{a+s}{a^2+\omega^2+2as+s^2}, \quad G''(s)=\frac{\omega b}{a^2+\omega^2+2as+s^2}. \tag{13}$$

Avînd în vedere că $p\ll\omega$, în calcule se pot folosi relațiile simplificate :

$$G'(s)=b\frac{a+s}{a^2+\omega^2+2as}, \quad G''(s)=\frac{\omega b}{a^2+\omega^2+2as}. \tag{14}$$

Dacă se efectuează descompunerea în fracții elementare și se utilizează corespondența între forma (11) și formele (13), în rezolvarea problemei de sinteză se poate evita etapa intermediară a determinării elementelor w_i ale transferanței $\hat{w}$. Asupra acestui procedeu de calcul se vor face în continuare cîteva precizări.

4. Pentru simplificarea calculelor sînt preferate formulele (14) și în acest caz expresia (9) conduce la :

$$G(s)=\frac{1}{2}\sum_{i=1}^{n}y_i\frac{s+a_i+\omega\,\mathrm{tg}\,\delta}{s+x_i}, \tag{15}$$

unde

$$y_i=\frac{b_i\cos\delta}{2a_i}, \quad x_i=\frac{a_i^2+\omega^2}{2a_i}.$$

Identificînd membrul drept al relației (15) cu forma impusă

$$G(s)=\frac{1}{2}\cdot\frac{\sum_{i=0}^{n}B_is^i}{\sum_{i=0}^{n}A_is^i}, \tag{16}$$

cu A_i și B_i constante, se determină mai întîi valorile x_i și de aici:

$$a_i = x_i \pm \sqrt{x_i^2 - \omega^2} \tag{17}$$

(se observă că $G(s)$ trebuie astfel impus încît să rezulte $x_i > \omega$).

Apoi, din sistemul format de $n+1$ ecuații se determină cele $n+1$ necunoscute $y_1, \ldots, y_n$ și tg δ; în final se stabilesc valorile pentru constantele b_i și astfel se găsește f.d.t. $W(s)$ dată de expresia (10).

De obicei este de dorit anularea componentei în cuadratură $G''(s)$. Întrucît această condiție este greu realizabilă se poate impune anularea sau minimizarea ei în regim staționar, cînd influența ei este cea mai nefavorabilă:

$$\lim_{s\to 0} G''(s) = 0 \quad \text{sau} \quad \left|\lim_{s\to 0} G(s)\right| = \min. \tag{18}$$

5. De multe ori este convenabil ca pentru sinteză să se utilizeze alte corespondențe între anumite forme elementare $G(s)$ și $W(s)$. De exemplu, corespondența

$$W(s) = \frac{s^2 + 2bs + \omega^2}{s^2 + 2as + \omega^2} \quad \leftrightarrow \quad G(s) = \frac{1}{2} \cdot \frac{s+b}{s+a} \tag{19}$$

este utilă întrucît permite calculul direct al unui circuit de corecție derivativ.

6. În unele probleme de sinteză se impune de la început structura circuitului de corecție, urmînd a se stabili unul sau mai mulți parametri ai acestuia. În astfel de cazuri se impune de obicei extremizarea unui indice de calitate. Calculele decurg similar cu cele prezentate în paragrafele precedente, utilizînd corespondențe de tipul celor indicate.

Exemplu. Se consideră un servomecanism de poziție pentru care partea fixată are f.d.t. $H(s) = K_m/[s(\tau_m s + 1)]$; se impune introducerea unui circuit de corelație de curent alternativ, astfel încît ansamblu MELD să asigure o compensare de tip derivativ, cu f.d.t. $G(s) = \frac{1}{2} \cdot \frac{\tau_1}{T} \cdot \frac{Ts+1}{\tau_1 s + 1}$. Cunoscînd ceilalți parametri ai sistemului, se cere determinarea constantei T și ulterior a circuitului corespunzător, care să asigure minimizarea indicelui de calitate

$$I = \int_0^\infty \varepsilon^2(t)\,dt,$$

$\varepsilon(t)$ fiind eroarea sistemului.

Expresia imaginii erorii în funcție de imaginea mărimii de intrare $x_i(s)$ este

$$\varepsilon(s) = x_i(s) \frac{1}{1 + Y(s)}.$$

unde $Y(s) = \frac{K(Ts+1)}{s(\tau_1 s + 1)(\tau_m s + 1)}$ este f.d.t. a circuitului deschis al sistemului (s-a notat $K = \frac{K_1}{T} = \frac{1}{T} \cdot \frac{\tau_1 K_m}{2}$). Utilizînd valorile tabelate [4], calculate pe baza integralei lui Parseval, pentru semnal de intrare treaptă rezultă:

$$I = \frac{K(K_1 + \rho^2 - 1) + \rho}{2K[\rho(K_1 + 1) - K]} \quad \text{cu} \quad \rho = \frac{\tau_1 + \tau_m}{\tau_1 \tau_m}.$$

Minimul acestei expresii se obţine pentru

$$K=\frac{\rho}{K_1+\rho^2-1}\left(-1+\sqrt{K_1^2+\rho^2+K_1\rho^2}\right)$$

şi de aici se deduce $T=K_1/K$.

În continuare, utilizînd corespondenţa (19), se deduce imediat f.d.t. $W(s)$ a circuitului de corecţie de curent alternativ.

Concluzii

1. Formularea şi rezolvarea problemelor de sinteză pentru sistemele automate cu modulaţie în amplitudine este similară cu cea a sistemelor obişnuite ; etapa specifică în astfel de cazuri o reprezintă stabilirea modelului matematic pentru elementul liniar care funcţionează la frecvenţă purtătoare, pornind de la modelul matematic al ansamblului modulator—element liniar—demodulator.

2. Utilizarea metodei simbolice de reprezentare a mişcărilor modulate prin elemente ale unor algebre peste corpul real permite stabilirea unor relaţii pe baza cărora se poate rezolva problema sintezei sistemelor automate cu modulaţie în amplitudine.

3. La sinteza acestor sisteme este utilă utilizarea unor corespondenţe între anumite forme elementare ale funcţiilor de transfer ale elementelor liniare care funcţionează la frecvenţă purtătoare şi respectiv ale ansamblurilor modulator—element liniar—demodulator corespunzătoare.

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AN ALGEBRAIC METHOD FOR THE SYNTHESIS OF AMPLITUDE-MODULATED CONTROL SYSTEMS

(Summary)

It is proposed a method for the synthesis of amplitude-modulated systems which is based on a symbolic representation of modulated movements by means of elements of algebrae over the real corpus. This category of problems is characterized by the process of determining the transfer function of a linear element which is traversed by modulated signals. The solution of the synthesis problem is given for the case where the structure of the linear element is initially unknown and for the case where this structure is imposed beforehand, so that a series of certain parameters must be first found out.

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D.K. 621.317.44

EXPERIMENTELLE UNTERSUCHUNG DER FUNKTIONIERUNG DER MAGNETISCHEN SONDEN IN KREISEN VON MAGNETISCHEN GEGENTAKTVERSTÄRKERN MIT SELBSTSÄTTIGUNG UND STEUERUNG DURCH EIN KONSTANTES MAGNETISCHES FELD

VON

PARASCHIVA HABA

1. In einer früheren Arbeit [1] wurde eine theorethische Analyse der Funktionierung der magnetischen Sonden in Kreisen von magnetischen Gegentaktverstärkern mit Selbstsättigung und Steuerung mit einem konstanten magnetischen Feld durchgeführt. In der vorliegenden Arbeit wollen wir die experimentellen Ergebnisse, die mit solchen Sonden erzielt worden sind, vorlegen.

Für Bau von magnetischen Sonden wurden Bleche aus Permenorm 5 000 Z der Dicke 0,1 mm in U-Form mit folgenden Kennzeichen benutzt:

$$\mu_{max} = 50\,000,\ H_c = 0{,}12\ \text{A/cm}\ \text{und}\ B_s = 1{,}55\ \text{T}.$$

Für experimentelle Untersuchungen wurden mehrere Sonden mit je zwei gleichen Spulen auf denselben Kern mit unterschiedlichen Windungszahlen hergestellt. Für jede von ihnen wurde die Spannung—Strom-Charakteristik $U=f(I)$ aufgenommen, um die Sättigungsspanung, die für den Betrieb der Sonden notwendig ist, bestimmen zu können.

In den Abb. 1 sind diese Charakteristiken für drei Sonden mit folgenden Windungszahlen: 1 000, 2 000 und 5 000 dargestellt.

Die experimentelle Untersuchung der genannten Sonden wurde mit Hilfe einer selbstgebauten Helmholtzspule durchgeführt. Das Schaltbild, das für experimentelle Messungen und Prüfungen benutzt wurde, ist in der Abb. 2 angegeben.

Mit diesem Kreis wurde zunächst der Betrieb der gegebenen magnetischen Sonden bei kleinen Steuerfeldern untersucht. Es stellte sich dabei heraus, daß es zwischen den experimentellen Ergebnissen und den theoretischen Erwartungen eine gute Übereinstimmung besteht. Es wurde nähmlich festgestellt, daß der Ausgangsstrom des Kreises, I, durch eine Beziehung der Form

$$(1) \qquad I = \frac{NfA}{R}\,\mu_{\Delta_{ef}}\mu_0 H_x$$

wiedergegeben weden kann, wobei: N—die Windungszahl der Spule, f—die Frequenz der Speisespannung U, A—die Durchschnittsfläche der Sonde, R—der gesamte Wiederstand eines Zweiges des Betriebskreises der Sonde, μ_{sef}—die effektive Permeabilität der Sonde, μ_0—die Vakuumper-

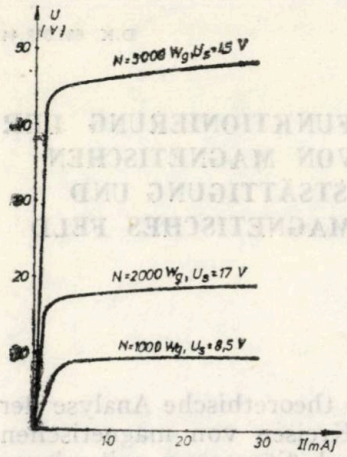


Abb. 1

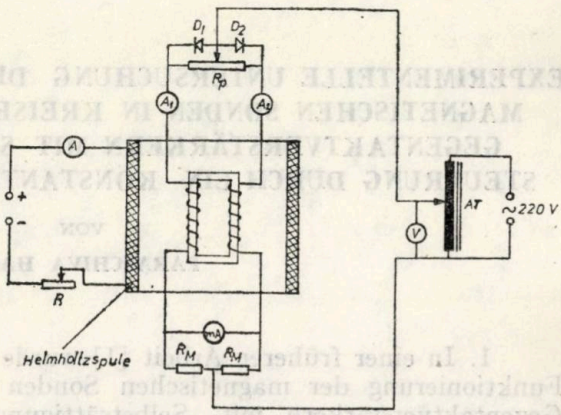


Abb. 2

meabilität und H_x—das Steuerfeld der Sonde darstellen. Man sieht also, daß der Ausgangsstrom des Kreises, I, von der angelegten Speisespannung unabhängig ist, während von dem angelegten Steuerfeld linear abhängt.

Diese Befunde sind in völliger Übereinstimmung mit den von G e y g e r in [2] bei Steuerfeldern von einiger Oe gefundenen Ergebnissen.

2. Da wir Geber mit magnetischen Sonden, gesteuert mit Permanentenmagneten, zu bauen beabsichtigen, haben wir die Untersuchung auf den gegebenen Kreis auch für solche Feldintensitäten fortgesetzt, die von uns benutzten Permanentenmagneten geliefert werden.

In den Abb. 3 *a* und 3 *b* sind die graphischen Darstellungen der beiden Zweigströme I_1 und I_2, sowie des ihren Resultierenden I in Abhängigkeit des angelegten Steuerfeldes H_x für verschiedene Werte der Speisespannung angegeben. Aus dieser Abbildung geht es klar hervor, daß in diesem Falle die genannten Ströme sowohl vom Steuerfeld als auch von der Speisespannung abhängen. Man sieht, daß bei Steuerfeldern, die größer als 18 A/cm sind, eine viel größere Speisespannung als die Sättigungsspannung notwendig ist.

3. Es wurde auch ein Studium dieses Kreises bei konstanten Steuerfeldern mit veränderlichen Speisespannungen für verschiedene Werte des R_M-Wiederstandes durchgeführt. Die entsprechenden Ergebnisse sind mittels der Charakteristiken $I=f(U)$ beim H_x=konst. und R_m=konst. in Vordergrund gestellt (Abb. 4 *a*, *b*, *c*). Hieraus geht es hervor, was auch am oben unterstrichen wurde, daß die Speisespannung erhöht werden muss, falls man bei größeren Steuerfeldern arbeiten sollte. Es ist weiterhin zu sehen, daß es für jeden gegebenen Kreis eine Grenzkurve der Charakte-

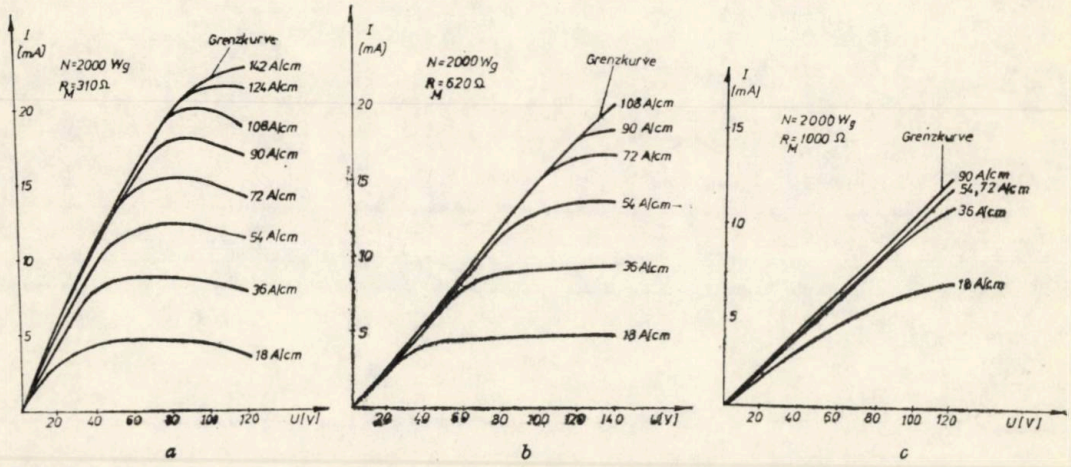

I (mA)
N=2000 Wg
R_M=310Ω
Grenzkurve
142 A/cm
124 A/cm
108 A/cm
90 A/cm
72 A/cm
54 A/cm
36 A/cm
18 A/cm
U[V]
a
N=2000 Wg
R_M=620Ω
108 A/cm
90 A/cm
72 A/cm
54 A/cm
36 A/cm
18 A/cm
b
N=2000 Wg
R_M=1000Ω
90 A/cm
54, 72 A/cm
36 A/cm
18 A/cm
c

Abb. 4

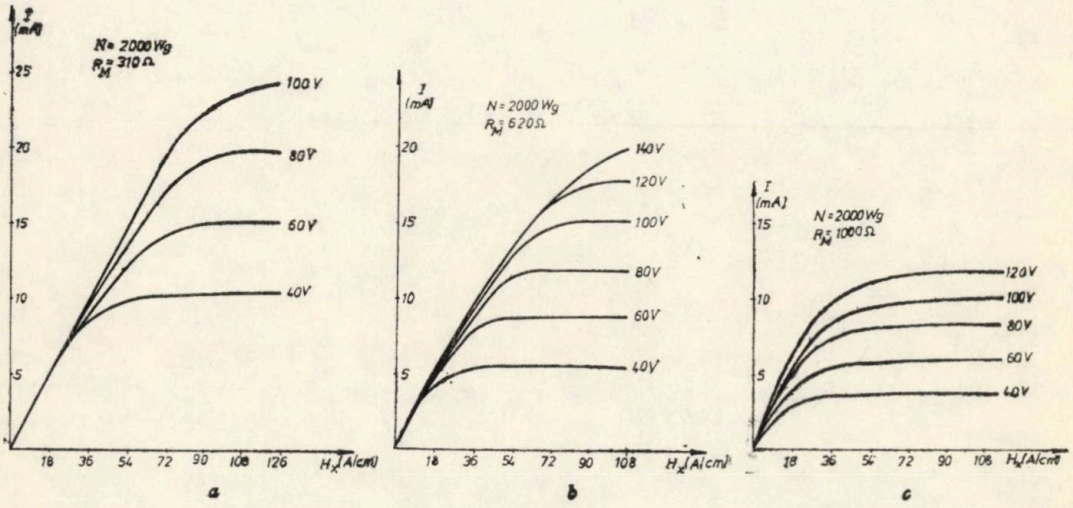
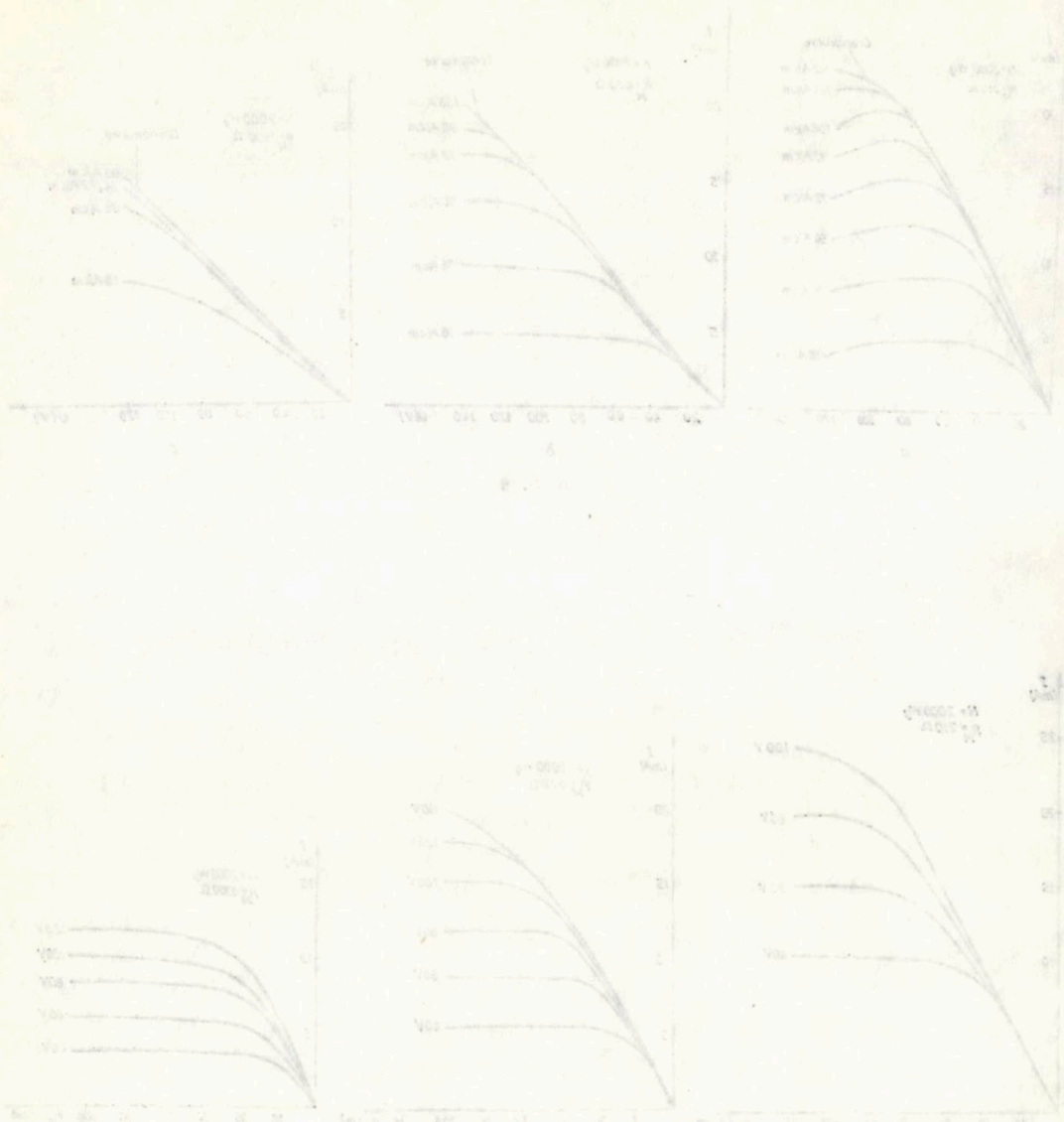

N= 2000 Wg
R_M= 310Ω
100 V
80 V
60 V
40 V
H_x[A/cm]
a
N= 2000 Wg
R_M= 620Ω
140 V
120 V
100 V
80 V
60 V
40 V
b
N=2000 Wg
R_M= 1000Ω
120 V
100 V
80 V
60 V
40 V
c

Abb. 5



ristik $I=f(U)$ gibt, die, falls sie überschritten wird, zu einem Arbeitsregime führt, bei dem der Strom I konstant bleibt, auch wenn das Steuerfeld erhöht wird. Es ist auch eine Abhängigkeit der Neigung der Charakteristiken $I=f(U)$ vom Wiederstand R_M festzustellen. In Verbindung da-

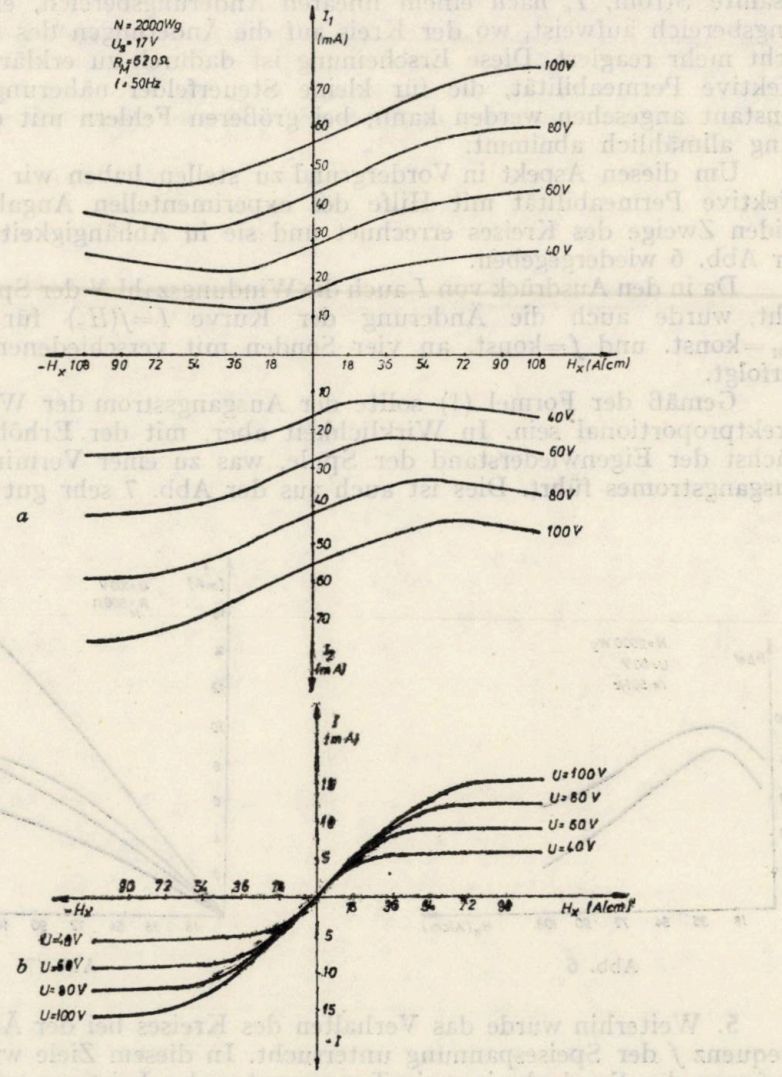


Abb. 3

mit kann man feststellen, daß diese Neigung um so stärker abnimmt, je stärker der Wiederstand R_M vermindert wird. Gleichzeitig wird die Grenzkurve für kleinere Steuerfelder erhalten.

4. In den Abb. 5 *a*, *b*, *c* sind die Characteristiken $I=f(H_x)$ für verschiedene Wiederstände R_M dargestellt. Hieraus folgt unmittelbar, daß sich

mit der Erhöhung des R_M-Wiederstandes auch der gesamte Wiederstand des Kreises, R, erhöhen und damit der resultierende Strom I abnehmen soll, wass auch experimentell beobachtet wird.

Die Analyse der gleichen Characteristiken zeigt uns auch, daß der gesamte Strom, I, nach einem linearen Änderungsbereich, einen Erhaltungsbereich aufweist, wo der Kreis auf die Änderungen des Steuerfeldes nicht mehr reagiert. Diese Erscheinung ist dadurch zu erklären, daß die effektive Permeabilität, die für kleine Steuerfelder näherungsweise als konstant angesehen werden kann, bei größeren Feldern mit derer Erhöhung allmählich abnimmt.

Um diesen Aspekt in Vordergrund zu stellen, haben wir die relative effektive Permeabilität mit Hilfe der experimentellen Angaben für die beiden Zweige des Kreises errechnet und sie in Abhängigkeit von H_x in der Abb. 6 wiedergegeben.

Da in den Ausdrück von I auch die Windungszahl N der Spulen hineingeht, wurde auch die Änderung der Kurve $I=f(H_x)$ für $U=$konst., $R_M=$konst. und $f=$konst. an vier Sonden mit verschiedenen N-Werten verfolgt.

Gemäß der Formel (1) sollte der Ausgangsstrom der Windungszahl direktproportional sein. In Wirklichkeit aber, mit der Erhöhung von N wächst der Eigenwiederstand der Spule, was zu einer Verminderung des Ausgangstromes führt. Dies ist auch aus der Abb. 7 sehr gut zu ersehen.

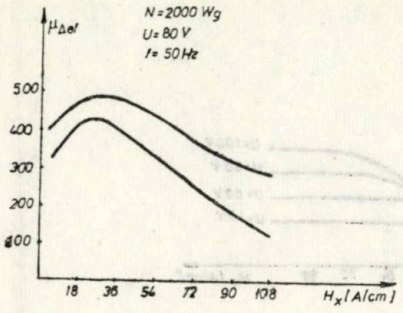


Abb. 6

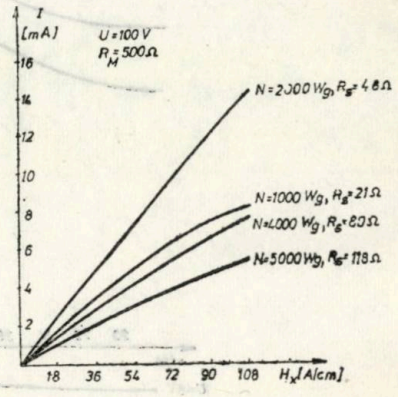


Abb. 7

5. Weiterhin wurde das Verhalten des Kreises bei der Änderung der Frequenz f der Speisespannung untersucht. In diesem Ziele wurde für die Speisung des Sondenkreises ein Tongenerator der Leistung von 10 W benutzt. Da sich mit der Erhöhung der Frequenz f auch die Sättigungsspannung erhöht, wurde das Signal vom Tongenerator durch einen Leistungsverstärker auf die Sonde angelegt.

In der Abb. 8 sind die Zweigsströme I_1 und I_2 sowie der resultierende Strom, I, in Abhängigkeit von dem Steuerfeld H_x für $f=200$ Hz, $R_M=$ $=620\,\Omega$ und $U=150$ V dargestellt. Die Speisespannung U wurde hierbei

deshalb erhöht. da sich mit der Erhöhung der Frequenz auch die Sättigungsspannung erhöht. In diesem Falle $U_s = 68$ V.

Vergleicht man die angegebenen Charakteristiken mit denen, die mit Hilfe der gleichen Sonde bei der Speisespannung $U = 100$ V erhalten wurden, so sieht man, daß sich mit der Zunahme der Frequenz f auch die Neigung diesen Charakteristiken erhöht, was eben den theoretischen Erwartungen entspricht.

6. Da es gemäß (1) zwischen Strom und Frequenz eine lineare Korrelation besteht, wurde dieses Aspekt weiterhin auch experimentell untersucht. Die entsprechenden Ergebnisse sind durch die in der Abb. 9 dargelegten Kurven wiedergegeben worden.

Die Untersuchungen wurden mit einer Sonde mit $N = 2\,000$ Wdg, $R_M = 310\,\Omega$ und bei $H_x = 44{,}5$ A/cm durchgeführt.

Die Analyse dieser Kurven zeigt uns einen linearen Abhängigkeitsbereich zwischen I und f. Dies deutet darauf hin, daß die untersuchte Sonde

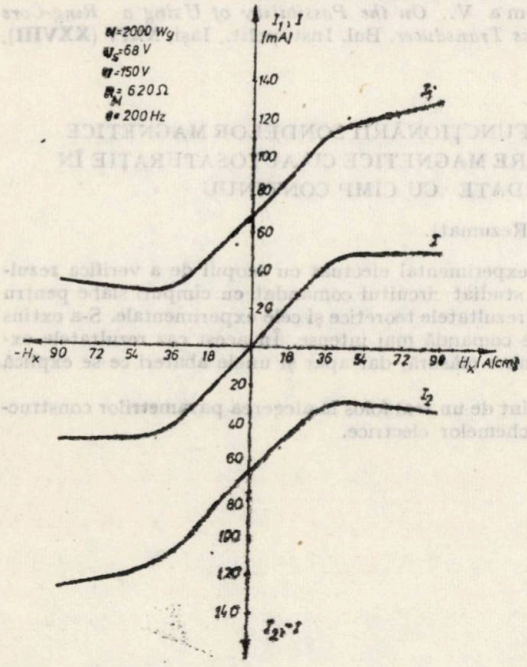


Abb. 8

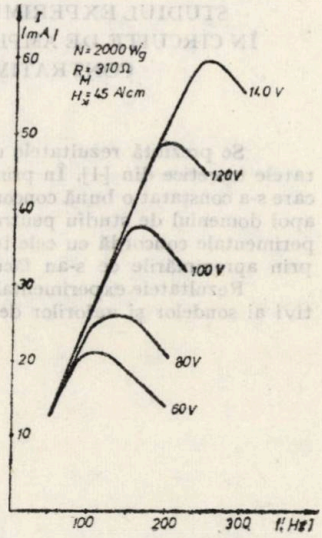


Abb. 9

auch als Frequenzgeber mit linearer Charakteristik benutzt werden kann. Man sieht weiterhin, daß falls die Frequenz f erhöht werden soll, muß auch die Speisespannung derentsprechend erhöhen, da sich die Sättigungsspannung mit wachsender Frequenz auch zunimmt.

Mit Hilfe der in den Abb. 3...9 angegebenen Kurven können also die Baudaten eines elektrischen Kreises mit magnetischen Sonde bestimmt werden.

7. Zusammenfassend darf man sagen, daß die von uns erhaltenen experimentellen Ergebnisse auch bei größeren Steuerfeldern mit den theoretischen Erwartungen in guter Weise übereinstimmen, auch wenn bei diesen Feldern, wegen der benutzten theoretischen Näherungen, bestimmte Abweichungen auftreten.

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Technische Hochschule, Jassy,
Lehrstuhl für Elektrotechnik, elektrische Messungen und Maschinen

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STUDIUL EXPERIMENTAL AL FUNCŢIONĂRII SONDELOR MAGNETICE ÎN CIRCUITE DE AMPLIFICATOARE MAGNETICE CU AUTOSATURAŢIE ÎN CONTRATIMP, COMANDATE CU CÎMP CONTINUU

(Rezumat)

Se prezintă rezultatele unui studiu experimental efectuat cu scopul de a verifica rezultatele teoretice din [1]. În primul rînd s-a studiat circuitul comandat cu cîmpuri slabe pentru care s-a constatat o bună concordanţă între rezultatele teoretice şi cele experimentale. S-a extins apoi domeniul de studiu pentru cîmpuri de comandă mai intense. În acest caz rezultatele experimentale concordă cu cele teoretice în mare măsură, dar apar şi unele abateri ce se explică prin aproximările ce s-au făcut în [1].

Rezultatele experimentale obţinute sînt de un real folos la alegerea parametrilor constructivi ai sondelor şi valorilor de lucru ale schemelor electrice.

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C.Z. 621.396.61

SUR UNE POSSIBILITÉ DE GÉNÉRER LES FONCTIONS PÉRIODIQUES DE TEMPS

PAR

MIRCEA DIACONESCU et ELDORINA DOBREANU

1. Introduction

Les fonctions périodiques de temps constituent la classe des fonctions la plus répandue en technique. Elles décrivent la plupart des phénomènes et des processus rencontrés couramment. La reproduction des fonctions périodiques de temps devient ainsi nécessaire pour beaucoup d'applications comme par exemple : le modélage de certains processus sur le calculateur analogique ou le modélage physique à n'importe quelle échelle.

Pour la plupart des fonctions qui puissent apparaître, une reproduction tout à fait fidèle est pratiquement impossible. C'est pourquoi d'habitude on utilise une description approximative en remplaçant la fonction proprement-dite par des segments de droite. Le problème, qui c'est posé, est de trouver le système de description qui puisse conduire aux plus petites approximations possibles et celui qui soit le plus facile à programmer.

Un générateur de fonctions périodiques de temps qui permet une description par des segments horizontaux et verticaux pour des fonctions à période réglable dans des larges limites est présenté dans ce qui suit. On se propose à programmer comme amplitude les segments verticaux et aussi la durée des segments horizontaux.

2. Le fonctionnement selon le schéma bloc

Le schéma bloc du générateur est présenté dans la fig. 1 où l'on a utilisé les notations ci-dessous : *1* — générateur de tact, *2* — compteur en anneau, *3* — décodeur, *4* — programmateur, *5* — additioneur et amplificateur de puissance, *6* — bouton de commande pour le commencement du cycle.

Le générateur de tact est un générateur d'impulsions à fréquence réglable. Il commande le compteur *2* dont la capacité n correspond au nombre de segments horizontaux par les quels on va réaliser la fonction à pro-

grammer. Les n états possibles du compteur *2* sont décodifiés par le décodeur *3*. Le bouton de commande *6* sert à amener à zero le compteur *2* et pour établir le commencement du cycle.

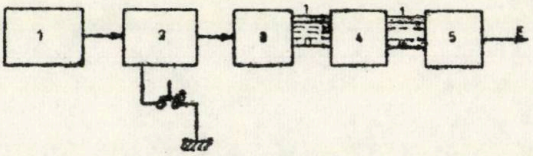


Fig. 1

Le programmateur *4* contient un nombre de n potentiomètres à l'aide des quels on programme l'amplitude en tension pour chacun des n segments horizontaux, qui à la fin décriveront la fonction désirée.

En chaque moment, grâce à la commande réalisée par le décodeur *3*, une seule tension programmée par le système *4* est saisie à la sortie correspondante ; c'est la tension qui par le numéro d'ordre correspond à l'état dans lequel se trouve le compteur *2*. De cette façon résulte que tour-à-tour chacune de ces tensions sera fournie à la sortie correspondante du programmateur *4*. L'additioneur-amplificateur *5* reprend tour-à-tour ces tensions recomposant à la sortie la fonction programmée. Par l'action sur les potentiomètres du système *4* il résulte qu'on peut programmer exactement la fonction désirée et par la fréquence du générateur *1* on peut régler la période.

Le seul désavantage qui puisse apparaître est le grand volume des liaisons que suppose le programateur *4*. Il est quand même possible de diminuer ce grand volume des liaisons en procédant comme l'on indique ci-dessous.

Pour quelques détails supplémentaires du schéma on va supposer qu'il s'agit d'un générateur qui permet le modélage d'une fonction par 40 segments horizontaux, respectivement verticaux avec la durée du cycle réglable entre 0, 01 sec. et 15 min. En ce cas on peut faire les considérations suivantes en ce qui concerne la manière de la réalisation pratique du générateur.

3. Le générateur de tact. Le compteur. Le décodeur

Le compteur a la valeur de la capacité égale à 40 et est réalisé par le montage en cascade d'un compteur décadique (DN) avec un compteur à 4 bits ($4N$).

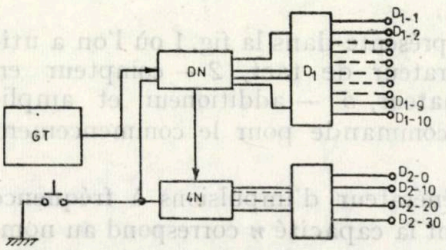


Fig. 2

Sachant que les périodes sont comprises entre 0,01 sec. et 15 min. de la sorte que pour chaque période soient nécessaire 40 impulsions du générateur de tact, la fréquence du générateur doit être réglée entre 4 000 Hz et 0,0444 Hz.

La solution pratique dépend premièrement de la précision avec

la quelle on doit réaliser la durée d'une période et peut avoir une diversité de formes à partir d'un simple multivibrateur à fréquence reglable jusqu'au générateur commandé par un cristal de quartz, suivi par des circuits de division numérique.

Le compteur (DN) et le compteur ($4N$) peuvent être réalisés à l'aide de deux circuits intégrés CDB 490 E, respectivement CDB 473 E. Le bouton de commande pour le commencement du cycle sera connecté en ce cas aux bornes R des compteurs ; en même temps il est nécessaire à utiliser deux décodeurs : D_1, qui décodifie les compteurs jusqu'à dix et D_2 qui décodifie les dizaines. Les décodeurs travaillent à signal de départ zero.

4. Le programmateur

Le schéma du programmateur est représenté dans la fig. 3, où pour simplifier ont été représentés en détails les circuits pour les segments *1,11,21, 31*, les autres groupes à quatre segments chacun ayant des circuits tout à fait identiques (sauf les sources qui sont uniques).

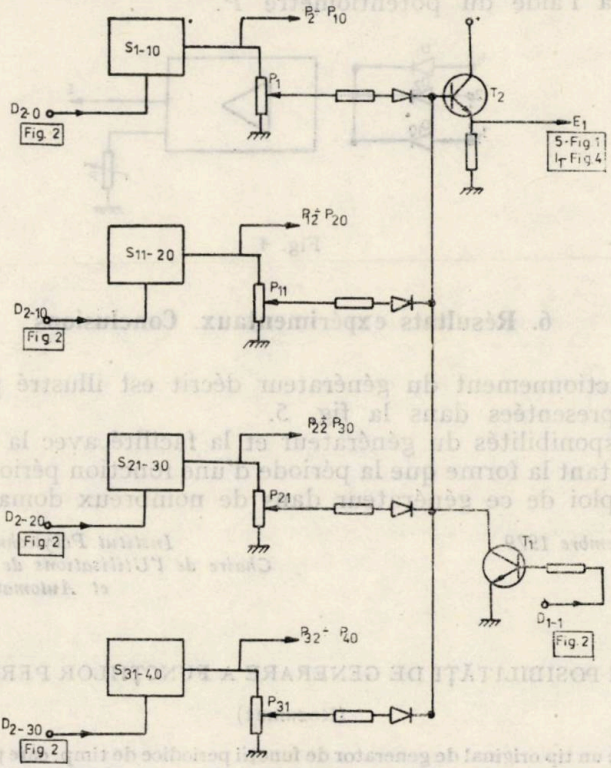


Fig. 3

Le transistor T_1 est bloqué quand le compteur *2* se trouve dans un des états *1, 11, 21, 31*, au reste étant saturé.

Les sources d'alimentation $S_1...10...S_{31}...40$ sont commandées par le décodeur de disaines ainsi que la source S_{1-10} fournie la tension de sortie quand le compteur se trouve dans un des états *1...10*, la source $S_{11}...S_{20}$, pour les états *11...20* et ainsi par la suite.

Dans ce cas pour l'état *1* le transistor T_1 est bloqué, le potentiomètre P_1 est alimenté et la tension du courseur de celui-ci s'obtient à la sortie E. Pour les états *2...10* le transistor T_1 est saturé et à la sortie E la tension est nulle. Dans l'état *11* le transistor T_1 est bloqué de nouveau. Maintenant la source $S_{11}...20$ est débloquée, le potentiomètre P_{11} est alimenté et la tension du courseur de celui-ci s'obtient à la sortie E.

5. L'additioneur-amplificateur de puissance

L'additioneur-amplificateur est representé dans la fig. 4. Il contient un circuit OU réalisé avec les diodes $D_1...D_{10}$, qui ramasse les tensions des sorties E (fig. 3) et l'amplificateur proprement-dit dont l'amplification peut être réglée à l'aide du potentiomètre P.

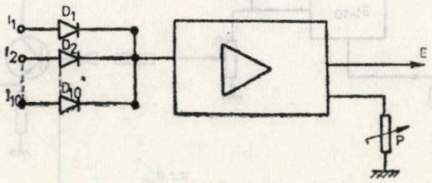


Fig. 4

6. Résultats expérimentaux. Conclusions

La fonctionnement du générateur décrit est illustré par les oscillogrammes representées dans la fig. 5.

Les disponibilités du générateur et la facilité avec la quelle on peut programmer tant la forme que la période d'une fonction périodique de temps permet l'emploi de ce générateur dans de nombreux domaines.

Reçue le 20 décembre 1979

Institut Polytechnique, Jassy,
Chaire de l'Utilisations de l'énergie électrique et Automations

ASUPRA UNEI POSIBILITĂŢI DE GENERARE A FUNCŢIILOR PERIODICE DE TIMP

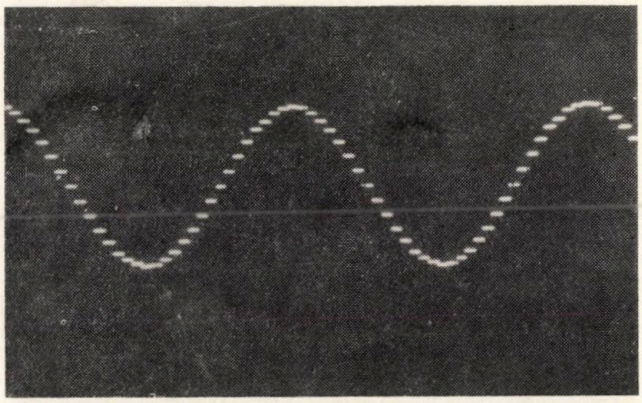
(Rezumat)

Se descrie un tip original de generator de funcţii periodice de timp, care permite o descriere prin segmente orizontale şi verticale pentru funcţii cu perioade reglabile în limite largi. Se programează ca amplitudine segmentele verticale şi ca durată segmentele orizontale.

Generatorul permite modelarea unei funcţii prin 40 segmente orizontale, respectiv verticale cu durata perioadei reglabilă între 0,01 sec...15 min. Se prezintă schema bloc şi funcţionarea blocurilor componente.

Se evidenţiază funcţionarea cu oscilograme obţinute pe un model experimental.

a



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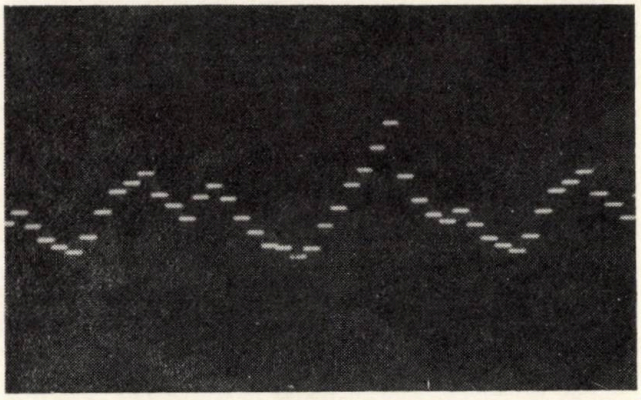

Fig. 5.

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C.D. 621.316.11

SUR L'EXPLOITATION OPTIMALE DES RÉSEAUX DE DISTRIBUTION

PAR

GH. CÎRŢINĂ et I. ŢIŢARU

1. Considérations générales

Pour une suite de mesures des puissances actives, réactives, tensions, etc., $\{x_i\}$, dans la détermination de l'intervalle de confiance on utilise la relation [1] :

$$P(|x| < x_\beta) = 2\Phi\left(\frac{x_\beta}{\sigma}\right), \tag{1}$$

où :

$$\Phi\left(\frac{x_\beta}{\sigma}\right) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\frac{x_\beta}{\sigma}} e^{-x^2/2}\, dx \tag{2}$$

— la fonction de répartion Laplace.

Si pour la limite x_β on prend un multiple de l'écart type (σ), $\nu\sigma$, on peut obtenir facilement les probabilités $P(\nu\sigma)$ [1] ;

$$x_\beta = \sigma, \quad P(\overline{x} - \sigma < x < \overline{x} + \sigma) \approx 0{,}683, \tag{3}$$

$$x_\beta = 2\sigma, \quad P(\overline{x} - 2\sigma < x < \overline{x} + 2\sigma) \approx 0{,}960, \tag{4}$$

$$x_\beta = 3\sigma, \quad P(\overline{x} - 3\sigma < x < \overline{x} + 3\sigma) \approx 0{,}997. \tag{5}$$

On peut interpréter ces valeurs dans le sens que dans 68,3%, 96%, respectivement 99,7% des cas la valeur x se trouvera dans l'intervalle respectif.

Dans ce qui suit on va considérer l'intervalle de confiance $(\overline{x} - \nu\sigma, \overline{x} + \nu\sigma)$ et on va étudier l'influence du paramètre ν sur la compensation de l'énergie réactive, la consommation du matériel conducteur, le réglage de la tension, etc. De même, on analyse la corrélation de la compensation réactive et du réglage de la tension dans l'exploitation optimale des réseaux de distribution, en proposant un nouveau schéma de réglage optimal de la tension.

2. Influence du niveau de compensation de l'énergie réactive sur l'économie d'énergie et du matériel conducteur

Compte tenu que le transit réactif a une contribution importante dans la détermination du niveau des pertes totales d'énergie d'un réseau, la situation actuelle de l'énergie dans le monde (caractérisée par les prix du pétrole augmentant sans cesse) impose la réexamination sévère de la compensation de l'énergie réactive. Certainement, la première conclusion qui s'impose est de rendre près de minimum les circulations des puissances réactives par la compensation des celles-ci. La réalisation de cet objectif est possible parce que les condensateurs représentent des sources réactives économiques dans les réseaux à moyenne tension et même à basse tension.

Avec une probabilité $P(\nu\sigma)$ on va considérer la compensation du chaque noeud jusqu'au niveau de la charge réactive minima $Q_j^c = Q_j^{\min}$. Donc les puissances nominales des condensateurs sont :

$$Q_{nj}^c = Q_j^{\min} = (\overline{Q}_j - \nu\sigma_{Q_j})\left(\frac{U_n}{\overline{U} + \nu\sigma_U}\right)^2, \quad (j = 1, 2, \ldots, n). \tag{6}$$

La puissance fournie par des condensateurs dépend du niveau de la tension aux bornes. Pour les charges minima les puissances réactives sont :

$$Q_j^{\min} = (\overline{Q}_j - \nu\sigma_{Q_j}), \quad (j = 1, 2, \ldots, n) \tag{7}$$

et pour les charges maxima :

$$Q_j^{\max} = (\overline{Q}_j - \nu\sigma_{Q_j})\left(\frac{\overline{U} - \nu\sigma_U}{\overline{U} + \nu\sigma_U}\right)^2, \quad (j = 1, 2, \ldots, n). \tag{8}$$

Dans les relations ci-dessus on a noté par $\overline{Q}_j$, $\overline{U}$ – les puissances réactives moyennes, respectivement, la tension moyenne, calculées à partir de m valeurs relevées en 24 heures.

L'économie d'énergie, dans T jours, par compensation, a comme expression :

$$T(\Delta P - \Delta P_c) = \frac{mT}{U_n^2} \sum_{j=1}^{n} R_j [\overline{Q}_j^2 - \nu^2 \sigma_{Q_j}^2], \quad m = 24, \tag{9}$$

dans le cas d'un réseau radial quelconque avec les noeuds numérotés de 0 à n. L'étiquette de chaque ligne sera celle du noeud du côte droit.

Pour pouvoir apprécier l'influence du paramètre ν sur l'économie d'énergie il est nécessaire d'analyser l'expression :

$$\Delta E(\nu) = \frac{\overline{Q}^2 - \nu^2\sigma_Q^2}{\overline{P}^2 + \overline{Q}^2 + \sigma_P^2 + \sigma_Q^2}\, 100, \ [\%]. \tag{10}$$

Les valeurs de la fonction $E(\nu)$ sont données, en pourcentages, dans le tableau 1, pour $\sigma_P = \sigma_Q = 5\%$, $P = Q = 100\%$.

Tableau 1

	1	2	3	4	5
$\Delta E(\nu)$, [%]	50	49,5	49	48	47

De même, il est important à souligner l'influence de la compensation sur la section économique des conducteurs, $\Delta S(\nu)$;

$$\Delta S(\nu) = S_1 - S_2 = \frac{K_1}{K_2} \cdot \frac{100}{\sqrt{3}U_n} \left(\sqrt{P^2 + Q^2} - \sqrt{P^2 + (2\nu)^2 \sigma_Q^2} \right), [\%] =$$

$$= 100 \left(\sqrt{2} - \sqrt{1 + \frac{\nu^2}{100}} \right), [\%]. \tag{11}$$

La diminution de la section économique des conducteurs, $\Delta S(\nu)$, en fonction du paramètre ν est donnée dans le tableau 2.

Tableau 2

	1	2	3	4	5
$\Delta S(\nu)$, [%]	41	39	37	33	29

On constate que pour les réseaux urbains, où la consommation réactive est presque constante en temps et représente 60...90% du niveau de la puissance active, l'économie du matériel conducteur est assez importante.

3. L'influence de la compensation de l'énergie réactive sur le réglage optimal de la tension dans un réseau de distribution

Les variations lentes de tension qui affectent les réseaux de distribution peuvent être compensées par les régleurs en charge associés aux transformateurs HT/MT, RT. La commande des ces régleurs est assurée par des régulateurs automatiques de tension triphasés (RATT). Par l'adjonction à ces régulateurs des blocs de compoundage (RC), il est possible d'adapter la valeur de la tension à la charge du réseau. Le réglage de la tension dans un réseau électrique est acceptable lorsque simultanément la valeur moyenne $\overline{u}_j$ des écarts relatifs de tension et l'écart type, σ_{uj}, sont si réduits que possible.

Suivant ce processus il est nécessaire de connaître non seulement les charges actives, réactives et les chutes de tension dans le réseau fictif, mais aussi les coefficients de corrélation entre ces grandeurs. Un amélioration de ce processus est proposée en [3], par l'introduction d'une impédance image variable en temps, qui permet d'obtenir un meilleur réglage de la tension. La succession en temps des valeurs de l'impédance est assurée par un registre à récirculation (RC).

Si l'on va considérer la compensation jusqu'au niveau de la charge minima $Q_j^c = Q_j^{min} \approx \overline{Q_j} - 2\sigma_{Q_j}$, la faible circulation réactive n'entraîne que des très faibles chutes de tension dans le réseau à MT et ceci pour deux raisons : a) la plus grande partie de l'énergie réactive est produite là où l'on utilise ; b) la réactance des lignes de distribution est, dans l'ensemble, inférieure à leur résistance. Par conséquent ;

$$\Delta U_j' = \frac{R_j P_j}{\sqrt{3}\, U_n}. \tag{12}$$

Le schéma de réglage de la tension, au niveau du jeu de barres BT du transformateur HT/MT, T, est représenté dans la fig. 1. En général, le dispositif de compensation DC reproduit une chute de tension proportionnelle à celle qui existe entre le poste HT/MT et le point fictif qui est censé représenter le mieux la tension de l'ensemble du réseau MT [2].

En excluant la zone d'insensibilité du régulateur de type intégral (RI), pour le régime stationnaire du processus de réglage (fig. 1) on a ;

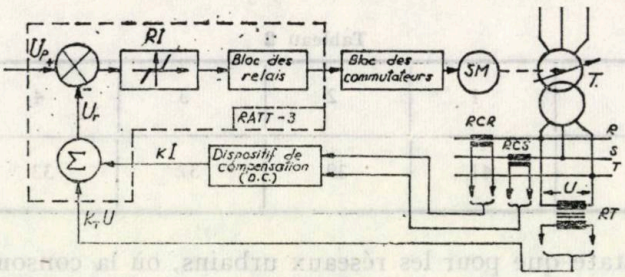

Fig. 1

$$U_p = U_r = K_T U - KI, \quad \text{ou} \tag{13}$$

$$U = \frac{1}{K_T} U_p + \frac{K}{K_T} I = U_p^* + SI, \tag{14}$$

où : U_p^* — la valeur de consigne de la tension ; $S = K/K_T$ — le statisme du réglage.

Pour obtenir une tension constante, $U_2 = U_f$, à l'extrémité de la ligne fictive, on choisit le produit SI égal à la chute de tension de cette ligne. Si la compensation de la chute de tension n'est pas parfaite il en résulte des variations de la tension U_2 qui peuvent être contrôlées par le choix adéquat du statisme S. La modification automatique du statisme S en fonction du temps est assurée par le dispositif de compensation DC, dont le schéma est présenté dans la fig. 2.

Le dispositif de compensation proposé contient un générateur de cadence, de fréquence f, qui, par le diviseur de fréquence (f/N), réalise le déplacement de l'inscription initiale dans le régistre à récirculation RC. Le nombre de cellules du registre RC correspond au nombre des intervalles de temps p, pour lesquels la compensation et le statisme sont fixées à des valeurs imposées. Pour obtenir les p états distincts il est nécessaire de réaliser la relation ;

(15) $$\frac{pN}{f} = 24.3600 \text{ sec.}$$

Dans ce but on choisit des valeurs adéquates pour la fréquence f et le facteur N.

Si initialement l'inscription dans le registre RC est $(1, \underbrace{0,...,0}_{p-1})$, pour l'intervalle de temps j, quand le contenu des cellules du registre RC sera 0, sauf la cellule j qui aura 1, dans le circuit de sortie du RC sera introduite

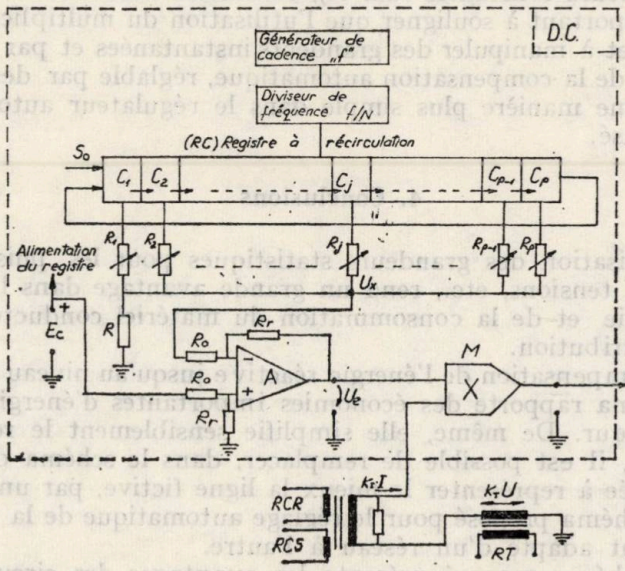

Fig. 2

la résistance $R_j + R$. La valeur du statisme des caractéristiques de réglage dans l'intervalle j est assurée par la résistance R_j.

Dans le schéma représenté dans la fig. 2 l'amplificateur A assure tant l'adaptation des signaux de sortie du registre RC, que la dépendence linéaire du statisme en fonction de la résistance R_j. En effet, avec les notations de la fig. 2 on peut écrire ;

(15) $$U_x = E_c \frac{R}{R + R_j},$$

(16) $$U_e = \frac{R_r}{R_0}(E_c - U_x) = \frac{R_r R_j}{R_0(R + R_j)} E_c,$$

(17) $$U_r = K_T U - K_I K_M U_e I = K_T U - K_I K_M \frac{R_r R_j}{R_0(R + R_j)} E_c I,$$

où : K_M – le facteur de proportionalité du multiplicateur M.

À partir des relations (14) et (17) on déduit le statisme pour l'intervalle de temps j :

$$S_j = \frac{K_I}{K_T} K_M \frac{R_r R_j}{R_0(R+R_j)} E_c. \tag{18}$$

Si $R \gg R_j$ la relation (18) devient ;

$$S_j = \frac{K_I}{K_T} K_M K_p E_c R_j = \alpha R_j, \tag{19}$$

où : $K_p = R_r/R_0$ — le facteur de transfert de l'amplificateur A, α — le produit des grandeurs constantes sauf R_j, considérée variable.

Il est important à souligner que l'utilisation du multiplicateur analogique M permet à manipuler des grandeurs instantanées et par conséquent, l'introduction de la compensation automatique, réglable par des intervalles de temps, d'une manière plus simple dans le régulateur automatique de tension triphasé.

4. Conclusions

1. L'utilisation des grandeurs statistiques pour les puissances actives, réactives, tensions, etc., rend un grande avantage dans l'analyse des pertes d'énergie et de la consommation du matériel conducteur dans un réseau de distribution.

2. La compensation de l'énergie réactive jusqu'au niveau de la charge réactive minima rapporte des économies importantes d'énergie et de matériel conducteur. De même, elle simplifie sensiblement le réglage de la tension. Ainsi, il est possible de remplacer, dans le schéma du DC, l'impédance, censée à représenter le mieux la ligne fictive, par une résistance.

3. Le schéma proposé pour le réglage automatique de la tension peut être facilement adapté d'un réseau à l'autre.

4. Le schéma proposé présente les avantages des circuits intégrés.

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et Automations

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ASUPRA EXPLOATĂRII OPTIMALE A REȚELELOR DE DISTRIBUȚIE

(Rezumat)

Se studiază influența intervalului de încredere asupra compensării energiei reactive, a reglajului de tensiune etc. Se pune în evidență corelația dintre compensarea optimală a energiei reactive și reglajul optimal al tensiunii, propunîndu-se o nouă schemă de reglaj automat al tensiunii în rețelele de distribuție, bazată pe un dispozitiv nou de compensare a căderii de tensiune pe ansamblul rețelei.

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C.D. 621.43.044

SUR QUELQUES ASPECTS DE L'UTILISATION DU SYSTÈME D'ALLUMAGE DU TYPE CAPACITIF

PAR

TRAIAN VIDRAŞCU et AL. PAPP

1. Il est bien connu [1] que dans le cas des moteurs à combustion interne, dans des conditions de basse temperature et de grande humidité il s'avère nécessaire d'appliquer une tension de 25 000 V entre les électrodes de la bougie pour assurer l'apparition de l'étincelle. C'est seulement dans ces conditions que le niveau énergétique des molecules du combustible atteint une valeur suffisament haute pour permettre à celles-ci de se dissocier en ions et de pouvoir produire l'allumage du combustible par leur collision avec les molécules d'oxygène. L'étincelle qui fait son apparition entre les électrodes de la bougie exerce son action sur le niveau énergétique des molecules du combustible tant par une radiation à rayons ultraviolets que par une augmentation locale de la température.

L'énergie cédée par l'étincelle doit être grande au commencement pour réaliser l'allumage d'une quantité suffisante de combustible, ce qui permet la propagation de la combustion dans toute la masse du combustible.

Si l'énergie de l'étincelle est insuffisante, la quantité de combustible qui s'allume est petite et la flamme s'éteint sans se propager dans la masse du combustible [2]. Le mélange combustible—air ne brule pas complètement et le moteur fonctionne dans des mauvaises conditions (il consomme beaucoup et la puissance est insuffisante). Le fonctionnement du moteur dans ces conditions défavorables peut être évité par la production aux électrodes des bougies d'une étincelle puissante qui soit indépendente de la vitesse du moteur.

Une telle étincelle peut être fournie par le systéme d'allumage du type capacitif.

2. Dans ce qui suit nous tacherons à établir les valeurs de la tension qui doit charger le condensateur du système capacitif pour obtenir aux bougies une tension de 25 000 V, pour divers rapports de transformation de la bobine d'induction.

Nous commençons par établir quelques relations fondamentales concernant le fonctionnement du système d'allumage du type capacitif.

Le système d'allumage électrique du type capacitif est constitué, en principe, par un condensateur C_p (fig. 1) chargé à la tension U_p par l'intermède d'un converteur transistorisé à partir de la batterie électrique de l'auto [3]. Le condensateur C_p se décharge à l'aide d'un thyristor convenablement commandé, dans le bobinage primaire de la bobine d'induction. Dans le secondaire se produit une haute tension, dont la valeur dépend de divers facteurs (rapport de transformation de la bobine, admittance de charge).

On néglige l'élément conductif de l'admittance de charge et on ne tient compte que de son caractère susceptif, c'est-à-dire capacitif, dans notre cas. On évalue à 100 pF la valeur de la capacité totale qui existe aux bornes du secondaire. On atribue 50 pF à la capacité parasite du bobinage

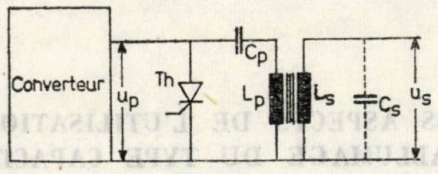

Fig. 1

secondaire de la bobine et 50 pF au distributeur, aux fils de connexions et à l'énergie des bougies [1].

3. Tout d'abord on va calculer l'énergie emmagasinée dans le primaire et transmise au secondaire.

Selon la fig. 1 l'énergie emmagasinée dans le primaire de la bobine est de nature électrostatique, c'est-à-dire :

$$W_p = \frac{1}{2} C_p U_p^2 \tag{1}$$

et l'énergie transmise au secondaire est donneé, selon [1], par :

$$W_s = \frac{1}{2} (C_s + C_p/n^2) U_s^2, \tag{2}$$

où n est le rapport de transformation de la bobine d'induction.

Un autre élément qui doit être mentionné c'est le rendement η, qui dépend des pertes de la bobine, à savoir :

$$\eta = \frac{W_s}{W_p}. \tag{3}$$

À partir des relations (1), (3) on obtient la valeur de la tension U_p à laquelle le condensateur C_p doit être chargé :

$$U_p = \frac{U_p}{n} \sqrt{\frac{C_p + n^2 C_s}{C_p}}. \tag{4}$$

À l'aide des relation (1) et (4) on a calculé la tension U_p qui doit charger le condensateur C_p et l'énergie accumulée par celui-ci (W_p) pour obtenir aux électrodes des bougies une tension de 25 000 V dans les cas où $C_p = 1...2$ μF et $n = 50$; 100 et 200. On a admis $C_s = 100$ pF et $\eta = 0{,}5$. Les valeurs obtenus sont presentées dans le tableau 1.

4. Avant d'analyser les résultats présentés dans ce tableau il faut préciser que la valeur de l'énergie qui doit être fournie aux bougies pour assurer leur fonctionnement dans les conditions imposées est de 32 m J. Compte tenu de cette remarque on observe que les cas 1 et 2, ne sont pas favorables

à une utilisation pratique, notamment à cause de la grande sollicitation du converteur, sans parler de la grande valeur de U_p. De même, dans le cas 4, la tension U_p est acceptable, mais l'énergie demandée au converteur est trop grande. Par contre le cas 3 est acceptable pour l'utilisation pratique.

Tableau 1

Nr.	C_p μF	n	U_p V	W_p mJ
1	1	50	790,5	312
2	2	50	750	562
3	1	100	500	125
4	2	100	433	187
5	1	150	425	90
6	2	150	344	118
7	1	200	395	78
8	2	200	306	94

Dans les cas 5, 6 et 8 les résultats sont nettement plus favorables tant en ce qui concerne U_p (qui ne dépasse pas 425 V) que la valeur de l'énergie (qui ne depasse pas 118 mJ). Les valeurs minimales sont $U_p=306$ V et $W_p=78$ mJ. Ces performances ont été réalisées dans la présence d'une bobine d'induction avec $n=150$ et $n=200$.

Il résulte que les meilleurs résultats sont fournis par les installation munies avec une bobine d'induction qui possède un grand rapport de transformation ($n=200$ ou, au moins, $n=150$).

En comparant ces résultats et ceux publiés dans la littérature [4] ($U_p=400$ V, $n=100$, $W_p=80$ mJ) on trouve, à l'aide de la relation (4), que la tension du secondaire est $U_s=20\,000$ V. Cette valeur peut assurer un fonctionnement normal du moteur à chaud mais peut présenter des difficultés au démarage à froid. La situation est pire lors de l'utilisation d'une bobine ayant un rapport de transformation $n=50$. Dans ce cas il résulte $U_s=12\,500$ V, valeur pratiquement inutilisable pour le démarage d'un moteur avec un grand taux de compression.

Conclusions

L'étude du fonctionnement du systéme d'allumage du type capacitif equipé avec des bobines d'induction possédant un rapport de transformation compris entre 50 et 200, un rendement $\eta=0,5$, la capacité de charge du secondaire en étant de 100 pF et la valeur de la haute tension $U_s=25\,000$ V conduit aux conclusions suivantes ;

1. La plus favorable valeur du rapport de transformation de la bobine est $n=200$, éventuellement $n=150$.

2. Il est possible que le système d'allumage électrique du type capacitif, ne soit pas capable à donner satisfaction dans le cas des vehicules

qui sont équipés avec une bobine d'induction ayant un rapport de transformation $n=100$. Dans ces cas $U_r=400$ V, $U_s=20\,000$ V et le moteur peut fonctionner normalement à chaud, mais peut présenter des difficultés au démarage à froid.

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ASUPRA UNOR ASPECTE PRIVIND UTILIZAREA SISTEMULUI DE APRINDERE ELECTRICĂ DE TIP CAPACITIV

(Rezumat)

Se studiază utilizarea sistemului capacitiv la bobine de inducţie avînd raportul numărului de spire secundar/primar (n) cuprins între 50 şi 200. Admiţînd că energia acumulată în condensatorul sistemului se transmite în secundar cu un randament relativ mic, de circa 50%, se găseşte că valoarea tensiunii necesară la condensatorul sistemului este de 25 000 V pe o sarcină $C_s=100$ pF. Din calcule rezultă că cea mai adecvată bobină de inducţie este cea care are $n=200$, urmată apoi de cea cu $n=150$. Studiul întreprins explică cazurile pentru care la unele autovehicule sistemul de aprindere electrică de tip capacitiv nu dă rezultatele aşteptate.

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RECENZII

BOOK REVIEWS РЕЦЕНЗИИ

GUIAŞU S., *Information Theory with Applications*. McGraw Hill, Inc., N.Y., 1977, 439 pp.

The book, an up-to-date treatise on information theory, is divided into five parts : Entropy, Probabilistic Theory of the Transmission of Information, Algebraic Coding, Applications to Statistical Inference and Applications to Classification Theory, Pattern-recognition Theory and Game Theory. Each part is followed by comments and exercises. At the end of the book the reader will find a large amount of bibliographic titles and also an author index and a subject index. Parts of the book can be used for special courses on entropy, transmission of information and coding theory. The self-contained character makes the book useful not only for mathematicians, information theorists and engineers but also for physicists and geneticists.

V. Munteanu

WEBER H., *Laplace-Transformation für Ingenieure der Elektrotechnik*. B. G. Teubner Verlag, Stuttgart, 1978, 197 S.

Als mathematisches Instrument für die Lösung einiger Probleme der Elektrotechnik unter dem Gesichtspunkte Zeit—Frequenz erweist sich die Laplace-Transformation nicht nur als ein moderner sondern auch als ein unentberlicher Weg.

Aber der Ingenieur, als Mann der physikalischen Wirklichkeiten, beherrscht oftmals nicht so leicht die abstrakte Sprache der Mathematik, die ihm brauchbar ist.

Man muß daher eine gemeinsame Sprache finden — allgemeinerweise vereinfacht — die dem Ingenieur zugänglich ist und das beabsichtigt das vorliegende Buch.

Der Verfasser definiert schrittweise, in erstem Kapitel, die Fourier-Reihe, die Fourier-Integrale, die Fourier-Transformation (Direkte- und Inverse-Fourier-Transformation) und schließlich die Laplace-Transformation, als verallgemeinerte Transformation der letzten. Die Laplace- Transformation besitzt eine größere Aussagekraft dank ihrer besseren Konvergenz-Eigenschaften. In demselben Kapitel zeigt der Verfasser die Rechnungsmöglichkeiten der inversen Laplace-Transformierten mittels des umständlichen Verfahrens der Reziduen, nach der Funktionstheorie.

Also versucht der Verfasser im zweiten Kapitel die Rechnung der Laplace-Transformation zu vereinfachen mittels einiger Sätze, die mit nicht anspruchsvollen Möglichkeiten beweisbar sind.

In dem dritten und letzten Kapitel gibt es einige Anwendungen dieser Sätze, von jetzt an als strenge Transformationsregel — insbesondere im Bereich von Differentialgleichungen und von Systemen der Differentialgleichungen — betrachtet.

Es gibt auch am Ende des Buches, als Hilfsmittel, einige Korrespondenzen Bild-Funktion/Original-Funktion und die Laplace-Transformierten einzelner Impulse und periodischer Zeitfunktionen.

In einer nicht zu anspruchsvollen Sprache geshrieben, überzeugt das Buch dank seiner Beweiseinfachheit und eigener Auswahl der Anwendungen, so daß der Studierende enthusiastisch ausschreien könnte : „Die Laplace-Transformation ? Keine Problem !".

P. Pogăngeanu

PAPOULIS A., *Signal Analysis*. Mc Graw Hill, Inc., N. Y., 1977, 431 pp.

The book is divided into 12 chapters grouped into three parts, namely : Part One : Signals, systems and transforms ; Part Two :Selected topics and Part Three : Data smoothing and spectral estimation.

In the first part of the book, over 5 chapters (Introduction, Discrete systems, Fourier analysis, Continuous systems and Digital processing of analog signals), analog and discrete signals and linear systems are presented by means of Fourier integral. The use of Dirac distribution (as in Papoulis' first book, *The Fourier Integral and Its Applications*, McGraw-Hill, New York, 1962) leads to important advantages as rigour, efficiency, simplicity and, last but not least, elegancy.

The second part covers selected topics and contains 3 chapters (Bandlimited functions, Factorization, Windows, Hilbert transforms and Frequency modulation, uncertainty, ambiguity).

The third part contains the last 4 chapters : Stochastic processes, Data smoothing, Ergodicity, correlation estimators, Fourier transforms of random signals and Spectral estimation.

The author's style is remarkable being very clear although concentrate, elegant and didactic. The material is selfcontained and at the end of the book the reader finds 94 problems and their solutions too.

L. Goraş

EDWARDS D.F.A., *Electronic Measurement Techniques*. Butterworths, London, 1971.

This book gives us, in one volume, informations about electronic measurement techniques with a wiew to selecting and using the electronic measurement instruments.

The book has 18 chapters and 377 pages. In the first chapter it is presented the international system of units (S.I.), notions concerning measuring methods and errors. The next three chapters give us briefly the classical measuring instruments used for the direct and alternating current. The classical measuring methods are the subject of another three chapters where the author analyses the apparatus and null methods, destinated to measure the characteristic quantities of magnetic and dielectric materials as well as of certain nonelectric quantities. The remaining chapters presents and analyses various electronic instruments, namely : analog and digital voltmeters, oscilloscopes and their use, frequencemeters, apparatus working on the principle of resonance, signal generators. There are also presented the measuring methods of the characteristics of the transistors, of the integrated circuits and amplifiers as well as the measuring methods at very high frequency.

Each chapter ends with bibliographical recommendations which permit the reader to widen his area of information in electronics. The examples and practical exercices at the end of each chapter are very instructive for the readers, some of them being completely solved and the others having the final results.

Due to the simple, brief and clear style, D.F.A. Edwards book is very useful for those who want to learn the technique of electronic measurement.

D. Bărbulescu

KAMKE D., KRÄMER K., *Physikalische Grundlagen der Masseinheiten*. B. G. Teubner Verlag, Stuttgart, 1977, 218 S.

Die Meßtechnik ist ungetrennt von ihr quantitativen Anblick, durch die Maßeinheiten-Systeme ausgedrückt.

Fast alle Bücher der Meßtechnik (insbesondere die die auf elektrischem Wege verwirklicht ist) behandeln das Problem der Maßeinheiten überwiegend theoretisch.

Die Verfasser dieses Buches versuchen, dagegen, sich mit den physikalischen Grundlagen der praktischen Realisierung der Maßeinheiten zu beschäftigen und zwar mit den Normalien der Basis-Einheiten.

Man weiß daß das Internationale System der Masseinheiten (S.I.) herrschend geworden ist nach einem langen und verschlungenen Versuchsvorgang, oftmals im Widerspruch mit dem gesunden Menschenverstand (s. CGS-Kompromisse).

Erwerbung der ganzen Menschheit — sowohl in der Physik als auch in der Technik — scheint es als notwendig für S.I. die Kenntnis der Technologie der Verkörperung seiner sieben

Basis-Einheiten, weil diese allen abgeleiteten Einheiten übriger bekannter physikalischer Größen zu Grunde liegen.

Da die Stabilität, die Reproduzierbarkeit und die Genauigkeit der Normalien zeitlich immer verbessert worden ist, behandeln die Verfasser das Problem unter dem historischen Gesichtspunkte, was methodisch ein positives Merkmal ist.

Es scheint daher als fortschrittliche Aspekte der Anschluß einiger Normalien (Länge und Zeit, z. B.) an Erscheinungen des Mikrokosmosses (physikalisch-atomische) statt der Erscheinungen des Makrokosmosses (astronomische).

Es handelt auch über die Auswertung der letzten Entdeckungen der Physik (Laser, Maser, J o s e p h s o n-Effekt, u.s.w.) zum Aufbau der möglichst getreuen Normalien.

Als Studium- und Erforschungsinstrument ist das leicht verständliche und reich illustrierte Buch oftmals mit vielen bibliographischen Referenzen angemerkt und das erlaubt die Vertiefung, nach Wunsch, irgendeines Problems.

P. Pogângeanu

OLSEN G. H., *Electronics. A Course Book for Students.* Newnes, Butterworths, London, 1980.

The book, at its fifth edition, is intended for the unspecialist readers in electronics but which passes the foundations of electrical engineering.

The author, professor at the Polytechnic Institute in Newcastle, England, succeded in presenting, only in a single volume, the main electronic components and devices giving the proper importance both theoretical and practical aspects. Thus he can offer the reader a very important and useful instructive material.

The volume contains eight chapters. The first three ones, after a brief introduction, deal with the passive components and the time-response of the circuits containing such components, connected to certain sinusoidal and rectangular signal sources.

The next chapter is devoted to semiconductor components and it offers a comprehensive treatment of the rectifier, Zener and tunnel diodes, and J—FET and MOS—FET transistors.

In the fifth chapter the reader may find useful concepts concerning the construction and behaviour of the main classical types of measuring instruments, methods which allow him to analyse and verify the performances of the electronic components and devices, by measurements.

In the last three chapters, which hold more than half of the volume, the supply-sources, electronic and amplifiers, important components of different types of electronic instruments are described and analysed. Among the mentioned chapters, the seventh is the largest one, the electronic amplifiers being here widely studied. The author starts from the amplifier, studies the parameters of the transistors and their equivalent circuits, the multi-stage-amplifiers analysing specially the feedback and its effects upon the performances of the amplifiers. The last chapter deals with electronic oscillators and different types of modern transistored and integrated circuits signal-generators.

In addition the book contains five appendix very useful to the reader to follow the text. The last appendix, concerning the basic matrix theory, is a very useful one. Besides the author gives the reader important lists of references at the end of each chapter.

It is to be remarked the fact that, besides the theoretical treatment of all the problems tackled, the author presents many practical diagrams very useful to those who want to achieve performed electronic devices.

The book, at a high level, rich in content, is a modern and important instrument of information and documentation in electronics.

D. Bărbulescu

CLAYTON G. B., *Operational Amplifiers.* Second ed., Butterworths, London — Boston 1979, 410 pp.

This book covers a complete systematized material concerning the operational amplifier devices which provide a great employment in the rapidly developing area of electronic instrumentation.

The author, principal lecturer at the Liverpool Polytechnics, England, a well known, research worker in the domain of operational amplifier circuits, offers to the reader the theoretical concepts necessary for understanding the operational amplifier's behaviour in different applications.

The work provides the reader with the terminology used into manufacturer's data sheet to specify the amplifier's performances, with the practical knowledge necessary to select the most adequate amplifier for a particular application.

In comparison with the first edition, issued in 1971, this edition is revised and completed with references concerning the new operational amplifier types with improved performances and the new circuit types, in which the first ones are incorporated.

First, the author presents the ideal operational amplifier and some basic circuits using this amplifier.

It follows the explanation of the significance of practical operational amplifiers.

A special chapter is reserved to the measurement of the operational amplifier parameters. The most part of the work, i.e. five chapters, presents the various practical applications (linear and non-linear circuits).

The last chapter insists on the practical considerations which occurr in achievement and employment of the real operational amplifier circuits.

Each chapter contains numerical exercices directly related to practical design considerations.

In the book the theoretical aspects of the problem together with practical results of the author's experience are studied, constituting an important reference work and an application handbook for work-shop engineers or/and electronic instrument designers.

C. Mărcuță

BLITZ J., KING W. G., ROGERS D. G., *Electrical, Magnetic and Visual Methods of Testing Materials.* Butterworths, London, 1969, 202 pp.

Electrical, Magnetic and Visual Methods of Testing Materials, by J. Blitz, W. G. King and D. G. Rogers, published by Butterworths, is included in a three volume series written for the technician's help, covering the non-distructive methods of testing materials.

The work contains about 200 pages divided in six chapters, an introduction, an appendix and an index for the technical terms used, dealing especially with magnetic and optical testings, everithing being presented in an attractive and comprehensive manner, containing a lot of drawings diagrams and photos.

The main chapters of the work : *Eddy Current Methods, Magnetic Particle Testing-Procedure, Magnetic Particle Testing-Equipment* and *Penetrant Methods* contain numerous practical using instructions the respective equipment, variant of achievment, all of them being based on the theory presented at the beginning of every chapter.

Some of the methods, known for a long time, are brought up-to-date, just owing to the performances of industrialy or laboratory equipment which are available in USA and Great Britain, when the work issued.

The adding bibliography of every chapter helps the interested ones with a more detailed information.

As an example, in the appendix, the „Magnatest Q Förster" equipment, interpretation of the results obtained with it and other necessary characteristics for an efficient using are described in detail.

The other two chapters : *Electrical and Magnetic Methods of Testing Materials — General* and *Optical Methods of Testing*, which are shorter and more general, permit the understanding of the classical methods and of deficienties determination by means of either electrical, magnetic and optical measurements by some other lessknown methods (D. C. Flux Methods, Microwaves Techniques, Holography).

In a field where, theoretically, it seemed that the classical methods of testing materials should be eliminated, this work represents a rehabilitation of these presenting numerous practical instructions.

The work proves to be a useful source for the technicians who work in the domain of electricity or mechanics, to whom it offers simple and cheep ways of testing materials by nondistructive methods.

Tr. Balan

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PROPAGATION DU CHAMP ÉLECTROMAGNÉTIQUE DANS UNE TÔLE FERROMAGNÉTIQUE CONDUCTRICE, À FAIBLE NONLINÉARITÉ, EN RÉGIME QUASISTATIONNAIRE HARMONIQUE

PAR

HUGO ROSMAN

Dédiée au professeur Dr. D. I. MANGERON à l'occasion de son 75-e anniversaire

1. Introduction

L'étude de la propagation du champ électromagnétique dans une tôle ferromagnétique conductrice, tout en tenant compte du caractère nonlinéaire du milieu, d'une part présente un grand intérêt pratique mais d'autre part implique des difficultés analytiques quelque fois considérables. Le désir de disposer d'une méthode accésible quant aux procédés de calcul a mobilisé, dans les dernières années, divers chercheurs qui ont orienté leurs recherches dans cette direction. On peut mentionner, dans ce sens, les résultats obtenus par Bailsford et Burgess [1], Reiche [2], Kesavamurthy et Rajagopalan [3], Kuschel et collaborateurs [4], Krasovitsky et Kurilo [5]. Toutefois il est nécessaire à souligner les contributions remarquables apportés dans la même direction par Fl. Stănciulescu [6], [7], qui a utilisé, avec succès, dans l'étude de la propagation du champ électromagnétique dans une tôle ferromaqnétique conductrice, mince, la méthode de la convolution complexe.

Les résultats acquis par les auteurs ci-dessus concordent d'une manière satisfaisante avec ceux obtenus par voie expérimentale mais se heurtent souvent à de grandes difficultés quant à leur mise en valeur, compte tenu du caractère complexe et difficile à manipuler des relations établies. Une tentative, que nous l'espérons réussie, de dépasser ces difficultés, est représentée par ce qui suit. Dans ce but on va élaborer une théorie assez simple et toutefois suffisamment exacte de la propagation du champ électromagnétique *lentement variable*, dans une tôle ferromagnétique conductrice à *faible nonlinéarité*. Les deux restrictions supplémentaires sont capables à maintenir un cadre assez large pour le domaine d'applicabilité du procédé proposé, en permettant, tout comme l'on a montré dans les Notes antérieures [8], [9], à adapter, pour le cas des milieux électromagnétiques nonlinéaires, le concept d'*inertialité* utilisé à l'étude des circuits électriques nonlinéaires.

Les prémises adoptées dans l'élaboration de l'étude proposée sont les suivantes [8], [9] :

a) Les vecteurs du champ électromagnétique varient, en fonction du temps, selon des lois sinusoïdales.

b) La caractéristique de magnétisation du milieu, en valeurs efficaces, est décrite par la relation :

$$B = \mu_1 H - \mu_3 H^3, \tag{1}$$

tandis que celle en valeurs instantanées est considérée linéaire (ce qui justifie, au fond, l'hypothèse introduite au point précédent).

c) La perméabilité magnétique du milieu, nonlinéaire, peut être approximée à l'aide de l'expression :

$$\mu = \mu_1 - \frac{3}{4}\mu_3 H_m^2, \tag{2}$$

où H_m est l'amplitude de l'intensité du champ magnétique.

d) La conductibilité σ de la tôle ferromagnétique est considérée constante et indépendante de l'état du champ électromagnétique dans la tôle.

2. Les équations du champ électromagnétique dans une tôle ferromagnétique conductrice, à faible nonlinéarité, en régime quasistationnaire harmonique

On considère une tôle ferromagnétique conductrice ayant l'épaisseur $d=2a$ (Fig. 1), le reste de ses dimensions (pratiquement infinies) en dépassant sensiblement son épaisseur, située dans un champ électromagnétique ayant à la surface de la tôle seulement une composante tangentielle, H_a, qui oscille en fonction du temps selon une loi sinusoïdale avec une fréquence suffisamment basse et ayant la valeur efficace complexe $\underline{H}_a$.

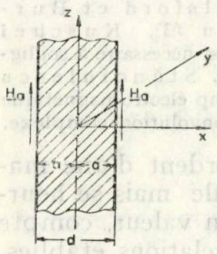

Fig. 1

L'équation différentielle vérifiée par la composante $\underline{H}_z$ de l'intensité du champ magnétique (d'ailleurs la seule qui existe) dans un point situé à l'intérieur de la tôle est identique avec celle satisfaite par la même composante dans un point situé à l'intérieur du demi-espace conducteur infini, en des conditions similaires [8] :

$$\frac{d^2\underline{H}_z}{dx^2} - j\omega\sigma\mu_1\underline{H}_z + j\frac{3}{2}\omega\sigma\mu_3\underline{H}_z^3 = 0. \tag{3}$$

Ici, tout comme dans [9], on a introduit l'approximation $H_m \approx \sqrt{2}H_z$, en général acceptable, qui permet à réduire l'étude de la propagation du champ électromagnétique à l'intégration d'une équation différentielle de type Duffing [10]. Afin d'obtenir la solution de cette équation on a adopté la *méthode de la perturbation* [11], en déterminant en dehors de la solution de l'équation génératrice et les solutions de la première et de la deuxième approximation. Dans ce but il s'avère avantageux à introduire la notation :

$$\lambda = \frac{3}{2}\omega\sigma\mu_3 \tag{4}$$

qui peut être considéré comme un petit paramètre car dans le cas des faibles nonlinéarités μ_3 a une valeur réduite.

Si l'on tient compte de (4) l'équation différentielle (3) devient :

$$\frac{d^2\underline{H}_z}{dx^2} - j\omega\sigma\mu_1\underline{H}_z + j\lambda\underline{H}_z^3 = 0, \tag{5}$$

ayant la solution :

$$\underline{H}_z=\underline{H}_{z1}+\lambda\underline{H}_{z2}+\lambda^2\underline{H}_{z3}, \tag{6}$$

où $\underline{H}_{z1}$ représente la solution de l'équation génératrice, $\underline{H}_{z2}$ — la solution de la première et $\underline{H}_{z3}$ — la solution de la deuxième approximation. Évidemment, $\underline{H}_z$ donné par (6) doit satisfaire l'équation (5) de sorte que :

$$\frac{d^2\underline{H}_{z1}}{dx^2}+\lambda\frac{d^2\underline{H}_{z2}}{dx^2}+\lambda^2\frac{d^2\underline{H}_{z3}}{dx^2}-j\omega\sigma\mu_1(\underline{H}_{z1}+\lambda\underline{H}_{z2}+\lambda^2\underline{H}_{z3})+ \tag{7}$$

$$+j\lambda(\underline{H}_{z1}+\lambda\underline{H}_{z2}+\lambda^2\underline{H}_{z3})^3=0.$$

2.1. Solution de l'équation génératrice

L'équation génératrice est de la forme :

$$\frac{d^2\underline{H}_{z1}}{dx^2}-j\omega\sigma\mu_1\underline{H}_{z1}=0, \tag{8}$$

en admettant la solution [12] :

$$\underline{H}_{z1}=\underline{H}_a\frac{ch\underline{\gamma}x}{ch\underline{\gamma}a}, \qquad (\underline{H}_a=\underline{H}_{z1}(-a)=\underline{H}_{z1}(a)), \tag{9}$$

avec

$$\underline{\gamma}=\sqrt{j\omega\sigma\mu_1}=\alpha+j\alpha \tag{10}$$

— la constante de propagation complexe et

$$\alpha=\sqrt{\frac{\omega\sigma\mu_1}{2}} \tag{11}$$

— la constante d'atténuation (de phase).

2.2. Solution de l'équation de la première approximation

Si l'on identifie dans (7) les termes en λ on obtient l'équation différentielle de la première approximation :

$$\frac{d^2\underline{H}_{z2}}{dx^2}-j\omega\sigma\mu_1\underline{H}_{z2}+j\underline{H}_{z1}^3=0, \tag{12}$$

ou, compte tenu de (9) :

$$\frac{d^2\underline{H}_{z2}}{dx^2}-j\omega\sigma\mu_1\underline{H}_{z2}=-j\underline{H}_a^3\frac{ch^3\underline{\gamma}x}{ch^3\underline{\gamma}a}. \tag{13}$$

La solution de cette équation peut être obtenue, par exemple, à l'aide de la méthode de la variation des constantes, en résultant de la forme (v. Annexe 1) :

$$\underline{H}_{z2}=j\frac{\underline{H}_a^3}{16\underline{\gamma}^2ch^3\underline{\gamma}a}\left[-\frac{1}{2}ch3\underline{\gamma}x+\left(2ch^2\underline{\gamma}a-\frac{3}{2}+6\underline{\gamma}a\,th\underline{\gamma}a\right)ch\underline{\gamma}x-6\underline{\gamma}x\,sh\underline{\gamma}x\right]. \tag{14}$$

2.3. Solution de l'équation de la deuxième approximation

Si l'on utilise le même procédé que celui appliqué dans le paragraphe précédent on identifie en (7) les termes en λ^2, obtenant, par cette voie, l'équation différentielle de la deuxième approximation :

$$\frac{d^2\underline{H}_{z3}}{dx^2} - j\omega\sigma\mu_1\underline{H}_{z3} + j3\underline{H}_{z1}^2\underline{H}_{z2} = 0. \tag{15}$$

Compte tenu de (9) et (14) l'équation (15) devient :

$$\frac{d^2\underline{H}_{z3}}{dx^2} - j\omega\sigma\mu_1\underline{H}_{z3} = \frac{3\underline{H}_a^5 \operatorname{ch}^2\underline{\gamma}x}{16\underline{\gamma}^2\operatorname{ch}^5\underline{\gamma}a}\left[-\frac{1}{2}\operatorname{ch}3\underline{\gamma}x + \left(2\operatorname{ch}^2\underline{\gamma}a - \frac{3}{2} + 6\underline{\gamma}a\operatorname{th}\underline{\gamma}a\right)\operatorname{ch}\underline{\gamma}x - 6\underline{\gamma}x\operatorname{sh}\underline{\gamma}x\right]. \tag{16}$$

Si l'on utilise et dans ce cas la méthode de la variation des constantes (Annexe 2) on peut obtenir la solution de l'équation (16) sous la forme :

$$\underline{H}_{z3} = \frac{3}{64}\cdot\frac{\underline{H}_a^5}{\underline{\gamma}^4\operatorname{ch}^5\underline{\gamma}a}\left[-\frac{1}{48}\operatorname{ch}5\underline{\gamma}x + \frac{1}{2}\left(\frac{1}{2}\operatorname{ch}^2\underline{\gamma}a - 1 + \frac{3}{2}\underline{\gamma}a\operatorname{th}\underline{\gamma}a\right)\operatorname{ch}3\underline{\gamma}x + \left(-\frac{2}{3}\operatorname{ch}^4\underline{\gamma}a + \frac{7}{3}\operatorname{ch}^2\underline{\gamma}a - \frac{67}{48} - \frac{1}{2}\underline{\gamma}a\operatorname{th}\underline{\gamma}a + \frac{3}{2}\underline{\gamma}^2a^2 - 3\underline{\gamma}a\frac{\operatorname{sh}^3\underline{\gamma}a}{\operatorname{ch}\underline{\gamma}a} - 9\underline{\gamma}^2a^2\operatorname{th}^2\underline{\gamma}a\right)\operatorname{ch}\underline{\gamma}x - \frac{3}{4}\underline{\gamma}x\operatorname{sh}3\underline{\gamma}x - \frac{3}{2}\underline{\gamma}^2x^2\operatorname{ch}\underline{\gamma}x + (3\operatorname{ch}^2\underline{\gamma}a - 1 + 9\underline{\gamma}a\operatorname{th}\underline{\gamma}a)\underline{\gamma}x\operatorname{sh}\underline{\gamma}x\right]. \tag{17}$$

2.4. La solution générale

Si l'on substitue en (6) les expressions (9), (14) et (17) de $\underline{H}_{z1}$, $\underline{H}_{z2}$ et, respectivement, $\underline{H}_{z3}$ et l'on tient compte, toutefois, de (4), on obtient :

$$\underline{H}_z = \underline{H}_a\frac{\operatorname{ch}\underline{\gamma}x}{\operatorname{ch}\underline{\gamma}a} + \frac{3}{32}\cdot\frac{\mu_3}{\mu_1}\cdot\frac{\underline{H}_a^3}{\operatorname{ch}^3\underline{\gamma}a}\left[-\frac{1}{2}\operatorname{ch}3\underline{\gamma}x + \left(2\operatorname{ch}^2\underline{\gamma}a - \frac{3}{2} + 6\underline{\gamma}a\operatorname{th}\underline{\gamma}a\right)\operatorname{ch}\underline{\gamma}x - 6\underline{\gamma}x\operatorname{sh}\underline{\gamma}x\right] - \frac{27}{256}\cdot\frac{\mu_3^2}{\mu_1^2}\cdot\frac{\underline{H}_a^5}{\operatorname{ch}^5\underline{\gamma}a}\left[-\frac{1}{48}\operatorname{ch}5\underline{\gamma}x + \frac{1}{2}\left(\frac{1}{2}\operatorname{ch}^2\underline{\gamma}a - 1 + \frac{3}{2}\underline{\gamma}a\operatorname{th}\underline{\gamma}a\right)\operatorname{ch}3\underline{\gamma}x + \left(-\frac{2}{3}\operatorname{ch}^4\underline{\gamma}a + \frac{7}{3}\operatorname{ch}^2\underline{\gamma}a - \frac{67}{48} - \frac{1}{2}\underline{\gamma}a\operatorname{th}\underline{\gamma}a + \frac{3}{2}\underline{\gamma}^2a^2 - 3\underline{\gamma}a\frac{\operatorname{sh}^3\underline{\gamma}a}{\operatorname{ch}\underline{\gamma}a} - 9\underline{\gamma}^2a^2\operatorname{th}^2\underline{\gamma}a\right)\operatorname{ch}\underline{\gamma}x - \frac{3}{4}\underline{\gamma}x\operatorname{sh}3\underline{\gamma}x - \frac{3}{2}\underline{\gamma}^2x^2\operatorname{ch}\underline{\gamma}x + (3\operatorname{ch}^2\underline{\gamma}a - 1 + 9\underline{\gamma}a\operatorname{th}\underline{\gamma}a)\underline{\gamma}x\operatorname{sh}\underline{\gamma}x\right]. \tag{18}$$

Pour déterminer l'expression de l'intensité du champ électrique on utilise l'équation :

$$\text{rot } \underline{\mathbf{H}} = \sigma \underline{\mathbf{E}} \tag{19}$$

et l'on tient compte, en outre, que dans le cas considéré $\underline{\mathbf{H}} = \mathbf{k}\underline{H}_z$, $\underline{\mathbf{E}} = \mathbf{j}\underline{E}_y$ de sorte que

$$\underline{E}_y = -\frac{1}{\sigma} \cdot \frac{\mathrm{d}\underline{H}_z}{\mathrm{d}x}. \tag{20}$$

Tous calculs effectués on obtient :

$$\begin{aligned}\underline{E}_y = -\frac{\underline{\gamma}}{\sigma}\Bigg\{ & \frac{\underline{H}_a}{\mathrm{ch}\underline{\gamma}a}\,\mathrm{sh}\underline{\gamma}x + \frac{3}{32}\cdot\frac{\mu_3}{\mu_1}\cdot\frac{\underline{H}_a^3}{\mathrm{ch}^3\underline{\gamma}a}\Bigg[-\frac{3}{2}\,\mathrm{sh}3\underline{\gamma}x + \Bigg(2\,\mathrm{ch}^2\underline{\gamma}a - \\ & -\frac{15}{2} + 6\underline{\gamma}a\,\mathrm{th}\underline{\gamma}a\Bigg)\mathrm{sh}\underline{\gamma}x - 6\underline{\gamma}x\,\mathrm{ch}\underline{\gamma}x\Bigg] - \frac{27}{256}\cdot\frac{\mu_3^2}{\mu_1^2}\cdot\frac{\underline{H}_a^5}{\mathrm{ch}^5\underline{\gamma}a}\Bigg[-\frac{5}{48}\,\mathrm{sh}5\underline{\gamma}x + \\ & +\frac{3}{4}(\mathrm{ch}^2\underline{\gamma}a - 3 + 3\underline{\gamma}a\,\mathrm{th}\underline{\gamma}a)\mathrm{sh}\,3\underline{\gamma}x + \Bigg(-\frac{2}{3}\,\mathrm{ch}^4\underline{\gamma}a + \frac{16}{3}\,\mathrm{ch}^2\underline{\gamma}a - \frac{115}{48} + \\ & +\frac{17}{2}\underline{\gamma}a\,\mathrm{th}\underline{\gamma}a + \frac{3}{2}\underline{\gamma}^2a^2 - 3\underline{\gamma}a\,\frac{\mathrm{sh}^3\underline{\gamma}a}{\mathrm{ch}\underline{\gamma}a} - 9\underline{\gamma}^2a^2\,\mathrm{th}^2\underline{\gamma}a\Bigg)\mathrm{sh}\underline{\gamma}x - \frac{9}{4}\,\underline{\gamma}x\,\mathrm{ch}3\underline{\gamma}x - \\ & -\frac{3}{2}\,\underline{\gamma}^2x^2\,\mathrm{sh}\underline{\gamma}x + (3\mathrm{ch}^2\underline{\gamma}a - 4 + 9\underline{\gamma}a\,\mathrm{th}\underline{\gamma}a)\underline{\gamma}x\,\mathrm{ch}\underline{\gamma}x\Bigg]\Bigg\}.\end{aligned} \tag{21}$$

Dans les relations (18) et (21) les composantes $\underline{H}_z$ et $\underline{E}_y$ du champ magnétique, respectivement électrique, dans un point situé à l'intérieur d'une tôle ferromagnétique conductrice, ont été exprimées comme fonctions de l'intensité du champ magnétique à la surface de la tôle, $\underline{H}_a$, considéré comme orienté selon une direction tangentielle à cette surface. Cependant les mêmes composantes peuvent être exprimées comme fonctions de la valeur moyenne de l'induction magnétique dans la tôle, ce qui peut constituer un avantage dans certaines situations. Dans ce cas on détermine préalablement, la valeur moyenne du champ magnétique dans la tôle, $\underline{H}_{\mathrm{med}}$, à l'aide de la relation :

$$\underline{H}_{\mathrm{med}} = \frac{1}{2a}\int_{-a}^{+a} \underline{H}_z\,\mathrm{d}x \tag{22}$$

en résultant (v. Annexe 3) :

$$\begin{aligned}\underline{H}_{\mathrm{med}} = & \frac{2}{\underline{\gamma}}\,\underline{H}_a\frac{\mathrm{sh}\underline{\gamma}a}{\mathrm{ch}\underline{\gamma}a} + \frac{3}{16}\cdot\frac{\mu_3}{\mu_1}\cdot\frac{\underline{H}_a^3}{\underline{\gamma}\mathrm{ch}^3\underline{\gamma}a}\Bigg(\frac{4}{3}\,\mathrm{sh}^3\underline{\gamma}a + 6\,\mathrm{sh}\underline{\gamma}a - \frac{6\underline{\gamma}a}{\mathrm{ch}\underline{\gamma}a}\Bigg) + \\ & +\frac{27}{256}\cdot\frac{\mu_3^2}{\mu_1^2}\cdot\frac{\underline{H}_a^5}{\underline{\gamma}\,\mathrm{ch}^5\underline{\gamma}a}\Bigg(\frac{12}{15}\,\mathrm{sh}^5\underline{\gamma}a + \frac{11}{3}\,\mathrm{sh}^3\underline{\gamma}a + \frac{19}{2}\,\mathrm{sh}\underline{\gamma}a + \frac{27}{2}\underline{\gamma}a\,\frac{\mathrm{sh}^2\underline{\gamma}a}{\mathrm{ch}\underline{\gamma}a} + \\ & +8\underline{\gamma}a\,\mathrm{ch}^3\underline{\gamma}a - 18\underline{\gamma}^2a^2\,\frac{\mathrm{sh}\underline{\gamma}a}{\mathrm{ch}^2\underline{\gamma}a}\Bigg).\end{aligned} \tag{23}$$

Sans doute, la solution de l'équation (23) par rapport à $\underline{H}_a$ se heurte, dans le cas général, à des difficultés considérables, qui peuvent être dépassées seulement dans des cas concrets, en utilisant des techniques numériques de calcul. Une fois déterminée l'intensité, $\underline{H}_a$, du champ magnétique, comme fonction de la valeur moyenne, $\underline{H}_{med}$, de celle-ci, on peut l'exprimer et en fonction de la valeur moyenne, B_{med}, de l'induction magnétique, à l'aide de la relation :

$$\underline{H}_{med} = \frac{\underline{B}_{med}}{\mu_1 - \frac{3}{4}\mu_3 H_{med}^2}, \tag{24}$$

où l'on a tenu compte de (1).

3. La cas des tôles minces

Pour une tôle mince, compte tenu que $|\underline{\gamma}a| \ll 1$, $|\underline{\gamma}x| \ll 1$, il est possible à retenir dans les développements en séries des fonctions trigonométriques hyperboliques qui interviennent dans les expressions (18) de $\underline{H}_z$ et (21) de $\underline{H}_y$ seulement un nombre fini de termes. L'approximation est tout à fait satisfaisante si dans ces développements on retient les termes en a ou x dont la puissance est au plus trois.

Les détails de calcul sont résumés dans l'Annexe 4, en obtenant finalement les relations :

$$\underline{H}_z = \frac{1+\alpha^4a^2x^2-j\alpha^2(a^2-x^2)}{1+\alpha^4a^4}\underline{H}_a + j\frac{3}{4}\omega\sigma\mu_3(a^2-x^2)\underline{H}_a^3\left(\frac{1-j3\alpha^2a^2}{1+9\alpha^4a^4} - \right. \tag{25}$$
$$\left. - \frac{27}{64}\cdot\frac{\underline{\mu}_3}{\mu_1}\cdot\frac{1-j5\alpha^2a^2}{1+25\alpha^4a^4}\underline{H}_a^3\right),$$

respectivement

$$\underline{E}_y = -j\omega\mu_1\underline{H}_a\left(\frac{1-j\alpha^2a^2}{1+\alpha^4a^4} - \frac{3}{2}\cdot\frac{\underline{\mu}_3}{\mu_1}\cdot\frac{1-j3\alpha^2a^2}{1+9\alpha^4a^4}\underline{H}_a^2 + \frac{81}{128}\cdot\frac{\underline{\mu}_3^2}{\mu_1^2}\cdot\frac{1-j5\alpha^2a^2}{1+25\alpha^4a^4}\underline{H}_a^4\right)x, \tag{26}$$

où α est donné par (11). Soient

$$\underline{H}_{zl} = \frac{1+\alpha^4a^2x^2-j\alpha^2(a^2-x^2)}{1+\alpha^4a^4}\underline{H}_a, \qquad \underline{E}_{yl} = -j\omega\mu_1\frac{1-j\alpha^2a^2}{1+\alpha^4a^4}x\underline{H}_a \tag{27}$$

les expressions des valeurs efficaces complexes des intensités du champ magnétique, respectivement électrique dans le cas d'une tôle à caractéristique de magnétisation linéaire ($\mu_1 = \mu$, $\mu_3 = 0$). On definit les déviations :

$$\underline{H}_z - \underline{H}_{zl} = j\frac{3}{4}\omega\sigma\mu_3\left(\frac{1-j3\alpha^2a^2}{1+9\alpha^4a^4} - \frac{27}{64}\cdot\frac{\underline{\mu}_3}{\mu_1}\cdot\frac{1-j5\alpha^2a^2}{1+25\alpha^4a^4}\underline{H}_a^2\right)(a^2-x^2)\underline{H}_a^3, \tag{28}$$

$$\underline{E}_y - \underline{E}_{yl} = j\frac{3}{2}\omega\mu_3\left(\frac{1-j3\alpha^2a^2}{1+9\alpha^4a^4} - \frac{27}{64}\cdot\frac{\underline{\mu}_3}{\mu_1}\cdot\frac{1-j5\alpha^2a^2}{1+25\alpha^4a^4}\underline{H}_a^2\right)\underline{H}_a^3x. \tag{29}$$

On peut observer que la déviation $\underline{H}_z - \underline{H}_{zl}$ varie selon une loi parabolique tandis que la déviation $\underline{E}_y - \underline{E}_{yl}$ varie linéairement en fonction de la dis-

tance x. Donc la déviation de l'intensité du champ magnétique par rapport à la valeur qui correspond au cas d'un milieu linéaire est maximum dans l'axe de la tôle tandis que celle du champ électrique est maximum sur les faces de la tôle. Quant aux déviations relatives des intensités du champ on peut les définir à l'aide des relations :

$$\underline{\varepsilon}_H = \frac{\underline{H}_z - \underline{H}_{zl}}{\underline{H}_{zl}}, \qquad \underline{\varepsilon}_E = \frac{\underline{E}_y - \underline{E}_{yl}}{\underline{E}_{yl}}. \tag{30}$$

Compte tenu, d'une part, des expresions (29), (30) et d'autre part de (27), (28), on obtient :

$$\begin{cases} \underline{\varepsilon}_H = \dfrac{\mathrm{j}\dfrac{3}{4}\omega\sigma\mu_3\left(\dfrac{1-\mathrm{j}3\alpha^2a^2}{1+9\alpha^4a^4} - \dfrac{27}{64}\cdot\dfrac{\underline{\mu}_3}{\mu_1}\cdot\dfrac{1-\mathrm{j}5\alpha^2a^2}{1+25\alpha^4a^4}\underline{H}_a^2\right)(a^2-x^2)\underline{H}_a^2}{\dfrac{1+\alpha^4a^2x^2-\mathrm{j}\alpha^2(a^2-x^2)}{1+\alpha^4a^4}}, \\ \underline{\varepsilon}_E = -\dfrac{\dfrac{3}{2}\cdot\dfrac{\underline{\mu}_3}{\mu_1}\left(\dfrac{1-\mathrm{j}3\alpha^2a^2}{1+9\alpha^4a^4} - \dfrac{27}{64}\cdot\dfrac{\underline{\mu}_3}{\mu_1}\cdot\dfrac{1-\mathrm{j}5\alpha^2a^2}{1+25\alpha^4a^4}\underline{H}_a^2\right)\underline{H}_a^2}{\dfrac{1-\mathrm{j}\alpha^2a^2}{1+\alpha^4a^4}}. \end{cases} \tag{31}$$

Il est intéressant à observer que tandis que la déviation relative de l'intensité du champ magnétique, $\underline{\varepsilon}_H$, dépend de x, la déviation relative de l'intensité du champ électrique, $\underline{\varepsilon}_E$, est indépendente de x, ayant la même valeur dans n'importe quelle position du point situé à l'interieur de la tôle.

4. Pertes par courants Foucault dans les tôles ferromagnétiques minces

Par l'unité de surface latérale d'une tôle ferromagnétique, située dans un champ magnétique tangentiel à celle-ci et ayant l'intensité $\underline{H}_a$, pénètre une puissance moyenne :

$$\frac{P}{A} = 2\,|\mathcal{R}e(\underline{\mathbf{E}}_a \times \underline{\mathbf{H}}_a^*)| = 2\,|\mathcal{R}e(\underline{E}_a \underline{H}_a^*)|. \tag{33}$$

Si l'on considère seulement le cas des tôles minces l'expression de l'intensité du champ électrique sur les faces de la tôle, $\underline{E}_a$, peut être déduite de (26) en effectuant la substitution $x = a$:

$$\underline{E}_a = -\mathrm{j}\omega\mu_1\underline{H}_a\left(\frac{1-\mathrm{j}\alpha^2a^2}{1+\alpha^4a^3} - \frac{3}{2}\cdot\frac{\underline{\mu}_3}{\mu_1}\cdot\frac{1-\mathrm{j}3\alpha^2a^2}{1+9\alpha^4a^4}\underline{H}_a^2 + \right. \\ \left. + \frac{81}{128}\cdot\frac{\underline{\mu}_3^2}{\mu_1^2}\cdot\frac{1-\mathrm{j}5\alpha^2a^2}{1+25\alpha^4a^4}\underline{H}_a^4\right)a. \tag{34}$$

En même temps on admet que l'intensité du champ magnétique, $\underline{H}_a$, représente l'origine des phases de sorte que :

$$(35)\quad \frac{P}{A}=\omega^2\mu_1^2\sigma a^2 H_a^2\left|\frac{1}{1+\alpha^4a^4}\cdot-\frac{9}{2}\cdot\frac{\mu_3}{\mu_1}\cdot\frac{H_a^2}{1+9\alpha^4a^4}+\frac{405}{128}\cdot\frac{\mu_3^2}{\mu_1^2}\cdot\frac{H_a^4}{1+25\alpha^4a^4}\right|.$$

Dans l'unité de volume de la tôle les pertes par courants Foucault sont données par la relation

$$(36)\quad p_v=\frac{1}{2}\omega^2\mu_1^2\sigma_a H_a^2\left|\frac{1}{1+\alpha^4a^4}-\frac{9}{2}\cdot\frac{\mu_3}{\mu_1}\cdot\frac{H_a^2}{1+9\alpha^4a^4}+\frac{405}{128}\cdot\frac{\mu_3^2}{\mu_1^2}\cdot\frac{H_a^4}{1+25\alpha^4a^4}\right|.$$

Une étude détaillée de l'expression obtenue sera effectuée dans une Note subséquente.

5. Conclusions

1. On étudie le processus de propagation du champ électromagnétique dans une tôle ferromagnétique conductrice, à faible nonlinéarité, en considérant le milieu comme ayant un caractère „inerti l", dans le sens proposé dans la Note précédente [8].

2. L'équation différentielle satisfaite par la valeur efficace complexe de l'intensité du champ magnétique dans la tôle en étant de la forme (3) et utilisant, pour l'intégration de cette équation, la méthode de la perturbation, on obtient la solution (18). Ensuite on déduit l'expression (21) de la valeur efficace complexe de l'intensité du champ électrique induit dans un point situé à l'intérieur de une tôle.

3. Dans le cas particulier des tôles minces on obtient les expressions (25), pour $\underline{H}_z$ et (26), pour $\underline{E}_y$. Ces expressions sont assez simples et permettent à analyser, sans difficultés, l'influence de la nonlinéarité du milieu.

4. On détermine l'expression (36) des pertes par courants Foucault dans l'unité de volume de la tôle.

Annexe 1

La solution de l'équation (13) est de la forme :

$$(A.\ 1)\qquad \underline{H}_{z2}=\underline{A}_2e^{-\underline{\gamma}x}+\underline{B}_2e^{\underline{\gamma}x},$$

où $\underline{A}_2$ et $\underline{B}_2$ sont des constantes d'intégration que résultent comme solutions du système :

$$(A.\ 2)\qquad \begin{cases}\underline{A}_2'e^{-\underline{\gamma}x}+\underline{B}_2'e^{\underline{\gamma}x}=0,\\ -\underline{\gamma}\,\underline{A}_2'e^{-\underline{\gamma}x}+\underline{\gamma}\underline{B}_2'e^{\underline{\gamma}x}=-j\underline{H}_a^3\dfrac{\mathrm{ch}^3\underline{\gamma}x}{\mathrm{ch}^3\underline{\gamma}a}.\end{cases}$$

On obtient :

$$(A.\ 3)\qquad \begin{cases}\underline{A}_2=\dfrac{j\underline{H}_a^3}{16\underline{\gamma}^2\,\mathrm{ch}^3\underline{\gamma}a}\left(\dfrac{1}{4}e^{4\underline{\gamma}x}+\dfrac{3}{2}e^{2\underline{\gamma}x}+3\underline{\gamma}x-\dfrac{1}{2}e^{-2\underline{\gamma}x}\right)+\underline{M}_2,\\ \underline{B}_2=-\dfrac{j\underline{H}_a^3}{16\underline{\gamma}^2\,\mathrm{ch}^3\underline{\gamma}a}\left(\dfrac{1}{2}e^{2\underline{\gamma}x}+3\underline{\gamma}x-\dfrac{3}{2}e^{-2\underline{\gamma}x}-\dfrac{1}{4}e^{-4\underline{\gamma}x}\right)+\underline{N}_2,\end{cases}$$

où M_2 et N_2 sont des constantes d'intégration. Compte tenu des expression (A. 3) la solution (A. 1) de l'équation de la première approximation devient :

$$\text{(A. 4)} \qquad \underline{H}_{z2}=\frac{\mathrm{j}\underline{H}_a^3}{16\underline{\gamma}^2\,\mathrm{ch}^3\underline{\gamma}a}\left(-\frac{1}{2}\,\mathrm{ch}3\underline{\gamma}x+3\,\mathrm{ch}\underline{\gamma}x-6\underline{\gamma}x\,\mathrm{sh}\underline{\gamma}x\right)+\underline{M}_2\mathrm{e}^{-\underline{\gamma}x}+\underline{N}_2\mathrm{e}^{\underline{\gamma}x}.$$

On peut déterminer les constantes $\underline{M}_2$, $\underline{N}_2$ à l'aide des conditions à la limite :

$$\text{(A. 5)} \qquad \underline{H}_{z2}(-a)=\underline{H}_{z2}(a)=0$$

en obtenant

$$\text{(A. 6)} \qquad \underline{M}_2=\underline{N}_2=\frac{\mathrm{j}\underline{H}_a^3}{32\underline{\gamma}^2\,\mathrm{ch}^4\underline{\gamma}a}\left(\frac{1}{2}\,\mathrm{ch}3\underline{\gamma}a-3\,\mathrm{ch}\underline{\gamma}a+6\underline{\gamma}a\,\mathrm{ch}\underline{\gamma}a\right).$$

Par conséquent

$$\text{(A. 7)} \qquad \underline{H}_{z2}=\frac{\mathrm{j}\underline{H}_a^3}{16\underline{\gamma}^2\mathrm{ch}^3\underline{\gamma}a}\left[-\frac{1}{2}\,\mathrm{ch}3\underline{\gamma}x+\left(2\,\mathrm{ch}^2\underline{\gamma}a-\frac{3}{2}+6\underline{\gamma}a\,\mathrm{th}\underline{\gamma}a\right)\mathrm{ch}\underline{\gamma}a-6\underline{\gamma}x\,\mathrm{sh}\underline{\gamma}x\right].$$

Annexe 2

L'équation différentielle (15) admet la solution :

$$\text{(A.8)} \qquad \underline{H}_{z3}=\underline{A}_3\mathrm{e}^{-\underline{\gamma}x}+\underline{B}_3\mathrm{e}^{\underline{\gamma}x},$$

avec $\underline{A}_3$, $\underline{B}_3$ — constantes d'intégration dont les expressions résultent comme solutions du système :

$$\text{(A. 9)} \qquad \begin{cases} \underline{A}_3\mathrm{e}^{-\underline{\gamma}x}+\underline{B}_3\mathrm{e}^{\underline{\gamma}x}=0, \\ -\underline{\gamma}\underline{A}_3\mathrm{e}^{-\underline{\gamma}x}+\underline{\gamma}\underline{B}_3\mathrm{e}^{\underline{\gamma}x}=\dfrac{3\underline{H}_a^5\,\mathrm{ch}^2\underline{\gamma}x}{16\underline{\gamma}^2\,\mathrm{ch}^3\underline{\gamma}x}\left[-\dfrac{1}{2}\,\mathrm{ch}3\underline{\gamma}x+\left(2\mathrm{ch}^2\underline{\gamma}a-\dfrac{3}{2}+6\underline{\gamma}a\,\mathrm{th}\underline{\gamma}a\right)\mathrm{ch}\underline{\gamma}x-6\underline{\gamma}x\,\mathrm{sh}\underline{\gamma}x\right]. \end{cases}$$

Dans une première étape on détermine :

$$\text{(A. 10)} \qquad \begin{cases} \underline{A}_3'=-\dfrac{3}{64}\cdot\dfrac{\underline{H}_a^5}{\underline{\gamma}^3\,\mathrm{ch}^5\underline{\gamma}x}\left[-\dfrac{1}{8}\,\mathrm{e}^{6\underline{\gamma}x}-\dfrac{1}{4}\,\mathrm{ch}4\underline{\gamma}x+\dfrac{1}{2}\left(\mathrm{ch}^2\underline{\gamma}a-1+3\underline{\gamma}a\,\mathrm{th}\underline{\gamma}a\right)\mathrm{e}^{4\underline{\gamma}x}+\right. \\ \quad+\left(\mathrm{ch}^2\underline{\gamma}a-\dfrac{5}{4}+3\underline{\gamma}a\,\mathrm{th}\underline{\gamma}a\right)\mathrm{ch}2\underline{\gamma}x+\left(\mathrm{ch}^2\underline{\gamma}a-\dfrac{5}{4}+3\underline{\gamma}a\,\mathrm{th}\underline{\gamma}a\right)\mathrm{e}^{2\underline{\gamma}x}+3\underline{\gamma}x\,\mathrm{ch}2\underline{\gamma}x- \\ \quad\left.-3\underline{\gamma}x\,\mathrm{e}^{2\underline{\gamma}x}\,\mathrm{sh}\,2\underline{\gamma}x-3\underline{\gamma}x\,\mathrm{e}^{-2\underline{\gamma}x}+\dfrac{3}{2}\underline{\gamma}a\,\mathrm{ch}^2\underline{\gamma}a-\dfrac{5}{4}+\dfrac{9}{2}\underline{\gamma}a\,\mathrm{th}^2\underline{\gamma}a\right], \\ \underline{B}_3'=\dfrac{3}{64}\cdot\dfrac{\underline{H}_a^5}{\underline{\gamma}^3\mathrm{ch}^5\underline{\gamma}a}\left[-\dfrac{1}{4}\,\mathrm{ch}4\underline{\gamma}x+\dfrac{1}{2}\left(\mathrm{ch}^2\underline{\gamma}a-1+3\underline{\gamma}a\,\mathrm{th}\underline{\gamma}a\right)\mathrm{e}^{-2\underline{\gamma}x}+\left(\mathrm{ch}^2\underline{\gamma}a-\right.\right. \\ \quad-\dfrac{5}{4}+3\underline{\gamma}a\,\mathrm{th}\underline{\gamma}a\Big)\mathrm{ch}2\underline{\gamma}x+\left(\mathrm{ch}^2\underline{\gamma}a-\dfrac{5}{8}+3\,\mathrm{th}\underline{\gamma}a\right)e^{-2\underline{\gamma}x}-\dfrac{1}{8}\mathrm{e}^{-6\underline{\gamma}x}-3\underline{\gamma}x\,\mathrm{ch}2\underline{\gamma}x- \\ \quad\left.-3\underline{\gamma}x\mathrm{e}^{-2\underline{\gamma}x}+3\underline{\gamma}x\mathrm{e}^{-2\underline{\gamma}x}\,\mathrm{sh}\,2\underline{\gamma}x+\dfrac{3}{2}\underline{\gamma}a\,\mathrm{ch}^2\underline{\gamma}a-\dfrac{5}{4}+\dfrac{9}{2}\underline{\gamma}a\,\mathrm{th}^2\underline{\gamma}a\right]. \end{cases}$$

Pour déterminer les primitives de celles-ci on tient compte en préalable que :

$$\text{(A. 11)} \qquad \begin{cases} \displaystyle\int x\,\mathrm{ch}2\underline{\gamma}x\,\mathrm{d}x=\frac{1}{2\underline{\gamma}}\left(x\,\mathrm{sh}2\underline{\gamma}x-\frac{1}{2\underline{\gamma}}\,\mathrm{ch}2\underline{\gamma}x\right), \\ \displaystyle\int x\mathrm{e}^{2\underline{\gamma}x}\,\mathrm{sh}2\underline{\gamma}x\,\mathrm{d}x=\frac{1}{4}\left(\frac{1}{2\underline{\gamma}}\,x\mathrm{e}^{4\underline{\gamma}x}-\frac{1}{8\underline{\gamma}^2}\,\mathrm{e}^{4\underline{\gamma}x}-x^2\right), \\ \displaystyle\int x\mathrm{e}^{-2\underline{\gamma}x}\,\mathrm{sh}2\underline{\gamma}x\,\mathrm{d}x=\frac{1}{4}\left(\frac{1}{2\underline{\gamma}}\,\mathrm{e}^{-4\underline{\gamma}x}+\frac{1}{8\underline{\gamma}^2}\,\mathrm{e}^{-4\underline{\gamma}x}+x^2\right), \\ \displaystyle\int x\mathrm{e}^{2\underline{\gamma}x}\,\mathrm{d}x=\frac{1}{2\underline{\gamma}}\left(x-\frac{1}{2\underline{\gamma}}\right)\mathrm{e}^{2\underline{\gamma}x},\quad \int x\mathrm{e}^{-2\underline{\gamma}x}\,\mathrm{d}x=-\frac{1}{2\underline{\gamma}}\left(x+\frac{1}{2\underline{\gamma}}\right)\mathrm{e}^{-2\underline{\gamma}x}. \end{cases}$$

Tous calculs effectués on obtient :

$$(A,\ 12)\quad \begin{cases} \underline{A}'_3 = -\dfrac{3}{128}\cdot\dfrac{\underline{H}_a^5}{\underline{\gamma}^4\mathrm{ch}^5\underline{\gamma}a}\left[-\dfrac{1}{24}\mathrm{e}^{6\underline{\gamma}x}-\dfrac{1}{8}\mathrm{sh}4\underline{\gamma}x+\dfrac{1}{4}\left(\mathrm{ch}^2\underline{\gamma}a-\dfrac{1}{4}+3\underline{\gamma}a\,\mathrm{th}\underline{\gamma}a\right)\mathrm{e}^{4\underline{\gamma}x}+\right.\\ +\left(\mathrm{ch}^2\underline{\gamma}a-\dfrac{5}{4}+3\underline{\gamma}a\,\mathrm{th}\underline{\gamma}a\right)\mathrm{sh}2\underline{\gamma}x+\left(\mathrm{ch}^2\underline{\gamma}a+\dfrac{7}{8}+3\underline{\gamma}a\,\mathrm{th}\underline{\gamma}a\right)\mathrm{e}^{2\underline{\gamma}x}+3\left(\underline{\gamma}x\,\mathrm{sh}\,2\underline{\gamma}x-\right.\\ \left.-\dfrac{1}{2}\mathrm{ch}2\underline{\gamma}x\right)-\dfrac{3}{4}\underline{\gamma}x\left(\dfrac{1}{2}\mathrm{e}^{4\underline{\gamma}x}-\underline{\gamma}x\right)-3\underline{\gamma}x\mathrm{e}^{2\underline{\gamma}x}+\left(3\,\mathrm{ch}^2\underline{\gamma}a-\dfrac{5}{2}+9\underline{\gamma}a\,\mathrm{th}\underline{\gamma}a\right)\underline{\gamma}x+\underline{M}_3,\\ \underline{B}'_3=\dfrac{3}{128}\cdot\dfrac{\underline{H}_a^5}{\underline{\gamma}^4\mathrm{ch}^5\underline{\gamma}a}\left[-\dfrac{1}{8}\mathrm{sh}4\underline{\gamma}x-\dfrac{1}{4}\left(\mathrm{ch}^2\underline{\gamma}a-\dfrac{1}{4}+3\underline{\gamma}a\,\mathrm{th}\underline{\gamma}a\right)\mathrm{e}^{-4\underline{\gamma}x}+\left(\mathrm{ch}^2\underline{\gamma}a-\right.\right.\\ -\dfrac{5}{4}+3\underline{\gamma}a\,\mathrm{th}\underline{\gamma}a\Big)\mathrm{sh}2\underline{\gamma}x-\left(\mathrm{ch}^2\underline{\gamma}a+\dfrac{7}{8}+3\underline{\gamma}a\,\mathrm{th}\underline{\gamma}a\right)\mathrm{e}^{-2\underline{\gamma}x}+\dfrac{1}{24}\mathrm{e}^{-6\underline{\gamma}x}-3\left(\underline{\gamma}x\,\mathrm{sh}2\underline{\gamma}x-\\ \left.-\dfrac{1}{2}\mathrm{ch}2\underline{\gamma}x\right)-\dfrac{3}{4}\underline{\gamma}x\left(\dfrac{1}{2}\mathrm{e}^{4\underline{\gamma}x}+\underline{\gamma}x\right)-3\underline{\gamma}x\mathrm{e}^{-2\underline{\gamma}x}+\left(3\,\mathrm{ch}^2\underline{\gamma}a-\dfrac{5}{2}+9\underline{\gamma}a\,\mathrm{th}\underline{\gamma}a\right)\underline{\gamma}x\Big]+\underline{N}_3, \end{cases}$$

où $\underline{M}_3$ et $\underline{N}_3$ sont des constantes d'intégration. Si l'on substitue les expressions (A. 11) dans la solution (A. 8) on obtient :

$$(A.\ 13)\quad \begin{aligned} \underline{H}_{z3}=\frac{3}{64}\cdot\frac{\underline{H}_a^5}{\underline{\gamma}^4\mathrm{ch}^5\underline{\gamma}a}\Big[&\frac{1}{24}\mathrm{ch}5\underline{\gamma}x-\frac{1}{8}\mathrm{sh}4\underline{\gamma}x\,\mathrm{sh}\underline{\gamma}x-\frac{1}{4}\left(\mathrm{ch}^2\underline{\gamma}a-\frac{1}{4}+3\underline{\gamma}a\,\mathrm{th}\underline{\gamma}a\right)\mathrm{ch}3\underline{\gamma}x+\\ &+\left(\mathrm{ch}^2\underline{\gamma}a-\frac{5}{4}+3\underline{\gamma}a\,\mathrm{th}\underline{\gamma}a\right)\mathrm{sh}\underline{\gamma}x\,\mathrm{sh}2\underline{\gamma}x-\left(\mathrm{ch}^2\underline{\gamma}a+\frac{7}{8}+3\underline{\gamma}a\,\mathrm{th}\underline{\gamma}a\right)\mathrm{ch}\underline{\gamma}x-3\left(\underline{\gamma}x\,\mathrm{sh}2\underline{\gamma}x-\right.\\ &\left.-\frac{1}{2}\mathrm{ch}2\underline{\gamma}x\right)\mathrm{ch}\underline{\gamma}x-\frac{3}{2}\underline{\gamma}^2x^2\,\mathrm{ch}\underline{\gamma}x+\frac{3}{4}\underline{\gamma}x\,\mathrm{sh}\,3\underline{\gamma}x+3\underline{\gamma}x\,\mathrm{sh}\underline{\gamma}x+\left(3\,\mathrm{ch}^2\underline{\gamma}a-\frac{5}{2}+\right.\\ &\left.+9\underline{\gamma}a\,\mathrm{th}\underline{\gamma}a\right)\underline{\gamma}x\,\mathrm{sh}\underline{\gamma}x\Big]+\underline{M}_3\mathrm{e}^{-\underline{\gamma}x}+\underline{N}_3\mathrm{e}^{\underline{\gamma}x}. \end{aligned}$$

Compte tenu des identités :

$$(A.\ 14)\quad \begin{cases} \mathrm{sh}\,4\underline{\gamma}x\,\mathrm{sh}\underline{\gamma}x=\dfrac{1}{2}(\mathrm{ch}5\underline{\gamma}x-\mathrm{ch}3\underline{\gamma}x), & \mathrm{sh}2\underline{\gamma}x\,\mathrm{ch}\underline{\gamma}x=\dfrac{1}{2}(\mathrm{sh}3\underline{\gamma}x+\mathrm{sh}\underline{\gamma}x),\\ \mathrm{sh}\underline{\gamma}x\,\mathrm{sh}2\underline{\gamma}x=\dfrac{1}{2}(\mathrm{ch}3\underline{\gamma}a-\mathrm{ch}\underline{\gamma}x), & \mathrm{ch}2\underline{\gamma}x\,\mathrm{ch}\underline{\gamma}x=\dfrac{1}{2}(\mathrm{ch}3\underline{\gamma}x+\mathrm{ch}\underline{\gamma}x), \end{cases}$$

on peut écrire l'expression (A. 13) sous la forme plus simple

$$(A.\ 15)\quad \begin{aligned} \underline{H}_{z3}=\frac{3}{64}\cdot\frac{\underline{H}_a^5}{\underline{\gamma}^4\mathrm{ch}^5\underline{\gamma}a}\Big[&-\frac{1}{48}\mathrm{ch}5\underline{\gamma}x+\frac{1}{2}\left(\frac{1}{2}\mathrm{ch}^2\underline{\gamma}a-1+\frac{3}{2}\underline{\gamma}a\,\mathrm{th}\underline{\gamma}a\right)\mathrm{ch}3\underline{\gamma}x-\\ &-\frac{1}{2}\left(3\,\mathrm{ch}^2\underline{\gamma}a+\frac{1}{2}+9\underline{\gamma}a\,\mathrm{th}\underline{\gamma}a\right)\mathrm{ch}\underline{\gamma}x-\frac{3}{4}\underline{\gamma}x\,\mathrm{sh}3\underline{\gamma}x-\frac{3}{2}\underline{\gamma}^2x^2\,\mathrm{ch}\underline{\gamma}x+3\,\mathrm{ch}^2\underline{\gamma}a-1+\\ &+9\underline{\gamma}a\,\mathrm{th}\underline{\gamma}a\Big)\underline{\gamma}x\,\mathrm{sh}\underline{\gamma}x\Big]+\underline{M}_3\mathrm{e}^{-\underline{\gamma}x}+\underline{N}_3\mathrm{e}^{-\underline{\gamma}x}. \end{aligned}$$

Pour déterminer les constantes d'intégration $\underline{M}_3$, $\underline{N}_3$ on impose les conditions à la limite

$$(A.\ 16)\qquad \underline{H}_{z3}(-a)=\underline{H}_{z3}(a)=0.$$

Si l'on tient compte des relations :

$$\text{(A. 17)}\qquad \begin{cases} \operatorname{ch}5\underline{\gamma}a = 16\operatorname{ch}^5\underline{\gamma}a - 20\operatorname{ch}^3\underline{\gamma}a + 5\operatorname{ch}\underline{\gamma}a\,, \\ \operatorname{sh}5\underline{\gamma}a = 16\operatorname{sh}^5\underline{\gamma}a + 20\operatorname{sh}^3\underline{\gamma}a + 5\operatorname{sh}\underline{\gamma}a, \\ \operatorname{ch}3\underline{\gamma}a = 4\operatorname{ch}^3\underline{\gamma}a - 3\operatorname{ch}\underline{\gamma}a\,, \quad \operatorname{sh}3\underline{\gamma}a = 4\operatorname{sh}^3\underline{\gamma}a + 3\operatorname{sh}\underline{\gamma}a \end{cases}$$

il résulte

$$\text{(A. 18)}\qquad \underline{M}_3 = \underline{N}_3 = -\frac{3}{64}\cdot\frac{\underline{H}_a^5}{\underline{\gamma}^4\operatorname{ch}^5\underline{\gamma}a}\left(\frac{1}{3}\operatorname{ch}^4\underline{\gamma}a - \frac{23}{12}\operatorname{ch}^2\underline{\gamma}a + \frac{55}{96} - \frac{3}{4}\underline{\gamma}^2a^2 - 2\underline{\gamma}a\operatorname{th}\underline{\gamma}a + \right.$$
$$\left. + \frac{3}{2}\underline{\gamma}a\frac{\operatorname{sh}^3\underline{\gamma}a}{\operatorname{ch}\underline{\gamma}a} + \frac{9}{2}\underline{\gamma}^2a^2\operatorname{th}^2\underline{\gamma}a\right).$$

Finalement on obtient :

$$\text{(A. 19)}\qquad \underline{H}_{z3} = \frac{3}{64}\cdot\frac{\underline{H}_a^5}{\underline{\gamma}^4\operatorname{ch}^5\underline{\gamma}a}\left[-\frac{1}{48}\operatorname{ch}5\underline{\gamma}x + \frac{1}{2}\left(\frac{1}{2}\operatorname{ch}^2\underline{\gamma}a - 1 + \frac{3}{2}\underline{\gamma}a\operatorname{th}\underline{\gamma}a\right)\operatorname{ch}3\underline{\gamma}x + \right.$$
$$+\left(-\frac{2}{3}\operatorname{ch}^4\underline{\gamma}a + \frac{7}{3}\operatorname{ch}^2\underline{\gamma}a - \frac{67}{48} - \frac{1}{2}\underline{\gamma}a\operatorname{th}\underline{\gamma}a + \frac{3}{2}\underline{\gamma}^2a^2 - 3\underline{\gamma}a\frac{\operatorname{sh}^3\underline{\gamma}a}{\operatorname{ch}\underline{\gamma}a} - 9\underline{\gamma}^2a^2\operatorname{th}^2\underline{\gamma}a\right)\operatorname{ch}\underline{\gamma}x -$$
$$\left. - \frac{3}{4}\underline{\gamma}x\operatorname{sh}3\underline{\gamma}x - \frac{3}{2}\underline{\gamma}^2x^2\operatorname{ch}\underline{\gamma}x + (3\operatorname{ch}^2\underline{\gamma}a - 1 + 9\underline{\gamma}a\operatorname{th}\underline{\gamma}a)\,\underline{\gamma}x\operatorname{sh}\underline{\gamma}x\right].$$

Annexe 3

Si l'on substitue l'expression (18) de $\underline{H}_z$ dans l'intégrale de (22) et l'on tient compte toute fois que

$$\text{(A. 20)}\qquad \int x\operatorname{sh}k\underline{\gamma}x\,\mathrm{d}x = \frac{1}{k\underline{\gamma}}\left(x\operatorname{ch}k\underline{\gamma}x - \frac{1}{k\underline{\gamma}}\operatorname{sh}k\underline{\gamma}x\right),\ (k=1,\,3)$$

$$\int x^2\operatorname{ch}\underline{\gamma}x\,\mathrm{d}x = \frac{1}{\underline{\gamma}}\left(x^2 + \frac{2}{\underline{\gamma}^2}\right)\operatorname{sh}\underline{\gamma}x - \frac{2}{\underline{\gamma}^2}x\operatorname{ch}\underline{\gamma}x,$$

on obtient

$$\underline{H}_{\text{med}} = \frac{2}{\underline{\gamma}}\underline{H}_a\frac{\operatorname{sh}\underline{\gamma}a}{\operatorname{ch}\underline{\gamma}a} + \frac{3}{32}\cdot\frac{\underline{\mu}_3}{\mu_1}\cdot\frac{\underline{H}_a^3}{\underline{\gamma}\operatorname{ch}^3\underline{\gamma}a}\left(-\frac{1}{3}\operatorname{sh}3\underline{\gamma}a + 4\operatorname{ch}^2\underline{\gamma}a\operatorname{sh}\underline{\gamma}a - \frac{12\underline{\gamma}a}{\operatorname{ch}\underline{\gamma}a} + 9\operatorname{sh}\underline{\gamma}a\right) -$$
$$\text{(A. 21)}\qquad -\frac{27}{256}\cdot\frac{\underline{\mu}_3^2}{\mu_1^2}\cdot\frac{\underline{H}_a^5}{\underline{\gamma}\operatorname{ch}^5\underline{\gamma}a}\left(-\frac{1}{120}\operatorname{sh}5\underline{\gamma}a + \frac{1}{6}\operatorname{ch}^2\underline{\gamma}a\operatorname{sh}\underline{\gamma}a - \frac{1}{6}\operatorname{sh}3\underline{\gamma}a + \frac{1}{2}\underline{\gamma}a\frac{\operatorname{sh}\underline{\gamma}a}{\operatorname{ch}\underline{\gamma}a}\operatorname{sh}^3\underline{\gamma}a - \right.$$
$$-\frac{4}{3}\operatorname{ch}^4\underline{\gamma}a\operatorname{sh}\underline{\gamma}a - \frac{4}{3}\operatorname{ch}^2\underline{\gamma}a\operatorname{sh}\underline{\gamma}a - \frac{211}{24}\operatorname{sh}\underline{\gamma}a - 19\underline{\gamma}a\frac{\operatorname{sh}^2\underline{\gamma}a}{\operatorname{ch}\underline{\gamma}a} - 6\underline{\gamma}a\frac{\operatorname{sh}^4\underline{\gamma}a}{\operatorname{ch}\underline{\gamma}a} - 18\underline{\gamma}^2a^2\frac{\operatorname{sh}^3\underline{\gamma}a}{\operatorname{ch}^2\underline{\gamma}a} -$$
$$\left. - \frac{1}{2}\underline{\gamma}a\operatorname{ch}3\underline{\gamma}a - 6\underline{\gamma}a\operatorname{ch}\underline{\gamma}a + 6\underline{\gamma}a\operatorname{ch}^3\underline{\gamma}a + 18\underline{\gamma}^2a^2\operatorname{sh}\underline{\gamma}a + 2\operatorname{sh}\underline{\gamma}a\right).$$

Compte tenu des identités (A. 17) et tous calculs effectués on obtient finalement :

$$\underline{H}_{\text{med}} = \frac{2}{\underline{\gamma}}\underline{H}_a\frac{\operatorname{sh}\underline{\gamma}a}{\operatorname{ch}\underline{\gamma}a} + \frac{3}{8}\cdot\frac{\underline{\mu}_3}{\mu_1}\cdot\frac{\underline{H}_a^3}{\underline{\gamma}\operatorname{ch}^3\underline{\gamma}a}\left(\frac{2}{3}\operatorname{sh}^3\underline{\gamma}a + 3\operatorname{sh}\underline{\gamma}a - \frac{3\underline{\gamma}a}{\operatorname{ch}\underline{\gamma}a}\right) +$$
$$\text{(A. 22)}\qquad + \frac{27}{256}\cdot\frac{\underline{\mu}_3^2}{\mu_1^2}\cdot\frac{\underline{H}_a^5}{\underline{\gamma}\operatorname{ch}^5\underline{\gamma}a}\left(\frac{12}{15}\operatorname{sh}^5\underline{\gamma}a + \frac{11}{3}\operatorname{sh}^3\underline{\gamma}a + \frac{19}{2}\operatorname{sh}\underline{\gamma}a + \frac{27}{2}\underline{\gamma}a\frac{\operatorname{sh}^2\underline{\gamma}a}{\operatorname{ch}\underline{\gamma}a} + \right.$$
$$\left. + 8\underline{\gamma}a\operatorname{ch}^3\underline{\gamma}a - 18\underline{\gamma}^2a^2\frac{\operatorname{sh}\underline{\gamma}a}{\operatorname{ch}^2\underline{\gamma}a}\right)$$

Annexe 4

Si l'on suppose que la tôle ferromagnétique est suffisamment mince il est possible à accepter les approximations :

$$
\text{(A. 23)}\quad
\begin{cases}
\operatorname{ch}\underline{\gamma}x \approx 1+\dfrac{\underline{\gamma}^2x^2}{2},\ \operatorname{ch}\underline{\gamma}a \approx 1+\dfrac{\underline{\gamma}^2a^2}{2},\ \operatorname{sh}\underline{\gamma}x \approx \underline{\gamma}x+\dfrac{\underline{\gamma}^3x^3}{6},\ \operatorname{sh}\underline{\gamma}a \approx \underline{\gamma}a+\dfrac{\underline{\gamma}^3a^3}{6},\\
\operatorname{th}\underline{\gamma}a \approx \underline{\gamma}a-\dfrac{\underline{\gamma}^3a^3}{6},\ \operatorname{ch}^2\underline{\gamma}x \approx 1+\underline{\gamma}^2x^2,\ \operatorname{ch}^2\underline{\gamma}a \approx 1+\underline{\gamma}^2a^2,\ \operatorname{ch}^3\underline{\gamma}x \approx 1+\dfrac{3}{2}\underline{\gamma}^2x^2,\\
\operatorname{ch}^3\underline{\gamma}a \approx 1+\dfrac{3}{2}\underline{\gamma}^2a^2,\ \operatorname{sh}^3\underline{\gamma}a \approx \underline{\gamma}^3a^3,\ \operatorname{ch}^4\underline{\gamma}a \approx 1+2\underline{\gamma}^2a^2,\ \operatorname{ch}^5\underline{\gamma}a \approx 1+\dfrac{25}{2}\underline{\gamma}^2a^2,\\
\operatorname{sh}^5\underline{\gamma}a \approx 0,\ \operatorname{ch}\,3\underline{\gamma}x \approx 1+\dfrac{9}{2}\underline{\gamma}^2x^2,\ \operatorname{sh}\,3\underline{\gamma}x \approx 3\underline{\gamma}x+\dfrac{27}{6}\underline{\gamma}^3x^3,\ \operatorname{ch}\,5\underline{\gamma}x \approx 1+\dfrac{25}{2}\underline{\gamma}^2x^2,\\
\dfrac{\operatorname{sh}^3\underline{\gamma}a}{\operatorname{ch}\underline{\gamma}a} \approx \underline{\gamma}^3a^3,\ \operatorname{th}^2\underline{\gamma}a \approx \underline{\gamma}^2a^2,\ \dfrac{\operatorname{sh}^2\underline{\gamma}a}{\operatorname{ch}\underline{\gamma}a} \approx \underline{\gamma}^2a^2,\ \dfrac{\operatorname{sh}\underline{\gamma}a}{\operatorname{ch}^2\underline{\gamma}a} \approx \dfrac{2}{3}\underline{\gamma}a\dfrac{3-\underline{\gamma}^2a^2}{2+\underline{\gamma}^2a^2}.
\end{cases}
$$

Compte tenu de (A. 23) les expressions (18) et (21) deviennent :

$$
\text{(A. 24)}\quad \underline{H}_z=\underline{H}_a\frac{2+\underline{\gamma}^2x^2}{2+\underline{\gamma}^2a^2}+\frac{3}{2}\cdot\frac{\mu_3}{\mu_1}\cdot\frac{\underline{\gamma}^2\underline{H}_a^3}{2+3^2\underline{\gamma}a^2}(a^2-x^2)-\frac{81}{128}\cdot\frac{\mu_3^2}{\mu_1^2}\cdot\frac{\underline{\gamma}^2\underline{H}_a^5}{2+5\underline{\gamma}^2a^2}(a^2-x^2),
$$

respectivement

$$
\text{(A. 25)}\quad \underline{E}_y=-\frac{\underline{\gamma}^2}{\sigma}\underline{H}_a\left(\frac{2}{2+\underline{\gamma}^2a^2}-3\frac{\mu_3}{\mu_1}\cdot\frac{\underline{H}_a^2}{2+3\underline{\gamma}^2a^2}+\frac{81}{64}\cdot\frac{\mu_3^2}{\mu_1^2}\cdot\frac{\underline{H}_a^4}{2+5\underline{\gamma}^2a^2}\right)x.
$$

Enfin, si l'on tient compte des relations (10) et (11) on détermine les expressions (24) et (25)

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PROPAGAREA CÎMPULUI ELECTROMAGNETIC ÎNTR-O TOLĂ FEROMAGNETICĂ CONDUCTOARE, CU NELINIARITATE SLABĂ, ÎN REGIM CVASISTAŢIONAR ARMONIC

(Rezumat)

Se studiază propagarea cîmpului electromagnetic într-o tolă feromagnetică conductoare, cu neliniaritate slabă, în regim cvasistaţionar armonic, considerînd mediul ca avînd proprietăţi *inerţiale*, în sensul definit în Nota precedentă [8].

Ecuaţia diferenţială satisfăcută de valoarea efectivă complexă a intensităţii cîmpului magnetic în tolă fiind de forma (3) şi utilizînd, în vederea integrării acesteia, metoda perturbaţiei [11], se obţine soluţia (18). În baza acesteia se deduce şi expresia (21) a valorii efective complexe a intensităţii cîmpului electric indus într-un punct situat în interiorul tolei.

Se particularizează expresiile astfel obţinute pentru cazul tolelor subţiri obţinînd relaţiile, relativ simple, (25), pentru calculul lui $\underline{E}_z$ şi (26), pentru determinarea lui $\underline{H}_y$.

Se stabileşte expresia (36) a pierderilor prin curenţi turbionari în unitatea de volum a unei tole feromagnetice subţiri.

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THE EFFECT OF EDDY-CURRENTS ON THE STABILITY OF ATTRACTIVE ELECTROMAGNETIC LEVITATION SYSTEMS

BY

SAVEL MITREA, LORIN CANTEMIR and ILIE POPESCU

1. Introduction

It has been shown by the authors [1], [2] that there are some basic conditions which the control of attractive electromagnetic levitation systems must fulfil in order to achieve a good stability when perturbations are small. These conditions were derived under the assumption that eddy-currents in the ferromagnetic material were absent. The aim of the present paper is to show the way the control parameters are to be adjusted when eddy-currents are taken into account.

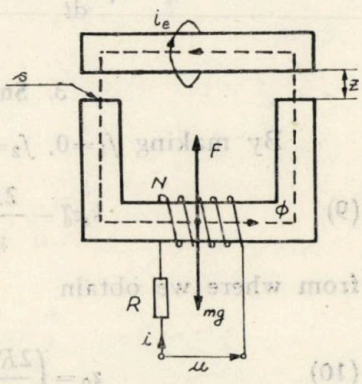

Fig. 1

2. The Electromagnet Equations

The electromagnet is sketched in Fig. 1. If the eddy-current i_e is absent the state variables may be chosen as the electromagnet current i, the gap z and the vertical velocity $v=\mathrm{d}z/\mathrm{d}t$ [2]. If eddy-currents are taken into account it is convenient to use the crossectional magnetic flux Φ instead of current i as the first state variable. In order to simplify the procedure we ignore leakage fluxes, magnetic potential drop over the ferromagnetic path and fringing. Also it is assumed that saturation does not occur. Under these conditions we can write the following equations:

$$(1)\qquad Ri+N\frac{\mathrm{d}\Phi}{\mathrm{d}t}=u, \qquad (2)\qquad R_e i_e+\frac{\mathrm{d}\Phi}{\mathrm{d}t}=0,$$

$$(3)\qquad \frac{\mu_0 S}{2z}(Ni+i_e)=\Phi, \qquad (4)\qquad m\frac{\mathrm{d}^2 z}{\mathrm{d}t^2}=mg-F,$$

$$F=\frac{\Phi^2}{\mu_0 s}. \tag{5}$$

By combining Eqs. (1)...(3) on one hand and Eqs. (4), (5) on the other hand we obtain respectively :

$$\frac{d\Phi}{dt}=\frac{u-2Rz\Phi/(\mu_0 sN)}{R/(NR_e)+N}, \quad \frac{d^2z}{dt^2}=g-\frac{\Phi^2}{m\mu_0 s}. \tag{6}$$

As it was shown in [1], [2] the system can be stabilized if voltage across the electromagnet terminals is of the form

$$u=k_z z^\gamma+k_v\frac{dz}{dt}, \tag{7}$$

where the first term in the right member represents a monotonically increasing function of the gap z and the second term, a linear function of vertical velocity.

By taking into account (7), the set of Eqs. (6) can be writen in normal form as follows :

$$\begin{cases} \dfrac{d\Phi}{dt}=\dfrac{k_z z^\gamma+k_v v-2Rz\Phi/(\mu_0 sN)}{R/(NR_e)+N}\triangleq f_1, \\ \dfrac{dz}{dt}=v\triangleq f_2, \quad \dfrac{dv}{dt}=g-\dfrac{\Phi^2}{m\mu_0 s}\triangleq f_3. \end{cases} \tag{8}$$

3. Small-Perturbation Stability

By making $f_1=0$, $f_2=0$, $f_3=0$ in (8) we get

$$k_z z_0^\gamma-\frac{2Rz_0\Phi_0}{\mu_0 sN}=0, \quad g-\frac{\Phi_0^2}{m\mu_0 s}=0, \tag{9}$$

from where we obtain

$$z_0=\left(\frac{2R}{k_z N}\sqrt{\frac{mg}{\mu_0 s}}\right)^{\frac{1}{\gamma-1}}, \quad \Phi_0=\sqrt{mg\,\mu_0 s}, \tag{10}$$

that represents the gap and the crossectional magnetic flux corresponding to the equilibrium point.

Stability of the system described by Eqs. (8) is studied for small perturbations in the vicinity of equilibrium point. As it is well known from nonlinear systems theory, the stability for small perturbations can be determined by studying the characteristic equation after linearization of the system equations in the vicinity of equilibrium point. By following this procedure we obtain the characteristic equation of the system în the form :

$$(11)\qquad \begin{vmatrix} \left.\frac{\partial f_1}{\partial \Phi}\right|_0 - \lambda & \left.\frac{\partial f_1}{\partial z}\right|_0 & \left.\frac{\partial f_1}{\partial v}\right|_0 \\ \left.\frac{\partial f_2}{\partial \Phi}\right|_0 & \left.\frac{\partial f_2}{\partial z}\right|_0 - \lambda & \left.\frac{\partial f_2}{\partial v}\right|_0 \\ \left.\frac{\partial f_3}{\partial \Phi}\right|_0 & \frac{\partial f_3}{\partial z} & \left.\frac{\partial f_3}{\partial v}\right|_0 - \lambda \end{vmatrix} = 0.$$

By expanding the determinant we obtain

$$(12)\qquad \lambda^3 + a_1\lambda^2 + a_2\lambda + a^3 = 0,$$

where

$$(13)\quad a_1 = \frac{2Rz_0}{N\mu_0 s[R/(NR_e)+N]},\quad a_2 = \frac{2k_v\sqrt{g/(m\mu_0 s)}}{R/(NR_e)+N},\quad a_3 = \frac{4gR(\gamma-1)}{N\mu_0 s[R/(NR_e)+N]}.$$

According to H u r w i t z's criterion, a first condition to be fulfilled in order that the system be stable is that all the above coefficients be positive. It is seen that coefficient a_3 is greater than zero only if $\gamma > 1$, a requirement that have been stressed upon in [1], [2] as well. The second condition, a third order system, must fulfil in order to be stable, according to the same criterion, is that the following inequality to take place:

$$(14)\qquad a_1a_2 - a_3 > 0.$$

By taking account of (13) relation (14) gives:

$$(15)\qquad k_v > \frac{(\gamma-1)[R/(NR_e)+N]\sqrt{mg\mu_0 s}}{z_0},$$

which shows clearly that eddy-currents have a destabilizing effect upon the system, consequently the amplification of velocity control signal has to be increased accordingly.

If an additional acceleration control signal is applied then relation (7) becomes

$$(16)\qquad u = k_z z^\gamma + k_v\frac{dz}{dt} + k_a\frac{d^2z}{dt^2}.$$

Computing again coefficients (13) and substituting for them into (14), one obtains:

$$(17)\qquad k_v > \frac{(\gamma-1)[R/(NR_e)+N]\sqrt{mg\mu_0 s}}{z_0 + (k_aN/R)\sqrt{g\mu_0 s/m}}.$$

Relation (17) shows that the utilization of acceleration control signal enables us to adjust the amplification of velocity control signal to convenient values. Moreover, as it was shown in [2], the use of acceleration control signal improves the versatility of control system so as to obtain a proper damping of the motion of electromagnet along the z axis.

4. Conclusions

The presence of eddy-currents in the ferromagnetic path of electromagnet requires the increase of amplification of velocity control signal. An additional acceleration control signal may be used in order to compensate for this increase.

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EFECTUL CURENȚILOR TURBIONARI ASUPRA STABILITĂȚII SUSTENTAȚIEI ELECTROMAGNETICE

(Rezumat)

Prezența curenților turbionari în circuitul magnetic al sustentației electromagnetice conduce la necesitatea creșterii amplificării buclei de comandă după viteză. Introducerea unui semnal de comandă suplimentar dependent de accelerație oferă posibilitatea ajustării acestei amplificări la valori convenabile.

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LANGSAME HETEROPARAMETRISCHE SCHWINGUNGEN IN FERRORESONANTEN ELEKTRISCHEN SERIENSCHALTKREISEN

VON

MIRCEA V. NEMESCU

1. Vorwort

Ein in der Praxis häufig begegneter elektrischer Schaltkreis ist der nichtlineare, seriengruppierte Spulen und Kondensatoren enthaltende *RLC*, ein Stromkreis in dem oft Resonanzverhältnisse nachgewiesen werden können.

Unter gewissen Bedingungen kann das Antwortsignal für solche Betriebsarten periodisch oder unperiodisch, mit sich langsam verändernder Amplitude und Phase erscheinen. Solche Schwingungen werden als langsame Schwingungen bezeichnet.

2. Theoretische Erwägungen

Führt man in einen ferroresonanten *RLC*-Serienschaltkreis mit nichtlinearem Induktor einen parametrisch variierenden Induktor ein (Abb. 1), so können im Stromkreis, unter gewissen Bedingungen, langsame Schwingungen entstehen. Verändert sich die Induktanz des parametrischen Induktors unter Einfluß eines externen Faktors, so heissen die im Stromkreis entstehenden Schwingungen langsame, heteroparametrische Schwingungen [8].

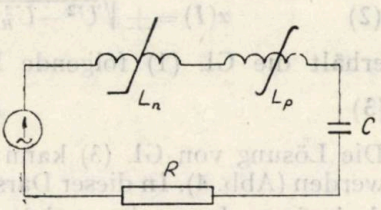

Abb. 1

Wählt man nun den parametrischen Induktor derart daß seine Änderungen sich nur im linearen Bereich bewegen, so erhält die Charakteristik $\psi(i)$ der beiden zusammengenommenen Induktoren, abgesehen von Hysteresiserscheinungen, die in Abb. 2 dargestellte Form, wo Kurve *1* dem Kleinstwert und Kurve *2* dem Höchstwert der Induktanz des Parameterinduktors entspricht. Die unterbrochene Linie stellt den Mittelwert der beiden Induktanzen dar. Man kann somit die beiden seriengeschalteten Induktoren als einen einzigen unlinearen, para-

metrischen Induktor betrachten (Abb. 3), dessen Charakteristik $\psi(i)$ zwischen den beinden Grenzwerten *1* und *2* variiert.

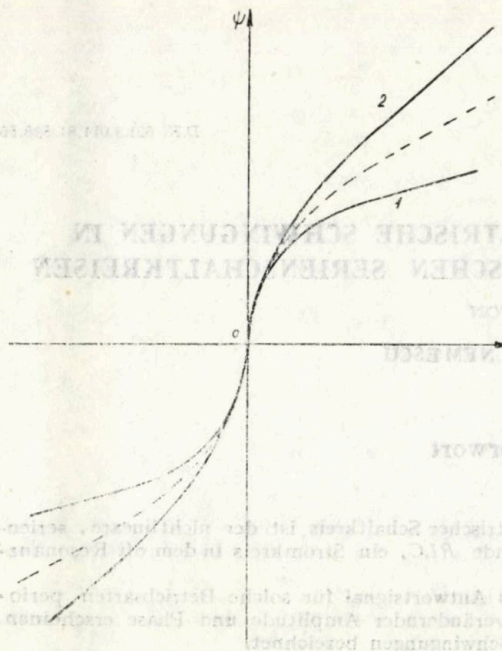

Abb. 2

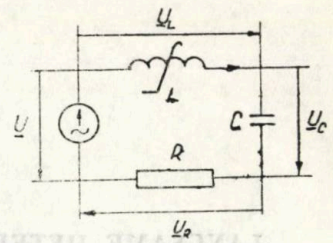

Abb. 3

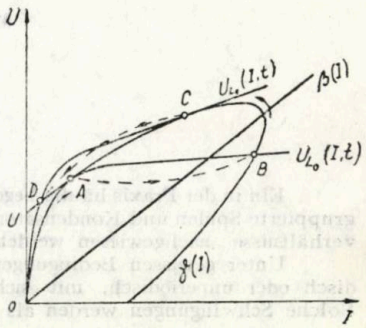

Abb. 4

Um den physischen Sinn der Erscheinung zu veranschaulichen, wird der Induktor fürs Erste als inertial angenommen. Unter dieser Bedingung kann der Effektivwert U der anliegenden Spannung unter folgende Form gebracht werden:

$$U^2=[U_L(I,t)-U_C(I)]^2+U_R^2 \tag{1}$$

oder, indem man die Begriffe teilt:

$$\alpha(I)=\pm\sqrt{U^2-U_R^2},\ \beta(I)=U_C(I)\ \text{und}\ \gamma(I)=\alpha(I)+\beta(I), \tag{2}$$

erhält die Gl. (1) folgende Form:

$$U_L(I,t)=\gamma(I). \tag{3}$$

Die Lösung von Gl. (3) kann durch Darstellung in Ebene (U, I) bestimmt werden (Abb. 4). In dieser Darstellung entspricht die Charakteristik $U_{L_0}(I,t)$ derjenigen Lage, in welcher die Induktanz des Parameterinduktors ihren Kleinstwert annimmt. Dieser Wert wird so angenommen, daß die Charakteristik $U_{L_0}(I,t)$, die dem nichtlinearen Parameterinduktor entspricht, als Tangente im Punkte A zur Ellipse $\gamma(I)$ zu liegen kommt. Als Ergebnis findet im Stromkreis der ferroresonante Sprung statt.

Wächst die Induktanz des Parameterinduktors an, so verschiebt sich die Lage der Charakteristik $U_L(I, t)$ der beiden Induktoren und die Stromamplitude und Phase [9] verändert sich. Wenn nun die Charakteristik $U_L(I, t)$ der beiden Induktoren für den Induktanzhöchstwert das Parameterinduktors den Wert $U_{L_1}(I, t)$ annimmt und im Punkte C tangent zur Ellipse $\gamma(I)$ zu liegen kommt, so findet im Schaltkreis ein Stromrücksprung statt. Punkt D entspricht dieser Betriebssituation (Abb. 4).

Wenn nun aus dieser Lage die Induktanz des Parameterinduktors zu fallen beginnt, bewegt sich der Funktionierungspunkt des Schaltkreises entlang der Kurve $\gamma(I)$ von D nach A, wo ein erneuter ferroresonanter Sprung stattfindet.

Aus dem oben gesagten geht hervor, daß, wenn bei kontinuierlicher Veränderung der Induktanz des Parameterinduktors unter Einfluß eines externen Faktors der Stromkreis abwechselnd aus dem resonanten Betriebszustand in den nichtresonanten versetzt wird, im Schaltkreis heteroparametrische Schwingungen entstehen. Die Dauer dieser Schwingungen wird von der Zeitspanne bestimmt, in der Funktionspunkt den Zyklus $ABCD$ durchläuft. Die Strecken AB und CD entsprechen den sprungförmigen Stromveränderungen, wodurch sich die sehr kurze Dauer erklärt, in der diese durchlaufen werden, eine Dauer, die im Vergleich zum restlichen Interval vernachlässigt werden kann. Die Strecken BC und DA werden in je einer Zeitspanne durchlaufen, die von der Induktanz des Parameterinduktors abhängig ist.

Für den Stromkreis aus Abb. 3 gilt folgendes Verhältnis:

$$Ri + \frac{1}{C}\int i\,dt + \frac{d\psi^*}{dt} = u, \tag{o}$$

in welchem: R — der dem Stromkreis entsprechende Widerstand, C — die Kondensatorkapazität des Stromkreises, ψ^* — der Gesamtfluß, der dem nichtlinearen Parameterinduktor entspricht, und u — die anliegende Spannung bedeutet.

Es wird die Charakteristik $i(\psi)$ der beiden Induktoren dem Mittelwert der Induktanz des Parameterinduktors angenähert und unter die Form eines Polynoms gebracht:

$$i = a_1\psi + a_3\psi^3 + a_5\psi^5. \tag{5}$$

Nimmt man für den Parameterinduktor folgende Regel an:

$$L_p = L_0(1 + m\cos\Omega t), \tag{6}$$

mit

$$L_0 = \frac{L_{max} - L_{min}}{2}, \quad m = \frac{L_{max} - L_{min}}{L_{max} + L_{min}}, \tag{7}$$

so ist der Gesamtfluß, der dem nichtlinearen Parameterinduktor entspricht folgendem Ausdruck zu entnehmen:

$$\psi^* = \psi - L_0 m(a_1\psi + a_3\psi^3 + a_5\psi^5)\cos\Omega t. \tag{8}$$

Bezeichnet man $L_1 = L_0 m$ und zieht man den Ausdruck (8) in Betracht, so erhält die Gl. (4) folgende Form:

$$Ri + \frac{1}{C}\int i\,dt + \frac{d}{dt}\left[\psi - L_1(a_1\psi + a_3\psi^3 + a_5\psi^5)\right] = u. \tag{9}$$

Betrachtet man Gl. (5) und die Tatsache, daß das Erregersignal $u = U_m \sin \omega t$ ist, so wird aus Gl. (9):

$$\begin{aligned}(1 - L_1a_1 - 3L_1a_3\psi^2 - 5L_1a_5\psi^4)\cos\Omega t\frac{d^2\psi}{dt^2} - \\ -(20L_1a_5\psi^3 + 6L_1a_3\psi)\cos\Omega t\left(\frac{d\psi}{dt}\right)^2 + [Ra_1 + 3a_3\psi^2 + 5a_5\psi^4 + \\ + (2L_1a_1\Omega + 6L_1a_3\Omega - 10L_1a_5\Omega\psi^4)\sin\Omega t]\frac{d\psi}{dt} + \left(\frac{a_1}{C} + L_1a_1\Omega^2\cos\Omega t\right)\psi + \\ + \left(\frac{a_3}{C} + L_1a_3\Omega^2\cos\Omega t\right)\psi^3 + \left(\frac{a_5}{C} + L_1a_5\Omega^2\cos\Omega t\right)\psi^5 = U_m\omega\cos\omega t.\end{aligned} \tag{10}$$

Die Integration der Gl. (10) wurde anhand der Merson-Methode numerisch durchgeführt.

3. Experimentalergebnisse

Der bei den Versuchen gebrauchte elektronische Schaltkreis ist in Abb. 5 *a* veranschaulicht, der unlineare Induktor ist als Toroid verwirklicht und der Parameterinduktor hat die in Abb. 5 *b* dargestellte Form.

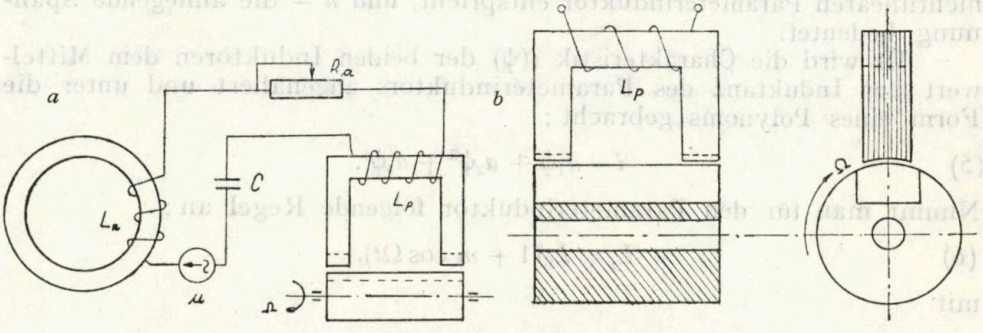

Abb. 5

Die Charakteristiken $i(\psi)$ des nichtlinearen Parameterinduktors sind in Abb. 6 dargestellt, wo die Charakteristik *1* dem Kleinstwert der Induktanz des Parameterinduktors und Kurve *2* deren Höchstwert entspricht. Die unterbrochene Linie veranschaulicht den Mittelwert der Induktanz des

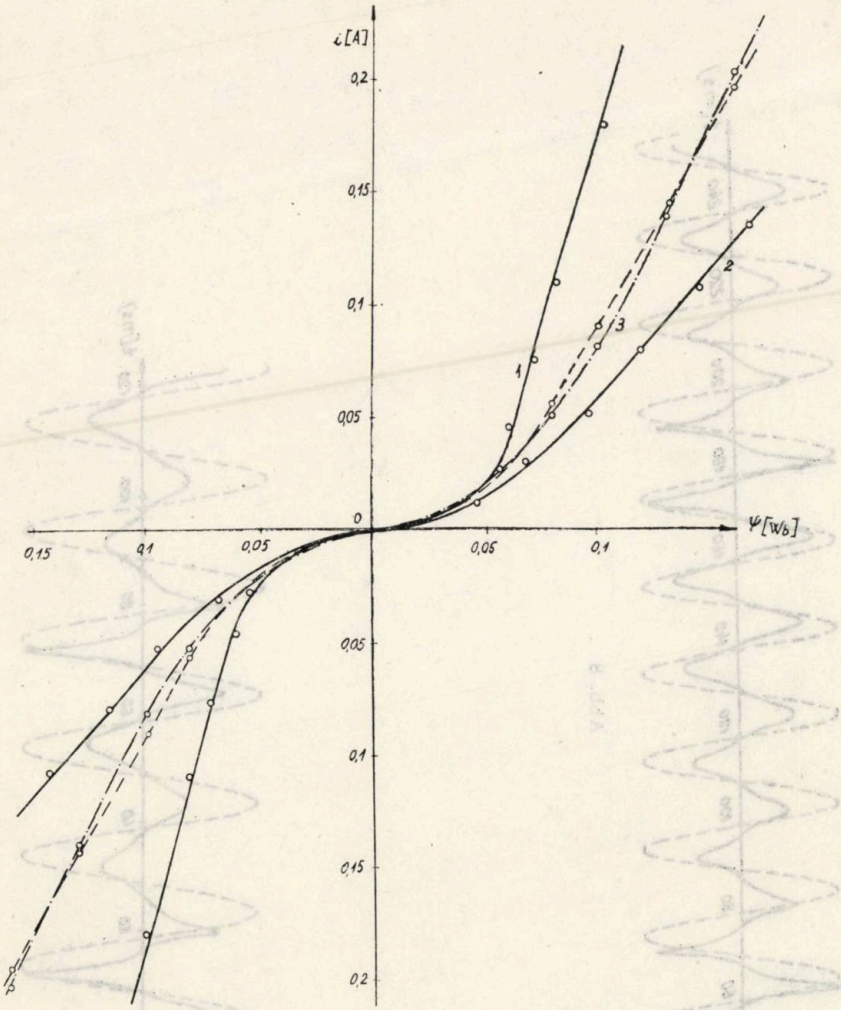
i [A]
0,2
0,15
0,1
0,05
0
0,15
0,1
0,05
0,05
0,1
0,05
0,1
0,15
0,2
Ψ [Wb]
1
2
3

Abb. 6

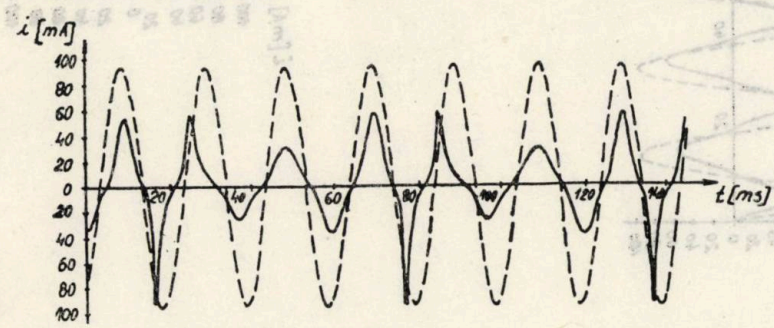
i [mA]
100
80
60
40
20
0
20
40
60
80
100
20
40
60
80
100
120
140
t [ms]

Abb. 7

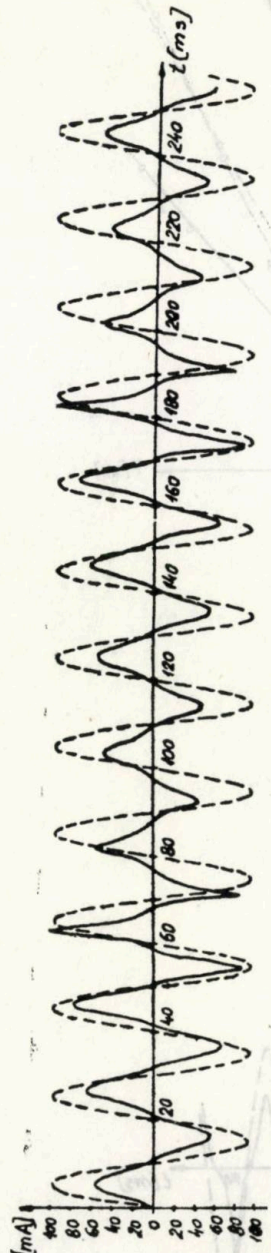

Abb. 8

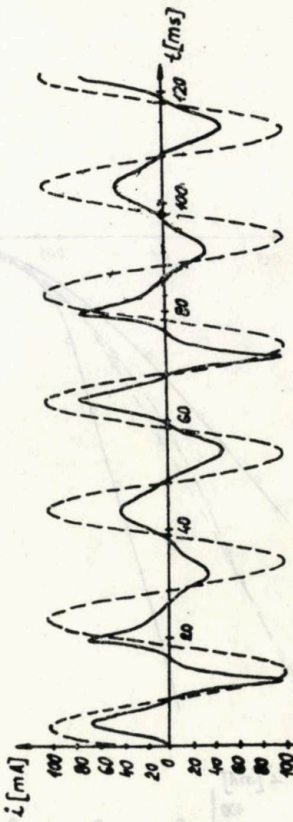

Abb. 9

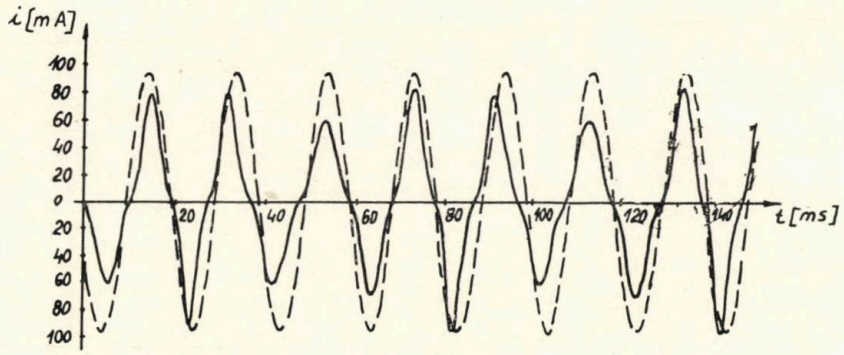

Abb. 10

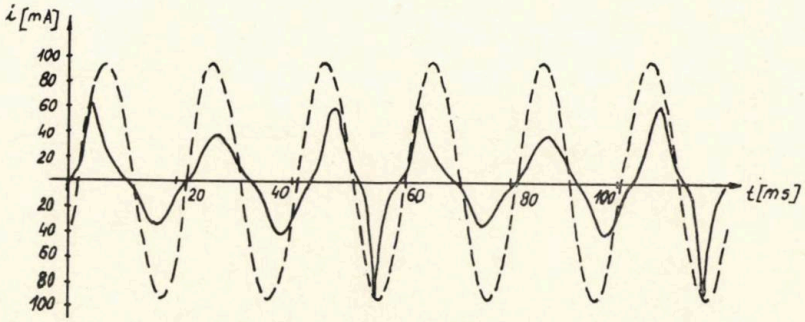

Abb. 11

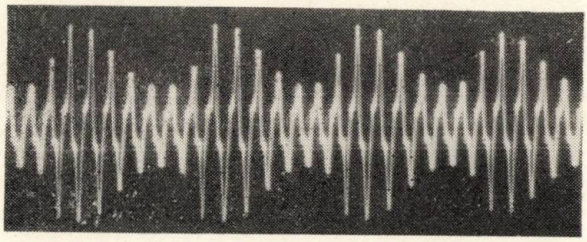

Abb. 12

Parameterinduktors. Die Koeffizienten des Annäherungspolynoms (5) wurden nach der Methode der kleinsten Quadrate berechnet und dabei wurde folgender Ausdruck erhalten:

$$i = 0{,}290757\,\psi + 62{,}2\,\psi^3 - 927{,}165\,\psi^5. \tag{11}$$

Die Charakteristik $i(\psi)$, die dieser Annäherung entspricht, ist in Abb. 6, Kurve *3*, dargestellt. Die Gl. (10) wurde unter der Bedingung Anfangswert gleich Null integriert, indem die Annäherung (11) benutzt wurde.

In Abb. 7 sind die Stromkreisschwingungen für folgende Parameterwerte gegeben: $R = 165{,}5\ \Omega$, $L_1 = 0{,}634$ H, $C = 5\ \mu$F, $\omega = 314$ rad/s, $\Omega = 104{,}8$ rad/s und $U_m = 8{,}73$ V.

Durch die Veränderung der Winkelgeschwindigkeit des Parameterinduktors auf $\Omega = 52{,}5$ rad/s und Konstanthalten der übrigen Parameter, erhalten die Stromkreisschwingungen die in Abb. 8 dargestellte Form. Man kann beobachten, daß das Herabsetzen der Winkelgeschwindigkeit Ω zum dementsprechenden Anwachsen der Schwingungsperiode im Schaltkreis führt. In Abb. 9 sind die Stromkreisschwingungen für $R = 185{,}3\ \Omega$ dargestellt, die übrigen Parameter bleiben konstant. Behalt der Widerstand seinen obigen Wert bei und die Induktanz erreicht $L_1 = 0{,}322$ H, so erhalten die Schwingungen die Form aus Abb. 10. Wird die Kapazität auf $C = 5{,}6\ \mu$F geändert, so erhält man die Schwingungen, wie sie in Abb. 11 dargestellt sind.

Die Stromkreisschwingungen aus Abb. 5 sind in Abb. 12 wiedergegeben, die Filmgeschwindigkeit beträgt dabei 100 mm/s. Die Stromkreisparameter sind: $R_a = 10\ \Omega$, $C = 5\ \mu$F, $U = 6{,}2$ V und $\omega = 314$ rad/s, die Drehgeschwindigkeit des Induktorrotors beträgt 300 rot/min. Es wird beobachtet, daß bei jeder Vorbeifahrt der Rotoraussparung des Parameterinduktors längs des fixen Teils, im Stromkreis ein ferroresonanter Sprung stattfindet. Somit stellt der in Abb. 5 *a*, *b* dargestellte Stromkreis einen neuen Typus von Übertragern oder statischen Umschaltern dar.

4. Schlußfolgerungen

Die theoretischen Erwägungen und die Experimentalergebnisse erlauben die Folgerung, daß das Erscheinen langsamer heteroparametrischen Schwingungen durch das Bestehen des ferroresonanten Sprungs und die Veränderung zumindest eines der Stromkreisparameter (Widerstand, Kapazität oder Induktanz) bestimmt wird. Es wird dabei bemerkt, daß für das Bestehen der Schwingungen die Änderung des Parameterelements mit einer kleineren Frequenz stattfinden muß als die des Erregersignals ($\Omega < \omega$). Die Periode der Schwingungen wird hauptsächlich durch die Periode des Parameterinduktors bestimmt, da die Form der Schwingungen von den Stromkreisparametern und der Variationsregel der Induktanz des Parameterinduktors abhängt.

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OSCILAȚII LENTE HETEROPARAMETRICE ÎN CIRCUITE ELECTRICE SERIE FERORESONANTE

(Rezumat)

Se studiază oscilațiile care apar în circuite electrice serie ferorezonante, la variația parametrică a inductanței circuitului. Este pus în evidență sensul fizic al fenomenului și sînt stabilite condițiile de apariție a acestor oscilații. Sînt deduse relații utile în analiza și proiectarea unor dispozitive neliniare, relații verificate atît numeric cît și experimental.

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ALMOST PERIODIC OSCILLATIONS IN THE MAGNETIC FREQUENCY DOUBLER

BY

NORBERT ZITRON and IUSTINA ZAHARIA

1. Introduction

Almost periodic oscillations in the magnetic frequency doubler are analysed in [1]. In the mentioned paper, the oscillation process is corectly presented from the physical point of view, but, in our opinion, the theoretical approach seems to be inadequate.

The aim of this paper is to establish the equations which permit a satisfactory analysis of the almost periodic oscillations in the magnetic frequency doubler.

2. Magnetic Frequency Doubler

The magnetic frequency doubler consists of two identical ferromagnetic cores with three windings (Fig. 1). The primary windings, having n_1 turns, are additionally series connected and are supplied from a sinusoidal voltage source. The tertiary windings, having n_0 turns, are differentially series connected through a low resistor R_0 to a direct constant source. The secondary windings, having n_2 turns are also differentially series connected. Corresponding to the theoretical results obtained in [2], in this last circuit occurs a voltage having two times the frequency of the supplied voltage. In order to study the almost periodic oscillations in the doubler, a capacitor C and a resistor R (fig. 1) are connected in parallel with the n_2 windings.

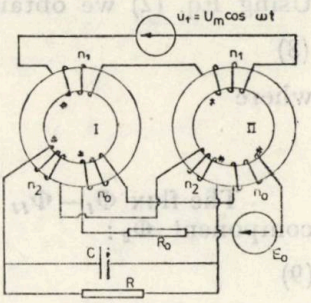

Fig. 1

3. Fundamental Equations Describing Almost Periodic Oscillations in the Magnetic Frequency Doubler

We make the following assumptions :

a) we neglect hysteretic properties of the cores, their characteristics being :

$$\sum_{l=0}^{2} n_L i_L = K_1 \Phi + K_3 \Phi^3 ; \tag{1}$$

b) we neglect the windings resistance ;

c) the sinusoidal voltage source, $E = U_m \cos \omega t$, is supposed to be applied at $t = 0$.

The differential equations for the primary circuit are :

$$n_1 \frac{\mathrm{d}(\Phi_I + \Phi_{II})}{\mathrm{d}t} = U_m \cos \omega t, \tag{2}$$

where Φ_I and Φ_{II} are the magnetic fluxes in the cores *I* and *II*, respectively.

The first Kirchhoff theorem applied to the resonant circuit gives :

$$i_2'' + i_R + i_C = 0. \tag{3}$$

Eq. (1) applied to the cores *I* and *II*, with the current directions represented in Fig. 1 gives :

$$n_1 i_1 + n_2 i_2'' + n_0 (i_0 + i_2''') = K_1 \Phi_I + K_3 \Phi_I^3, \tag{4}$$

$$n_1 i_1 - n_2 i_2'' - n_0 (i_0 + i_2''') = K_1 \Phi_{II} + K_3 \Phi_{II}^3. \tag{5}$$

Knowing that :

$$i_C = C \frac{\mathrm{d}u}{\mathrm{d}t} = C n_2 \frac{\mathrm{d}^2(\Phi_I - \Phi_{II})}{\mathrm{d}t^2}, \quad i_R = \frac{u}{R} = \frac{n_2}{R} \cdot \frac{\mathrm{d}(\Phi_I - \Phi_{II})}{\mathrm{d}t}, \tag{6}$$

Eqs. (3) may be transformed as follows :

$$C n_2 \frac{\mathrm{d}^2(\Phi_I - \Phi_{II})}{\mathrm{d}t^2} + \frac{n_2}{R} \cdot \frac{\mathrm{d}(\Phi_I - \Phi_{II})}{\mathrm{d}t} + i_2'' = 0. \tag{7}$$

Using Eq. (2) we obtain :

$$\Phi_I + \Phi_{II} = \Phi_m \sin \omega t. \tag{8}$$

where

$$\frac{U_m}{n_1 \omega} = \Phi_m.$$

The flux $\Phi_I - \Phi_{II}$ has a direct component Φ_0 and a second harmonic component Φ_2 :

$$\Phi_I - \Phi_{II} = \Phi_0 + \Phi_2(t). \tag{9}$$

By virtue of Eqs. (8) and (9), the fluxes Φ_I and Φ_{II} are :

$$(10)\qquad \Phi_I=\frac{1}{2}(\Phi_m\sin\omega t+\Phi_0+\Phi_2),\quad \Phi_{II}=\frac{1}{2}(\Phi_m\sin\omega t-\Phi_0-\Phi_2).$$

Substracting Eq. (5) from (4), Eq. (11) become :

$$(11)\qquad n_2i_2''+n_0i_0+n_0i_2'''=\frac{1}{2}[K_1(\Phi_I-\Phi_{II})+K_3(\Phi_I^3-\Phi_{II}^3)].$$

By substituting Eq. (10) into Eq. (11), we obtain :

$$n_2i_2''+n_0i_0+n_0i_2'''=\frac{1}{2}\left(K_1\Phi_0+\frac{3}{8}K_3\Phi_0\Phi_m^2+\frac{1}{4}K_3\Phi_0^3\right)+\frac{1}{2}\Bigg(K_1\Phi_2+$$

$$(12)\qquad +\frac{3}{4}K_3\Phi_0^2\Phi_2+\frac{3}{4}K_3\Phi_0\Phi_2^2+\frac{1}{4}K_3\Phi_2^3+\frac{3}{4}K_3\Phi_2\Phi_m^2\sin^2\omega t-$$

$$-\frac{3}{8}K_3\Phi_0\Phi_m\cos 2\omega t\Bigg),$$

from which it result :

$$i_2''=\frac{1}{n_2}\Bigg[-n_0i_0-n_0i_2'''+\frac{1}{2}\left(K_1\Phi_0+\frac{3}{8}K_3\Phi_0\Phi_m^2+\frac{1}{4}K_3\Phi_0^3\right)+$$

$$(13)\qquad +\frac{1}{2}\Bigg(K_1\Phi_2+\frac{3}{4}K_3\Phi_0^2\Phi_2+\frac{3}{4}K_3\Phi_0\Phi_2^2+\frac{1}{4}K_3\Phi_2^3+$$

$$+\frac{3}{4}K_3\Phi_2\Phi_m^2\sin^2\omega t-\frac{3}{8}K_3\Phi_0\Phi_m^2\cos 2\omega t\Bigg)\Bigg].$$

The differential equation for the direct current circuit leads to the relations :

$$(14)\qquad n_0\frac{d(\Phi_I-\Phi_{II})}{dt}+R_0(i_0+i_2''')=E_0,$$

$$(15)\qquad i_2'''=\frac{E_0}{R_0}-i_0-\frac{n_0}{R_0}\cdot\frac{d(\Phi_I-\Phi_{II})}{dt}.$$

By substituting Eq. (15) into Eq. (13) and then i_2'' into Eq. (7) we obtain :

$$\frac{d^2(\Phi_I-\Phi_{II})}{dt^2}+\frac{1}{Cn_2}\left(\frac{n_2}{R}+\frac{n_0^2}{n_2R_0}\right)\frac{d(\Phi_I-\Phi_{II})}{dt}+\frac{1}{2Cn_2^2}\Bigg(K_1\Phi_2+$$

$$+\frac{3}{4}K_3\Phi_0^2\Phi_2+\frac{3}{4}K_3\Phi_0\Phi_2^2+\frac{1}{4}K_3\Phi_2^3+\frac{3}{4}K_3\Phi_2\Phi_m^2\sin^2\omega t-$$

$$(16)\qquad -\frac{3}{8}K_3\Phi_0\Phi_m^2\cos 2\omega t\Bigg)+\frac{1}{2Cn_2^2}\left(K_1\Phi_0+\frac{3}{8}K_3\Phi_0\Phi_m^2+\frac{1}{4}K_3\Phi_0^3\right)-$$

$$-\frac{n_0}{Cn_2^2}\cdot\frac{E_0}{R_0}=0.$$

In a first approximation is possible to consider Φ_0 as being constant w.r.t. time. By virtue of Eq. (9), Eq. (16) may be transformed as follows :

$$\frac{d^2\Phi_2}{dt^2}+\frac{1}{Cn_2^2}\left(\frac{n_2}{R}+\frac{n_0^2}{n_2R_0}\right)\frac{d\Phi_2}{dt}+\frac{1}{2Cn_2^2}\left(K_1\Phi_2+\frac{3}{4}K_3\Phi_0^2\Phi_2+\right. \tag{17}$$

$$\left.+\frac{3}{4}K_3\Phi_0\Phi_2^2+\frac{1}{4}K_3\Phi_2^2+\frac{3}{4}K_3\Phi_2\Phi_m^2\sin^2\omega t-\frac{3}{8}K_3\Phi_0\Phi_m^2\cos 2\omega t\right)=0,$$

$$\frac{1}{2Cn_2^2}\left(K_1\Phi_0+\frac{3}{8}K_3\Phi_0\Phi_m^2+\frac{1}{4}K_3\Phi_0^3\right)-\frac{n_0}{Cn_2^2}\cdot\frac{E_0}{R_0}=0. \tag{18}$$

Using Eq. (18), Φ_0 can be determined.

We integrate the equation (17) using v a n d e r P o l's method [3], i.e. looking for a solution of the form :

$$\Phi_2\approx\Phi_{2m}(t)\cos\left[2\omega t+\theta(t)\right]=\Phi_{2m}(t)\cos 2\omega t\cos\theta(t)- \tag{19}$$

$$-\Phi_{2m}(t)\sin 2\omega t\sin\theta(t).$$

Substituting Eq. (19) into (17) and equating the coefficients of the terms containing $\sin(2\omega t+\theta)$ and $\cos(2\omega t+\theta)$ separately to zero, we obtain :

$$\begin{cases}\dfrac{d\Phi_{2m}}{dt}=-\dfrac{1}{2Cn_2}\left(\dfrac{n_2}{R}+\dfrac{n_0^2}{n_2R_0}\right)\Phi_{2m}-\dfrac{3K_3\Phi_m^2}{64\omega Cn_2^2}-\Phi_0\sin\theta,\\ \dfrac{d\theta}{dt}=\left(-\omega+\dfrac{K_1}{8\omega Cn_2^2}+\dfrac{3K_3\Phi_m^2}{64\omega Cn_2^2}\right)+\dfrac{3K_3}{32\omega Cn_2^2}\Phi_0^2+\\ \quad+\dfrac{3K_3}{128\omega Cn_2^2}\Phi_{2m}^2-\dfrac{3K_3\Phi_m^2}{64\omega Cn_2^2}\cdot\dfrac{\Phi_0}{\Phi_{2m}}\cos\theta.\end{cases} \tag{20}$$

When the almost periodic oscillations occur in the magnetic frequency doubler, the flux Φ_0 is a slowly varying function of time t, so from Eq. (14) we obtain :

$$n_0\frac{d\Phi_0}{dt}+R_0i_0=E_0. \tag{21}$$

Using Eq. (12), choosing $\Phi_2(t)=\Phi_{2m}(t)\cos[2\omega t+\theta(t)]$ and equating the coefficients of the terms containing a very low frequency f and a double frequency $2f$ we obtain for i_0 the expression :

$$n_0i_0=\frac{1}{2}\left(K_1\Phi_0+\frac{3}{8}K_3\Phi_0\Phi_m^2+\frac{1}{4}K_3\Phi_0^3\right)+\frac{3}{16}K_3\Phi_0\Phi_{2m}^2+\frac{3}{32}K_3\Phi_m^2\Phi_{2m}\cos\theta. \tag{22}$$

Substituting Eq. (22) into Eq. (2) it result :

$$\frac{d\Phi_0}{dt}+\frac{R_0}{n_0^2}\left[\frac{1}{2}\left(K_1\Phi_0+\frac{3}{8}K_3\Phi_0\Phi_m^2+\frac{1}{4}K_3\Phi_0^3\right)+\frac{3}{16}K_3\Phi_0\Phi_{2m}^2+\right. \tag{23}$$

$$\left.+\frac{3}{32}K_3\Phi_m^2\Phi_{2m}\cos\theta\right]=E_0.$$

In order to simplify the equations and to increase the generality of the results we introduce the following nondimentional variables :

$$y=\frac{\Phi_{2m}}{\Phi_m},\quad x=\frac{\Phi_0}{\Phi_m}.\quad \tau=\omega t. \tag{24}$$

Then, Eqs. (20) and (23) can be transformed as:

$$\begin{cases} \dfrac{dy}{d\tau}=-Ay-Bx\sin\theta, \\ \dfrac{d\theta}{d\tau}=K+D\left(4x^2+y^2-2\dfrac{x}{y}\cos\theta\right), \\ \dfrac{dx}{d\tau}=K_0-Ex-F(4x^3+2xy^2+y\cos\theta), \end{cases} \tag{25}$$

where we used the notations

$$\begin{cases} A=\dfrac{1}{2C\omega n_2}\left(\dfrac{n_2}{R}+\dfrac{n_0^2}{n_2R_0}\right),\quad B=\dfrac{3K_3\Phi_m^2}{64\omega^2Cn_2^2}, \\ K=-1+\dfrac{K_1}{8\omega^2Cn_2^2}+\dfrac{3K_3\Phi_m^2}{64\omega^2Cn_2^2},\quad D=\dfrac{3K_3\Phi_m^2}{128\,\omega^2Cn_2^2}, \\ K_0=\dfrac{E_0}{\omega\Phi_m},\quad E=\dfrac{R_0}{2n_0^2\omega}\left(K_1+\dfrac{3}{8}K_3\Phi_m^2\right),\quad F=\dfrac{3K_3R_0\Phi_m^2}{32n_0^2\omega}. \end{cases} \tag{26}$$

The equations (25) permit the study of all processes which occur in the circuit represented în Fig. 1.

4. Stability of the Oscillations

To perform a stability analysis it is necessary to introduce the non-linear functions $P(y,\theta,x)$, $Q(y,\theta,x)$ and $R(y,\theta,x)$ and so Eq. (25) becomes:

$$\begin{cases} \dfrac{dy}{d\tau}=-Ay-Bx\sin\theta=P(y,\theta,x), \\ \dfrac{d\theta}{d\tau}=K+D\left(4x^2+y^2-2\dfrac{x}{y}\cos\theta\right)=Q(y,\theta,x), \\ \dfrac{dx}{d\tau}=K_0-Ex-F(4x^3+2xy^2+y\cos\theta)=R(y,\theta,x). \end{cases} \tag{27}$$

We analyse the stability of the oscillations using the first method of Lyapunoff [3].

The periodic solution for which the components $y(\tau)$, $\theta(\tau)$ and $x(\tau)$ are constant is determined by:

$$P(y_s,\theta_s,x_s)=0,\quad Q(y_s,\theta_s,x_s)=0,\quad R(y_s,\theta_s,x_s)=0, \tag{28}$$

where the subscript s is used to designate the values of y, θ and x for the periodic solution.

We introduce the following notations:

$$\varepsilon = y - y_s, \quad \eta = \theta - \theta_s, \quad \sigma = x - x_s, \tag{29}$$

as small variations of y, θ, x from the steady-state values y_s, θ_s and x_s and determine whether these variations approach zero or not with time τ.

The small variations, ε, η and σ, must satisfy the differential equations:

$$\begin{gathered}\frac{d\varepsilon}{d\tau} = a_{11}\varepsilon + a_{12}\eta + a_{13}\sigma, \quad \frac{d\eta}{d\tau} = a_{21}\varepsilon + a_{22}\eta + a_{23}\sigma, \\ \frac{d\sigma}{d\tau} = a_{31}\varepsilon + a_{32}\eta + a_{33}\sigma,\end{gathered} \tag{30}$$

where

$$a_{11} = \left(\frac{\partial P}{\partial y}\right)_s = -A, \quad a_{12} = \left(\frac{\partial P}{\partial \theta}\right)_s = -Bx \cos\theta_s,$$

$$a_{13} = \left(\frac{\partial P}{\partial x}\right)_s = -B \sin\theta_s, \quad a_{21} = \left(\frac{\partial Q}{\partial y}\right)_s = 2D\left(y_s + \frac{x_s}{y_s^2}\cos\theta_s\right),$$

$$a_{22} = \left(\frac{\partial Q}{\partial \theta}\right)_s = 2D\frac{x_s}{y_s}\sin\theta_s, \quad a_{23} = \left(\frac{\partial Q}{\partial x}\right)_s = 2D\frac{4x_s y_s - \cos\theta_s}{y_s},$$

$$a_{31} = \left(\frac{\partial R}{\partial y}\right)_s = -F(4x_s y_s + \cos\theta_s), \quad a_{32} = \left(\frac{\partial R}{\partial \theta}\right)_s = Fy_s \sin\theta_s,$$

$$a_{33} = \left(\frac{\partial R}{\partial x}\right)_s = -E - F(12x_s^2 + 2y_s^2).$$

The characteristic equation of the system (30) is given by:

$$\begin{vmatrix} a_{11}-\lambda & a_{12} & a_{13} \\ a_{21} & a_{22}-\lambda & a_{23} \\ a_{31} & a_{32} & a_{33}-\lambda \end{vmatrix} = 0 \tag{31}$$

or

$$\lambda^3 + \alpha\lambda^2 + \beta\lambda + \gamma = 0, \tag{32}$$

where

$$\begin{gathered}\alpha = -(a_{11} + a_{22} + a_{33}), \quad \beta = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} + \begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix} + \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}, \\ \gamma = -\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix},\end{gathered} \tag{33}$$

The stability condition is given by the Routh–Hurwitz criterion [2], that is:

$$\alpha > 0, \quad (34.2) \quad \alpha\beta - \gamma > 0, \quad (34.3) \quad \gamma > 0. \tag{34.1}$$

The condition (3 4.1) is always fulfilled because

$$\alpha = 2A + E + 2F(6x_s^2 + y_s^2). \tag{35}$$

The inequations (34.2) and (34.3) permit to establish the stability conditions for the periodic solution. They are:

$$(36)\quad (\cos\theta_s - 4x_s y_s)\left\{BF[K+D(4x_s^2+y_s^2)](\cos\theta_s+4x_s y_s)-A^2F\frac{y_s}{x_s}\right\}>0,$$

$$(37)\quad [K+D(4x_s^2+y_s^2)](16x_s^2y_s^2-1)-By_s^2\sin^2\theta_s>0.$$

If, at least, one of the relations (36), (37) is not satisfied, almost periodic oscillations will occur.

5. Numerical Results

In order to justify the theoretical results, we numerically integrated the differential equations system (25), by M e r s o n's method, on a digital computer of the type Felix CE 256.

We used a magnetic frequency doubler with two permalloy cores, having the following characteristics:

$$(38)\quad \begin{cases} K_1=1.2\times10^4\ \text{A/Wb};\ K_3=1.5\times10^{12}\ \text{A/Wb}^3;\ n_1=300; \\ n_2=600;\ n_0=200;\ \Phi_m=1.35\times10^{-4}\ \text{Wb};\ R=1.7\times10^4\,\Omega; \\ R_0=1.7\times10^3\,\Omega;\ C=5\,\mu\text{F};\ E_0=2\text{V}. \end{cases}$$

We obtained:

$$A=0.0395,\ B=0.0073,\ K=-0.984,\ D=0.0036,$$
$$K_0=47.2,\ E=1.5,\ F=0.346.$$

The numerical results obtained for $y(t)$, $\theta(t)$, $x(t)$ and $s(t)=y(t)\cos[2\omega t+\theta(t)]$ are represented in Figs. 2 ... 5.

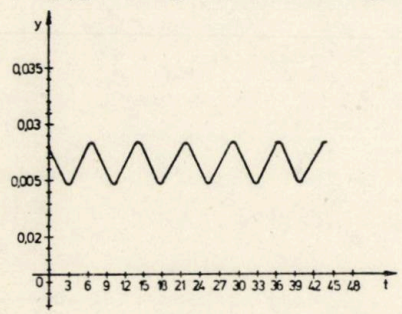

Fig. 2

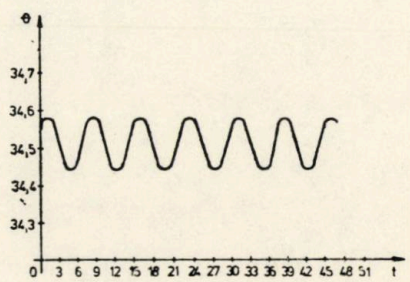

Fig. 3

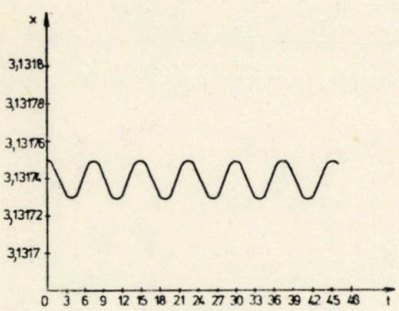

Fig. 4

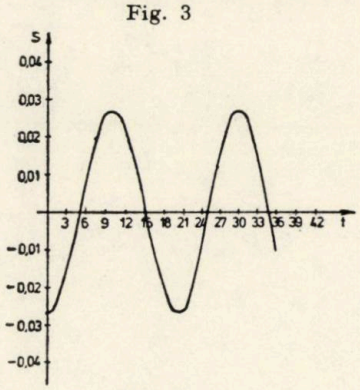

Fig. 5

From the curves represented in these figures it results that almost periodic oscillations occur in the magnetic frequency doubler, fact which was experimentally confirmed too.

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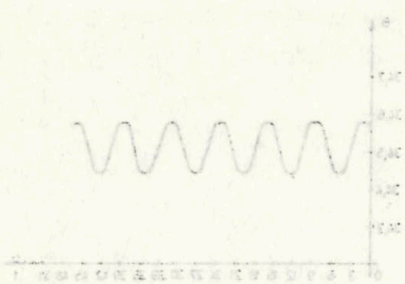
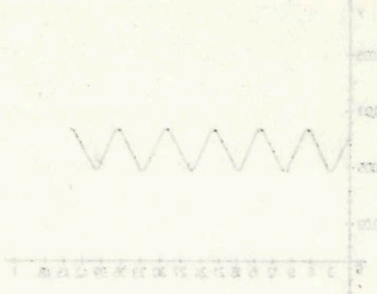
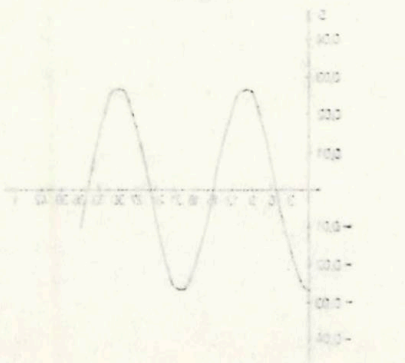
OSCILAŢII APROAPE PERIODICE ÎN DUBLORUL MAGNETIC DE FRECVENŢĂ

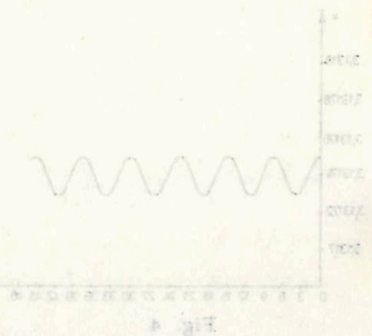
(Rezumat)

Se stabilesc ecuaţiile diferenţiale care permit analiza oscilaţiilor aproape periodice din dublorul magnetic de frecvenţă. Se efectuează studiul stabilităţii dispozitivului şi se prezintă rezultate numerice care justifică rezultatele teoretice obţinute.









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D.C. 538.551

AN EXISTENCE THEOREM OF THE NORMAL FORM FOR NONLINEAR NETWORKS IN TERMS OF TOPOLOGICAL CONSTRAINTS

BY

ADRIAN IOINOVICI

1. Introduction

Since there exist many networks that cannot be represented in the normal form, one of the basic problems in nonlinear circuits is to find sufficient conditions for networks which admit such a representation.

In other papers considering the above mentioned problem, the controlled sources are excluded [1, Theorem 3], the conditions of the theorems are difficult to check [2], [3], or these conditions are not expressed in terms of characteristic curves of the elements and the network topology [1, Theorem 1], [4], [5].

2. The Main Theorem

We introduce here an existence theorem of the normal form for a network $\mathcal{N}$ containing : independent sources, linear controlled sources and nonlinear resistances, capacitances, inductances.

Let $\mathcal{T}$ be any normal tree of $\mathcal{N}$ chosen as in [6]. Let us partition the set of network branches into eight disjoint subsets :

1. tree-independent voltage sources, E ;
2. tree-capacitances, Ct ;
3. tree-resistances, Rt ;
4. tree-inductances, Lt ;
5. link-capacitances, Cl ;
6. link-resistances, Rl ;
7. link-inductances, Ll ;
8. link-independent current sources, J.

(Each edge of the network graph for $\mathcal{N}$ represents a network element).

In view of the previous notation, the current and voltage vectors $(\mathbf{i}, \mathbf{v})$ are partitioned, as well as the fundamental cut-set matrix $(\mathbf{Q})$. Then, the governing equations can be written in the following partitioned form [6] :

$$(1)\qquad i_E=-Q_{EC}i_{Ct}-Q_{ER}i_{Rt}-Q_{EL}i_{Lt}-Q_{EJ}i_J\,,$$

$$(2)\qquad i_{Ct}=-Q_{CC}i_{Ct}-Q_{CR}i_{Rt}-Q_{CL}i_{Lt}-Q_{CJ}i_J\,,$$

$$(3)\qquad i_{Rt}=-Q_{RR}i_{Rt}-Q_{RL}i_{Lt}-Q_{RJ}i_J\,,\qquad (4)\quad i_{Lt}=-Q_{LL}i_{Lt}-Q_{LJ}i_J,$$

$$(5)\qquad v_{Ct}=Q'_{EC}v_E+Q'_{CC}v_{Ct},$$

$$(6)\qquad v_{Rt}=Q'_{ER}v_E+Q'_{CR}v_{Ct}+Q'_{RR}v_{Rt},$$

$$(7)\qquad v_{Lt}=Q'_{EL}v_E+Q'_{CL}v_{Ct}+Q'_{RL}v_{Rt}+Q'_{LL}v_{Lt},$$

$$(8)\qquad v_J=Q'_{EJ}v_E+Q'_{CJ}v_{Ct}+Q'_{RJ}v_{Rt}+Q'_{LJ}v_{Lt}.$$

Theorem. *If, for the circuit* $\mathcal{N}$, *the following hypotheses are satisfied*:

1. a) *The tree-capacitances are characterized by the functions*:

$$(9)\qquad q_{Ct}=f_{Ct}(v_{Ct})$$

(*where* q *denotes the capacitance charge*) *and the link-capacitances are linear time-invariable and characterized by the diagonal matrix* $\mathbf{C}_t$;

b) *the link-inductances are characterized by the functions*:

$$(10)\qquad \varphi_{Lt}=f_{Lt}(i_{Lt})$$

(*where* φ *denotes the inductance flux*) *and the tree-inductances are linear, time-invariable and characterized by the diagonal matrix* $\mathbf{L}_t$ (*no mutual inductances*);

c) *the resistive branches* (*including the controlled sources*) *are characterized by the equations*:

$$(11)\qquad \begin{cases} v_{Rt}=f_{Rt}(i_{Rt})+R_{tt}i_{Rt}+H_{Rt,Ct}v_{Ct}+H_{Rt,Lt}i_{Lt}+H_{Rt,E}v_E+H_{Rt,J}i_J\,,\\ v_{Rt}=R_{tt}i_{Rt}+R_{tt}i_{Rt}+H_{Rt,Ct}v_{Ct}+H_{Rt,Lt}i_{Lt}+H_{Rt,E}v_E+H_{Rt,J}i_J\,. \end{cases}$$

2. a) *The elements* a_{ij} *of the matrix*:

$$(12)\qquad \mathbf{A}=\mathbf{Q}_{CC}\mathbf{C}_t\mathbf{Q}'_{CC}$$

satisfy the inequality:

$$(13)\qquad a_{ii}\geqslant\sum_{\substack{j=1\\ j\neq i}}^{n_C}|\,a_{ij}\,|,\ \text{for}\ i=1,2,\ldots,n_C,$$

where n_C *is the number of tree-capacitances and* f_{Ct} *are strictly monotone increasing functions mapping* $(-\infty,\infty)$ *onto* $(-\infty,\infty)$;

b) *the elements* b_{ij} *of the matrix*:

$$(14)\qquad \mathbf{B}=\mathbf{Q}'_{LL}\mathbf{L}_t\mathbf{Q}_{LL}$$

satisfy the inequality:

$$(15)\qquad b_{ii}\geqslant\sum_{\substack{j=1\\ j\neq i}}^{n_L}|\,b_{ij}\,|,\ \text{for}\ i=1,2,\ldots,n_L,$$

where n_L *is the number of link-inductances and* f_{Lt} *are strictly monotone increasing functions mapping* $(-\infty,\infty)$ *onto* $(-\infty,\infty)$;

c) *the elements* d_{ij} *of the matrix*:

$$\mathbf{D}=\mathbf{Q}'_{RR}\mathbf{R}_{tt}\mathbf{Q}_{RR}-\mathbf{R}_{lt}\mathbf{Q}_{RR}-\mathbf{Q}'_{RR}\mathbf{R}_{tl} \tag{16}$$

satisfy the inequality:

$$d_{ii}\geqslant\sum_{\substack{j=1\\ j\neq i}}^{n_R}|\,d_{ij}\,|,\ \textit{for}\ i=1,2,\ldots,n_R, \tag{17}$$

where n_R *is the number of nonlinear link-resistances and* f_{Rl} *are strictly monotone increasing functions mapping* $(-\infty,\infty)$ *onto* $(-\infty,\infty)$;

3. *The functions* f_{Cl} *and* f_{Ll} *are differentiable, then the normal form exists for all possible values of current and voltage sources.*

Proof. From the condition 1 a), one can write:

$$\mathrm{q}_{Cl}+\mathbf{Q}_{CC}\mathrm{q}_{Cl}=\mathrm{f}_{Cl}(\mathrm{v}_{Cl})+\mathbf{Q}_{CC}\mathbf{C}_l\mathrm{v}_{Cl}.$$

Using (5) and (12), one obtains the equation:

$$\mathrm{f}_{Cl}(\mathrm{v}_{Cl})+\mathbf{A}\mathrm{v}_{Cl}=\mathrm{q}_{Cl}+\mathbf{Q}_{CC}\mathrm{q}_{Cl}-\mathbf{Q}_{CC}\mathbf{C}_l\mathbf{Q}'_{EC}\mathrm{v}_E. \tag{18}$$

In view of [7, Theorem 1], the condition 2 a) is sufficient to assure a unique solution of (18):

$$\mathrm{v}_{Cl}=\mathrm{g}_{Cl}(\mathrm{q}_{Cl}+\mathbf{Q}_{CC}\mathrm{q}_{Cl},\mathrm{v}_E). \tag{19}$$

From the condition 1 b), one can write:

$$\varphi_{Ll}-\mathbf{Q}'_{LL}\varphi_{Lt}=\mathrm{f}_{Ll}(\mathrm{i}_{Ll})-\mathbf{Q}'_{LL}\mathbf{L}_t\mathrm{i}_{Lt}.$$

Using (4) and (14), one obtains the equation:

$$\mathrm{f}_{Ll}(\mathrm{i}_{Ll})+\mathbf{B}\mathrm{i}_{Ll}=\varphi_{Ll}-\mathbf{Q}'_{LL}\varphi_{Lt}-\mathbf{Q}'_{LL}\mathbf{L}_t\mathbf{Q}_{LJ}\mathrm{i}_J. \tag{20}$$

In view of [7, Theorem 1] the condition 2 b) is sufficient to assure a unique solution of (20):

$$\mathrm{i}_{Ll}=\mathrm{g}_{Ll}(\varphi_{Ll}-\mathbf{Q}'_{LL}\varphi_{Lt},\mathrm{i}_J). \tag{21}$$

From the condition 1 c), using successively (6), (11), (3), one obtains finally the equation:

$$\begin{aligned}\mathrm{f}_{Rl}(\mathrm{i}_{Rl})+\mathbf{D}\mathrm{i}_{Rl}=&(\mathbf{Q}'_{ER}+\mathbf{Q}'_{RR}\mathbf{H}_{Rt,E}-\mathbf{H}_{Rl,E})\mathrm{v}_E+(\mathbf{Q}'_{RR}\mathbf{H}_{Rt,J}-\mathbf{Q}'_{RR}\mathbf{R}_{tt}\mathbf{Q}_{RJ}+\\ &+\mathbf{R}_{lt}\mathbf{Q}_{RJ}-\mathbf{H}_{Rl,J})\mathrm{i}_J+(\mathbf{Q}'_{CR}+\mathbf{Q}'_{RR}\mathbf{H}_{Rt,Ct}-\mathbf{H}_{Rl,Ct})\mathrm{v}_{Ct}+(\mathbf{Q}'_{RR}\mathbf{H}_{Rt,Ll}-\\ &-\mathbf{Q}'_{RR}\mathbf{R}_{tt}\mathbf{Q}_{RL}+\mathbf{R}_{lt}\mathbf{Q}_{RL}-\mathbf{H}_{Rlt,Ll})\mathrm{i}_{Ll}.\end{aligned} \tag{22}$$

In view of [7, Theorem 1], the condition 2 c) is sufficient to assure a unique solution of (22):

$$\mathrm{i}_{Rl}=\mathrm{g}_{Rl}(\mathrm{v}_E,\mathrm{i}_J,\mathrm{v}_{Ct},\mathrm{i}_{Ll}). \tag{23}$$

If one adds the hypothesis 3), it results the existence of the normal form for the circuit $\mathcal{N}$, the state variables being the elements of the vectors: $\mathrm{q}_{Cl}+\mathbf{Q}_{CC}\mathrm{q}_{Cl}$ and $\varphi_{Ll}-\mathbf{Q}'_{LL}\varphi_{Lt}$ [6].

3. Remarks

Remark 1. The theorem is also valid for circuits containing inductances characterized by the expressions:

$$\varphi_{Ll}=f_{Ll}(i_{Ll})+L_{lt}i_{Lt}, \qquad \varphi_{Lt}=L_{tl}i_{Ll}+L_{tt}i_{Lt}, \tag{10'}$$

the matrix **B** being then:

$$\mathbf{B}=\mathbf{Q}'_{LL}\mathbf{L}_{tt}\mathbf{Q}_{LL}-\mathbf{Q}'_{LL}\mathbf{L}_{tl}-\mathbf{L}_{lt}\mathbf{Q}_{LL}. \tag{14'}$$

Remark 2. If the resistive branches are characterized by the equations:

$$\begin{cases} i_{Rt}=f_{Rt}(v_{Rt})+G_{tl}v_{Rl}+H_{Rt,Ct}v_{Ct}+H_{Rt,Ll}i_{Ll}+H_{Rt,E}v_E+H_{Rt,J}i_J+ \\ \qquad +H_{Rt,Cl}v_{Cl}+H_{Rt,Lt}i_{Lt}, \\ i_{Rl}=G_{lt}v_{Rt}+G_{ll}v_{Rl}+H_{Rl,Ct}v_{Ct}+H_{Rl,Ll}i_{Ll}+H_{Rl,E}v_E+H_{Rl,J}i_J+ \\ \qquad +H_{Rl,Cl}v_{Cl}+H_{Rl,Lt}i_{Lt}, \end{cases} \tag{11'}$$

the condition 2c) can be formulated as follows:
the elements f_{ij} *of the matrix*:

$$\mathbf{F}=\mathbf{Q}_{RR}\mathbf{G}_{tl}\mathbf{Q}'_{RR}+\mathbf{G}_{tt}\mathbf{Q}'_{RR}+\mathbf{Q}_{RR}\mathbf{G}_{lt} \tag{16'}$$

satisfy the inequality:

$$f_{ii}\geq\sum_{\substack{j=1\\ j\neq i}}^{m_R}|f_{ij}|, \text{ for } i=1,2,\ldots,m_R, \tag{17'}$$

where m_R *is the number of nonlinear tree-resistances and* f_{Rt} *are strictly monotone increasing functions mapping* $(-\infty,\infty)$ *onto* $(-\infty,\infty)$.

Other similar cases can be studied.

Remark 3. According to [8], the condition 2) can be reformulated as follows:

The mapping f_{Ct}, f_{Ll}, f_{Rl}, *respectively, are continuous monotone increasing functions mapping* $(-\infty,\infty)$ *into* $(-\infty,\infty)$, *but the inequalities* (13), (15), (17), *respectively, become strictly inequalities.*

4. Conclusions

The hypotheses introduced by the theorem are simple and easily verifiable by inspection. They involve solely the examination of the nature of the characteristic curves of the nonlinear network elements and check whether some inequalities are satisfied by the elements of some matrices formed with the values of the linear network elements.

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O TEOREMĂ DE EXISTENȚĂ A FORMEI NORMALE A CIRCUITELOR ELECTRICE NELINIARE PE BAZĂ DE CRITERII TOPOLOGICE

(Rezumat)

Se enunță și se demonstrează o teoremă de existență a formei normale a ecuațiilor ce caracterizează un circuit neliniar *RLC*, incluzînd surse comandate liniare. Criteriile sînt exprimate în funcție de natura curbelor caracteristice ale elementelor de circuit și de topologia circuitului.

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SYMBOLIC REPRESENTATION OF SIGNALS ON HYPERSPACES

II. THE RESPONSE OF LINEAR SYSTEMS TO EXCITATIONS REPRESENTABLE SYMBOLICALLY

BY

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1. Introduction

Let $\mathcal{H}$ be the hyperspace direct sum of the complex field and n 4th order hypercomplex orthogonal spaces with a figurative point:

$$\hat{w}=(a_1+ja_2)\mathbf{E}_0+\sum_{p=1}^{n}(a_{1p}+ia_{2p}+ja_{3p}+ija_{4p})\mathbf{E}_p, \tag{1}$$

$$a_1, a_2\in\mathbf{R},\ a_{ip}\in\mathbf{R},\ (i=\overline{1,4},\ p=\overline{1,n})$$

and the multiplication table:

$$i^2=j^2=-1;\ (ij)^2=(ji)^2=1;\ \mathbf{E}_p\mathbf{E}_q=\delta_{pq}\mathbf{E}_p. \tag{2}$$

For the element (1) an exponential form was introduced in a previous paper [2]:

$$\hat{w}=Ae^{j\alpha}\mathbf{E}_0+\sum_{p=1}^{n}(A_{1p}+ijA_{2p})e^{i\beta_p}\mathbf{E}^{j\alpha_p}\mathbf{E}_p. \tag{3}$$

The transformation of (1) into (2) is realised by means of relations as:

$$\begin{cases} A=\sqrt{a_1^2+a_2^2},\ \operatorname{tg}\alpha=\dfrac{a_2}{a_1}; \\ A_{1p}, A_{2p}=\dfrac{1}{2}\left[\sqrt{(a_{1p}+a_{4p})^2+(a_{2p}-a_{3p})^2}\pm\sqrt{(a_{1p}-a_{4p})^2+(a_{2p}+a_{3p})^2}\right]; \\ \operatorname{tg}\beta_p=\dfrac{a_{4p}A_{1p}-a_{1p}A_{2p}}{a_{3p}A_{1p}+a_{2p}A_{2p}},\ \operatorname{tg}\alpha_p=\dfrac{a_{3p}A_{1p}+a_{2p}A_{2p}}{a_{1p}A_{1p}-a_{4p}A_{2p}}. \end{cases} \tag{4}$$

The way in which a large class of modulated signals (A.M., A.M. with a supressed carrier, S.S.B., S.S.B. with vestigial sideband, so as some amplitude and phase modulated signals) may be represented symbolically on this hyperspace with a fixed derivation operator has been rendered obvious in [2].

By means of an equivalence relation, a partition of this hyperspace into equivalence classes characterizing completely the signals representable symbolically has been realized.

The aim of this paper is to establish some propositions allowing to formulate the conditions in which multiplication, raising to a power and inversion modify or not the partition class of the elements entering those operations.

Those results are useful in studying the response of linear systems to excitations representable symbolically.

2. Basic Results

Like in [2], we shall denote with:

$$(5)\quad \begin{cases} C_s - \text{the class of special type elements in the hyperspace } \mathcal{H}; \\ C_r, (r=\overline{1, n-1}) - \text{the class of } r \text{ order arbitrary elements in the hyperspace } \mathcal{H}; \\ O'(O'') - \text{the class of 1st (2nd) species divisors of zero in the hyperspace } \mathcal{H}; \\ O'_r(O''_r), (r=\overline{1, n-1}) - \text{the class of } r \text{ order quasi divisors of zero of 1st (2nd) species on the hyperspace } \mathcal{H}. \end{cases}$$

Let $\widehat{w}_1=\sum_{k=0}^{n}\widehat{\alpha}_n \mathbf{E}_k$, $\widehat{w}_2=\sum_{k=0}^{n}\widehat{\beta}_k \mathbf{E}_k$ be two elements in where $\widehat{\alpha}_k$, $\widehat{\beta}_k$, $(k=\overline{0, n})$, represent the coordinates of the figurative points $\widehat{w}_1$, respectively $\widehat{w}_2$ in the hypercomplex space k.

Taking into account (2_3), one obtains:

$$(6)\quad \widehat{w}_1\widehat{w}_2=\sum_{k=0}^{n}\widehat{\gamma}_k \mathbf{E}_k,\ \widehat{\gamma}_k=\widehat{\alpha}_k\widehat{\beta}_k,\ (k=\overline{0, n}).$$

Proposition 1. *The following implication takes place*:

$$\widehat{w}_1\in C_{r_1}\wedge\widehat{w}_2\in C_{r_2}\Rightarrow\widehat{w}_1\widehat{w}_2\in C_l,\ (1\leqslant l\leqslant n).$$

Demonstration. If $\widehat{w}_1\in C_{r_1}$, then:

$$(7)\quad \widehat{\alpha}_p=\begin{cases} A_0 e^{j\varphi_{10}},\ p=0; \\ m_{1p}A_0 e^{i\psi_{1p}}e^{j\varphi_{10}},\ p\in\mathcal{P}_1,\ \text{card}\ \mathcal{P}_1=r_1; \\ (U_{1p}+ijV_{1p})e^{i\psi_{1p}}e^{j\varphi_{1p}},\ p\in\overline{\mathcal{P}}_1. \end{cases}$$

If $\widehat{w}_2\in C_{r_2}$, then:

$$(8)\quad \widehat{\beta}_p=\begin{cases} B_0 e^{j\varphi_{20}},\ p=0; \\ m_{2p}B_0 e^{i\psi_{2p}}e^{j\varphi_{20}},\ p\in\mathcal{P}_2,\ \text{card}\ \mathcal{P}_2=r_2; \\ (U_{2p}+ijV_{2p})e^{i\psi_{2p}}e^{j\varphi_{2p}},\ p\in\overline{\mathcal{P}}_2. \end{cases}$$

From (7), (8) and (6_2) it results:

$$(9)\quad \hat{\gamma}_p=\begin{cases} A_0B_0e^{j(\varphi_{10}+\varphi_{20})},\ p=0\ ; \\ m_{1p}m_{2p}A_0B_0e^{i(\psi_{1p}+\psi_{2p})}\,e^{j(\varphi_{10}+\varphi_{20})},\ p\in\mathcal{P}_1\cap\mathcal{P}_2\ ; \\ m_{1p}A_0(U_{2p}+ijV_{2p})e^{i(\psi_{1p}+\psi_{2p})}\,e^{j(\varphi_{10}+\varphi_{2p})},\ p\in\mathcal{P}_1\cap\overline{\mathcal{P}}_2\ ; \\ m_{2p}B_0(U_{1p}+ijV_{1p})e^{i(\psi_{1p}+\psi_{2p})}\,e^{j(\varphi_{1p}+\varphi_{20})},\ p\in\overline{\mathcal{P}}_1\cap\mathcal{P}_2\ ; \\ [U_{1p}U_{2p}+V_{1p}V_{2p}+ij(U_{1p}V_{2p}+V_{1p}U_{2p})]e^{i(\psi_{1p}+\psi_{2p})}\,e^{j(\varphi_{1p}+\varphi_{2p})}, \\ \qquad p\in\overline{\mathcal{P}}_1\cap\overline{\mathcal{P}}_2 \end{cases}$$

One may notice in (9) that generally speaking $\hat{w}_1\hat{w}_2\in C_l$, $0\leqslant l\leqslant n$. The conditions to be followed so that $\hat{w}_1\hat{w}_2$ should belong to a specified partition class may be divided into three different cathegories: a) regarding the sets $\mathcal{P}_1$, $\mathcal{P}_2$; b) concerning the real values characterising elements $\hat{w}_1$, $\hat{w}_2$; c) mixed conditions.

Having in view those conditions, the product of elements $\hat{w}_1\,\hat{w}_2$ belong to one of the partition classes (5_1), (5_2).

If $\hat{w}_1\in C_s$ and $\hat{w}_2\in C_s$, then $r_1=n$, $r_2=n$, $\mathcal{P}_1\cap\mathcal{P}_2=\{1, 2, ..., n\}$, $\overline{\mathcal{P}}_1\cap\cap\mathcal{P}_2=\mathcal{P}_1\cap\overline{\mathcal{P}}_2=\overline{\mathcal{P}}_1\cap\overline{\mathcal{P}}_2=\emptyset$.

Therefore, from (9) it results:

Consequence 1. *The product of two special type elements is a special type element.*

If $\hat{w}_1\in C_s$, $\hat{w}_2\in C_r$, then $\mathcal{P}_1\cap\mathcal{P}_2=\mathcal{P}_2$, $\mathcal{P}_1\cap\overline{\mathcal{P}}_2=\overline{\mathcal{P}}_2$, $\overline{\mathcal{P}}_1\cap\mathcal{P}_2=\overline{\mathcal{P}}_1\cap\cap\overline{\mathcal{P}}_2=\emptyset$.

It results:

Consequence 2. *The product of a special type element with an r order arbitrary one is an r order arbitrary element.*

Using Proposition 1 and mathematical induction it is easy to demonstrate:

Proposition 2. *The raising to a power of a certain element $\hat{w}\in\mathcal{H}$ does not change the partition class of the element.*

Regarding the divisors and quasi-divisors of zero in $\mathcal{H}$ the following propositions may be demonstrated:

Proposition 3. *The implication*:

$$\hat{w}_1\in C_{r1}\wedge\hat{w}_2\in O'_r(O''_r)\Rightarrow\hat{w}_1\,\hat{w}_2\in O'_r(O''_r),\ (\forall r=\overline{1,n},\ \forall r_1=\overline{1,n}),$$

takes place in $\mathcal{H}$.

For $r=n$, we obtain:

Consequence 3. *The product of a certain element belonging to the hyperspace $\mathcal{H}$ with a divisor of zero of 1st (2nd) species is a divisor of zero of 1st (2nd) species.*

It also takes place:

Proposition 4. *The product of a 1st species divisor of zero with a 2nd species divisor of zero is the element zero of the algebra.*

Demonstration.

$$\hat{w}_1 \in O' \Rightarrow \hat{w}_1 = \sum_{p=1}^{n} (a_p + ib_p + jb_p - ija_p)\mathbf{E}_p, \quad (a_p, b_p \in \mathbf{R} \; ; \; p = \overline{1, n}) \; ;$$

$$\hat{w}_2 \in O'' \Rightarrow \hat{w}_2 = \sum_{p=1}^{n} (c_p + id_p - jd_p + ijc_p)\mathbf{E}_p, \quad (c_p, d_p \in \mathbf{R} \; ; \; p = \overline{1, n}).$$

Effectuating the product and taking (2) into account, it results that $\hat{w}_1\hat{w}_2 = \hat{O}$, which is demonstrated by the proposition itself.

In other words, the 1st and 2nd species divisors of zero are *orthogonals*.

For an element $\hat{w} \in \mathcal{H}$, $\hat{w} \notin O_r$, $(r = \overline{1, n})$, $\hat{w}^{-1}$ is defined by relation :

$$\hat{w}\hat{w}^{-1} = \sum_{k=0}^{n} \mathbf{E}_k.$$

This way takes place :

Proposition 5. *For a certain element which is neither a divisor nor a quasi-divisor of zero, the inversion does not change the partition class of that element.*

The propositions stated above are enough to characterize the answer of linear systems with constant parameters to excitations representable symbolically.

Observation. With proposition 5 and observing that $\hat{w}_1/\hat{w}_2 = \hat{w}_1\hat{w}_2^{-1}$; $\hat{w}_2 \notin O_r$, $(r = \overline{1, n})$, the propositions above are also applicable to the ratio between two elements belonging to $\mathcal{H}$.

3. General Information Concerning the Response of the Linear Systems

Let be a linear system with constant parameters (to simplify the exposition, let us choose it with a unique input, respectively output) characterized by the Laplace transfer function $H(s)$.

If a signal $s_i(t)$, representable symbolically on the hyperspace $\mathcal{H}$ by element $\hat{S}_i$ with the derivation operator $\hat{d}$, is applied to the input of the system, then its response is representable symbolically on the same hyperspace by the element $\hat{S}_o$ given by relation [1] :

$$\hat{S}_o = \hat{H}\hat{S}_i. \tag{10}$$

The element $\hat{H}$ is obtained by replacing the variable s with the expression of the derivation operator $\hat{d}$ in that of the transfer function and shall be denoted as *transferance* or as *hypercomplex transfer function of the system.*

This transformation of the complex field (field of the variable s) in the hyperspace $\mathcal{H}$ (domain of the operator $\hat{d}$) shall be denoted from now on as *the transformation* $s \to \hat{d}$. There exist some remarkable properties of this

transformation (referring to the fact that to a class of transfer functions corresponds only one class of partitions on the hyperspace $\mathcal{H}$) which are to be pointed out in what follows.

Some ways of using the propositions from §2 to obtain information concerning the response of the linear systems shall also be exemplified.

If $s_i(t)$ is a A.M. signal, its representation on the hyperspace $\mathcal{H}$ becomes possible by means of a special type element :

$$s_i(t) \leftrightarrow \widehat{S}_i \in C_s.$$

a) If $\widehat{H} \in C_s$, it results from (10) and consequence 1 (§2) that $\widehat{S}_o = \widehat{H}\widehat{S}_i \in C_s$. As a result, the response of the system may be represented on $\mathcal{H}$ by means of a special type element so that it represents a purely amplitude modulated signal.

b) If $\widehat{H} \in C_r$, it results from (10) and consequence 2 (§2) that $\widehat{S}_o = \widehat{H}\widehat{S}_i \in C_r$. Accordingly, the response of the system is representable symbolically by means of an r order arbitrary element which represents symbolically an A.M. signal with parasite phase modulation for $n-r$ components (distorsions are introduced).

c) If $\widehat{H} \in O'_r(O''_r)$, it results from (10) and proposition 3 (§2) that $\widehat{S}_o = \widehat{H}\widehat{S}_i \in O'_r(O''_r)$ is representable on $\mathcal{H}$ by an r order quasi-divisor of zero of 1st (2nd) species. The response is a S.S.B. signal with upper (lower) sideband and residual lower (upper) sideband.

For $r=n$ (i.e. $H \in O'(O'')$) the response is a S.S.B. signal with upper (lower) sideband.

Observations. 1. Generally speaking $H(s)=M(s)/N(s)$ is a rational function of s. If $\widehat{M}(\hat{d}) \in O'_r(O''_r)$, it results from proposition 3 (§2) that $\widehat{H}(\hat{d}) \in O'_r(O''_r)$ only if $\widehat{N}(\hat{d}) \notin O'_r$ or O''_r.

If $\widehat{N}(\hat{d}) \in O'_r$ or O''_r a resonance with infinite values takes place inside the system. The signal response is representable symbolically on a hyperspace of larger dimensions.

2. The propositions established in [2] and the results obtained in §2 allow us to establish the immediate response in the time domain.

3. If the input signal is representable symbolically by an element from $\mathcal{H}$ which belongs to a class of partitions (5) other than $\mathcal{H}$, the propositions from §2 are enough to obtain information concerning the response.

4. The Partition of The Transferance Functions

Let be a linear system with constant parameters characterized by the Laplace transfer function :

$$(11) \qquad H(s) = \frac{\sum_{k=0}^{r} \alpha_k s^k}{\sum_{k=0}^{l} \beta_k s^k}, \quad (\alpha_k, \beta_k \in \mathbf{R}, \ \forall k = \overline{1, \max(r, l)}, \ |r-l| \leqslant 1).$$

Expression (11) may be written as :

$$H(s)=\prod_{k=0}^{m}\frac{A_{3k}s^2+A_{2k}s+A_{1k}}{B_{3k}s^2+B_{2k}s+B_{1k}}, \tag{12}$$

where A_{ik}, B_{ik}, $(i=\overline{1,3},\ k=\overline{0,m},\ m=\max(r,l))$ are functions of α_k, β_k, $(k=\overline{0,m})$ and some of A_{ik}, B_{ik} are zero.

Studying the steady-state response to excitations like [2] :

$$s(t)=Se^{\delta t}\left[1+\sum_{p=1}^{n}m_p e^{\delta_p t}\sin(\omega_p t+\psi_p)\right]\sin(\omega t+\varphi), \tag{13}$$

which may be represented symbolically on $\mathcal{H}$ by an element as :

$$\hat{s}=Se^{j\varphi}\mathbf{E}_0+\sum_{p=1}^{n}mpSe^{i\psi_p}e^{j\varphi_p}\mathbf{E}_p,$$

with the derivation operator :

$$\hat{d}=(\delta+j\omega)\mathbf{E}_0+\sum_{p=1}^{n}(\delta+\delta_p+i\omega_p+j\omega)\mathbf{E}_p, \tag{14}$$

the transferance of the system is :

$$\hat{H}(\hat{d})=\prod_{k=0}^{m}\frac{A_{3k}\hat{d}^2+A_{2k}\hat{d}+A_{1k}\hat{u}}{B_{3k}\hat{d}^2+B_{2k}\hat{d}+B_{1k}\hat{u}}, \tag{15}$$

with $\hat{u}=\sum_{k=0}^{n}\mathbf{E}_k$ as a unity element in $\mathcal{H}$.

Performing the calculi, we obtain :

$$\hat{H}=\prod_{k=1}^{m}\frac{a_{1k}+ja_{2k}}{b_{1k}+jb_{2k}}\mathbf{E}_0+\sum_{p=1}^{n}\prod_{k=1}^{n}\frac{a_{1k}^{(p)}+ia_{2k}^{(p)}+ja_{3k}^{(p)}+ija_{4k}^{(p)}}{b_{1k}^{(p)}+ib_{2k}^{(p)}+jb_{3k}^{(p)}+ijb_{4k}^{(p)}}\mathbf{E}_p, \tag{16}$$

where :

$$\begin{cases} a_{1k}=A_{3k}(\delta^2-\omega^2)+A_{2k}\delta+A_{1k},\ a_{2k}=\omega(2A_{3k}+A_{2k}),\\ a_{1k}^{(p)}=A_{3k}[(\delta+\delta_p)^2-\omega_p^2-\omega^2]+A_{2k}(\delta+\delta_p)+A_{1k},\\ a_{2k}^{(p)}=\omega_p[2A_{3k}(\delta+\delta_p)+A_{2k}],\ a_{3k}^{(p)}=\omega[2A_{3k}(\delta+\delta_p)+A_{2k}],\\ a_{4k}^{(p)}=2\omega\omega_pA_{3k}. \end{cases} \tag{17}$$

The relations for b_{ik}, $b_{jk}^{(p)}$, $(i=1,2\ ;\ j=\overline{1,4}\ ;\ p=\overline{1,n};\ k=\overline{1,m})$, have the same form as (17) replacing A_{ik} with B_{ik}.

Using equalities like [2] :

$$a_{1k}+ja_{2k}=A_ke^{j\varphi_k},\ a_1^{(p)}+ia_2^{(p)}+ja_3^{(p)}+ija_4^{(p)}=(A_1^{(p)}+ij\,A_2^{(p)})e^{i\psi_p}e^{j\varphi_p},$$

after a finite number of steps one obtains for $\hat{H}$ the expression :

$$\hat{H}=H_0e^{j\varphi_0}\mathbf{E}_0+\sum_{p=1}^{n}(H_1^{(p)}+ij\,H_2^{(p)})e^{i\psi^{(p)}}e^{j\varphi^{(p)}}\mathbf{E}_p. \tag{18}$$

We may easily impose to (18) conditions so that it should belong to one of the partition classes (5) :

$$(19)\qquad \begin{cases} \widehat{H} \in C_s \Rightarrow H_2^{(p)}=0,\ \varphi^{(p)}=\varphi_0,\ (\forall p=\overline{1,n}) ; \\ \widehat{H} \in O_r'(O_r'') \Rightarrow H_1^{(p)}=H_2^{(p)}(-H_2^{(p)}),\ (\forall p \in \mathcal{P},\ \text{card}\ \mathcal{P}=r). \end{cases}$$

The conditions (19) are, in fact, supplementary ones for α_k, β_k, $(k=\overline{1, \max(r, l)})$ in (11) depending on the quantities δ, ω, δ_p, ω_p, $(p=\overline{1, n})$, which belongs to the expression (14) of the derivation operator.

Relations (19_1) are also *conditions of undistorsion for the envelopes of the modulated signals by their transfer through linear systems*. In some concrete circumstances those conditions are obtained explicitely.

The hypercomplex answer of the system to excitations like (13) is :

$$\hat{S}_o=\widehat{H}\hat{S}_i=SH_0e^{j(\varphi+\varphi_0)}\mathbf{E}_0+\sum_{p=1}^{n} m_pS(H_1^{(p)}+ijH_2^{(p)})e^{i(\psi_p+\psi^{(p)})}e^{j(\varphi_p+\varphi^{(p)})}\mathbf{E}_p$$

and in the time domain :

$$s_o(t)=SH_0e^{\delta t}\sin(\omega t+\varphi+\varphi_0)+$$

$$+\sum_{p=1}^{n} m_pSH_1^{(p)}e^{(\delta+\delta_p)t}\sin(\omega_p t+\psi_p+\psi^{(p)})\sin(\omega t+\varphi_p+\varphi^{(p)})+$$

$$+\sum_{p=1}^{n} m_pSH_2^{(p)}e^{(\delta+\delta_p)t}\cos(\omega_p t+\psi_p+\psi^{(p)})\cos(\omega t+\varphi_p+\varphi^{(p)}).$$

In this way the facility to obtain the response in the time domain was pointed out.

For inputs which may be represented symbolically with the same derivation operator, the response in the time domain is obtained based on the propositions from [2].

Observation. The knowledge of the quantities interfearing in (18) $(H_0, \varphi_0, H_1^{(p)}, H_2^{(p)}, \psi^{(p)}, \varphi^{(p)} ; p=\overline{1, n})$ offer complete information concerning the steady-state response of the system to any excitation which may be represented symbolically with the same derivation operator. Those quantities depend both on the real quantities appearing in the expression of the derivation operator and on the parameters characterizing the considered system.

The knowledge of those dependences (with their possible graphical plotting) constitute *generalizations* for the frequency characteristics known in the harmonic case.

5. Conclusions

1. The symbolic representation of the modulated signals introduced in [2] allow the unitary treatment of the response of the linear systems with constant parameters by periodic, quasi-periodic or aperiodic excitations. The problem above is solved for A.M., A.M. with supressed carrier,

S.S.B., V.S.B. signals as well as for a class of amplitude and phase modulated signals.

Information concerning the behaviour of the system enter the generalized frequency response introduced in the present paper.

2. To find out the response of the systems to modulated excitations, the determination of the „low frequency equivalent system", usually leading to approximate solutions, is drawn apart.

3. The condition that a system should not distort the envelope of an A.M. signal is equivalent to the condition that the hypercomplex transfer function (the transferance) of the system should be a special type element in $\mathcal{H}$.

The present paper offers an algorithmic proceeding to impose the conditions of undistorsion for the envelope of the signal response concerning systems with specified L a p l a c e transfer functions. The proceeding also allows the evaluation of the distorsions introduced by a given system.

The paper points out for the first time that the divisors and quasi-divisors of zero in the hyperspace $\mathcal{H}$ characterize systems with low, up and band pass transfer functions.

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REPREZENTAREA SIMBOLICĂ A SEMNALELOR PE HIPERSPAŢII

II. Răspunsul sistemelor liniare la excitaţii reprezentabile simbolic

(Rezumat)

Se arată că metoda de reprezentare simbolică a semnalelor modulate, introdusă în [2], permite tratarea unitară a răspunsului sistemelor liniare cu parametri constanţi la excitaţii modulate. Se indică un procedeu algoritmic pentru impunerea condiţiilor de nedistorsionare a înfăşurătoarei semnalului răspuns, pentru sisteme cu funcţia de transfer L a p l a c e specificată. Procedeul permite de asemenea evaluarea distorsiunilor introduse de un sistem dat.

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STATE CONSTRAINTS AND ASYMPTOTIC STABILITY OF LINEAR CONSTANT DYNAMICAL SYSTEMS

BY

M. VOICU

1. Introduction

A problem of theoretical and practical interest in Systems Theory is to find in case of a linear constant dynamical system

$$\dot{x} = Ax, \qquad x(0) = x_0, \tag{1}$$

the conditions for the evolution matrix $A \triangleq (a_{ij})_{1 \leqslant i,j \leqslant n}$, so that $x(t) \in S \subset \subset R^n$, $t \geqslant 0$, where S is the prescribed state constraints set, often a compact set. Usually the state constraints set can be a n-dimensional interval, that is

$$x \in S \triangleq \prod_{i=1}^{n} [-\alpha_i, \alpha_i] \subset R^n, \ \alpha_i > 0, \ (i = 1, \ldots, n), \tag{2}$$

where Π is the Cartesian product.

The problem to solve in this paper is to delimit the A-matrix set, so that the solution of the system (1) have the form

$$x : [0, \infty) \to S, \ \forall x_0 \triangleq (x_{01}, \ldots, x_{0n}) \in S. \tag{3}$$

In this context it is necessary to establish a relation between a sufficient asymptotic stability condition and the fact that $x \in S$.

Problem (1), (3) is a Cauchy one with S as flow invariant set, for which the following theorem is available.

T h e o r e m 1, [1], [2]. *A necessary and sufficient condition for the existence of solution* (3) *of problem* (1) *is the following* :

$$\lim_{h \searrow 0} \frac{1}{h} d(x_0 + hAx_0 ; \ S) = 0, \quad \forall x_0 \in S, \tag{4}$$

where $d(x\,;\,S)\overset{\Delta}{=}\inf_{y\in S} d(x\,;\,y)$ *is the distance from* $x\in R^n$ *to* S.

2. Main Results

Theorem 2. *The necessary and sufficient conditions that system* (1) *have solution* (3) *are the following* :

$$-a_{ii}\geqslant\frac{1}{\alpha_i}\sum_{\substack{j=1\\ j\neq i}}^{n}|a_{ij}|\alpha_j,\quad (i=1,\ldots,n). \tag{5}$$

Proof. Relation (4), for the Euclidean distance, is true if and only if

$$\lim_{h\searrow 0}\frac{1}{h}\inf_{|y_i|\leqslant\alpha_i}\left|\sum_{i=1}^{n}\left(x_{0i}+h\sum_{j=1}^{n}a_{ij}x_{0j}-y_i\right)^2\right|^{1/2}=0 \tag{6}$$

for $\forall|x_{0i}|\leqslant\alpha_i$, $(i=1,\ldots,n)$, respectively, if and only if

$$\lim_{h\searrow 0}\frac{1}{h}\inf_{|y_i|\leqslant\alpha_i}\left|x_{0i}+h\sum_{j=1}^{n}a_{ij}x_{0j}-y_i\right|=$$

$$\left\{\begin{array}{l}\lim\limits_{h\searrow 0}\dfrac{1}{h}\left(-x_{0i}-h\sum\limits_{j=1}^{n}a_{ij}x_{0j}-\alpha_i\right),\ \text{for}\ \left(x_{0i}+h\sum\limits_{j=1}^{n}a_{ij}x_{0j}\right)<-\alpha_i,\\ 0,\quad\text{for}\ \left|x_{0i}+h\sum\limits_{j=1}^{n}a_{ij}x_{0j}\right|\leqslant\alpha_i,\\ \lim\limits_{h\searrow 0}\dfrac{1}{h}\left(x_{0i}+h\sum\limits_{j=1}^{n}a_{ij}x_{0j}-\alpha_i\right),\ \text{for}\ \left(x_{0i}+h\sum\limits_{j=1}^{n}a_{ij}x_{0j}\right)>\alpha_i,\\ \forall|x_{0i}|\leqslant\alpha_i,\ (i=1,\ldots,n).\end{array}\right\}=0, \tag{7}$$

It is easy to verify that relations (7) hold for $h>0$ sufficiently small, if and only if

$$\sum_{\substack{j=1\\ j\neq i}}^{n}a_{ij}x_{0j}-a_{ii}\alpha_i=0,\qquad\forall|x_{0j}|<\alpha_j,\qquad(j=1,\ldots,n\,;\,j\neq i), \tag{8 a}$$

$$\text{for}\quad\left(x_{0i}+h\sum_{j=1}^{n}a_{ij}x_{0j}\right)<-\alpha_i,\qquad x_{0i}=-\alpha_i,\qquad(i=1,\ldots,n)\,;$$

$$-\min_{|x_{0i}|\leqslant\alpha_i}(\alpha_i+x_{0i})\leqslant h\sum_{j=1}^{n}a_{ij}x_{0j}\leqslant\min_{|x_{0i}|\leqslant\alpha_i}(\alpha_i-x_{0i}),\quad\forall|x_{0j}|\leqslant\alpha_j,\quad(j=1,\ldots,n), \tag{8 b}$$

$$\text{for}\quad\left|x_{0i}+h\sum_{j=1}^{n}a_{ij}x_{0j}\right|\leqslant\alpha_i,\ \forall|x_{0i}|\leqslant\alpha_i,\ (i=1,\ldots,n);$$

$$\sum_{\substack{j=1\\ j\neq 1}}^{n}a_{ij}x_{0i}+a_{ii}\alpha_i=0,\qquad\forall|x_{0j}|<\alpha_j,\qquad(j=1,\ldots,n\,;\,j\neq i), \tag{8 c}$$

for $$\left(x_{0i}+h\sum_{j=1}^{n}a_{ij}x_{0j}\right)>\alpha_i, \qquad x_{0i}=\alpha_i, \qquad (i=1,\ldots,n).$$

Taking into account that $\min(\alpha_i \pm x_{0i})=0$ for $x_{0i}=\mp\alpha_i$, relations (8 *a*), (8 *b*), (8 *c*) hold for all $h>0$ if and only if

$$\sum_{\substack{j=1\\ j\neq i}}^{n}a_{ij}x_{0j}-a_{ii}\alpha_i\geqslant 0, \tag{9a}$$

$$\sum_{\substack{j=1\\ j\neq i}}^{n}a_{ij}x_{0j}+a_{ii}\alpha_i\leqslant 0, \tag{9b}$$

for $$\forall|x_{0j}|\leqslant\alpha_j, \qquad (i,j=1,\ldots,n,\ i\neq j).$$

S u f f i c i e n c y. If (5) are satisfied, one can make the evaluations

$$\sum_{\substack{j=1\\ j\neq i}}^{n}a_{ij}x_{0j}-a_{ii}\alpha_i\geqslant\sum_{\substack{j=1\\ j\neq i}}^{n}(a_{ij}x_{0j}+|a_{ij}|\alpha_j)\geqslant 0, \tag{10a}$$

$$\sum_{\substack{j=1\\ j\neq 1}}^{n}a_{ij}x_{0j}+a_{ii}\alpha_i\leqslant\sum_{\substack{j=1\\ j\neq i}}^{n}(a_{ij}x_{0j}-|a_{ij}|\alpha_j)\leqslant 0, \tag{10b}$$

for $$\forall|x_{0j}|\leqslant\alpha_j, \qquad (i,j=1,\ldots,n;\ i\neq j).$$

This means that (9 *a*), (9 *b*), and (4) respectively hold and the validity of (3) follows.

N e c e s s i t y. If (3) is true, it results that (4) and (9 *a*), (9 *b*) respectively are satisfied. Thus, from the inequalities (9 *a*) one can make the evaluations

$$a_{ii}\alpha_i\leqslant\min_{|x_{0j}|\leqslant\alpha_j}\left(\sum_{\substack{j=1\\ j\neq 1}}^{n}a_{ij}x_{0j}\right)=\sum_{\substack{j=1\\ j\neq 1}}^{n}\min_{|x_{0j}|\leqslant\alpha_j}(a_{ij}x_{0j})=\sum_{\substack{j=1\\ j\neq i}}^{n}(a_{ij}x_{0j})_{x_{0j}=-\alpha_j\operatorname{sgn}(a_{ij})}=$$

$$=-\sum_{\substack{j=1\\ j\neq i}}^{n}|a_{ij}|\alpha_j, \quad \forall|x_{0j}|\leqslant\alpha_j, \quad (i,j=1,\ldots,n\ ;\ i\neq j), \tag{11}$$

from where the inequalities (5) are to obtain.

Similarly one can proof that (9 *b*) implies (5).

For A-matrix set delimitation the system of inequations (5) is to solve, where a_{ij} are the unknown. Naturally this set of matrices is a part of the A-matrix set, for which system (1) is stable. We shall treat the problem of stability in the sequel.

T h e o r e m 3. *If the constants* $\alpha_i>0$, $(i=1,\ldots,n)$, *exist, so that inequalities* (5) *are satisfied, then system* (1) *is stable.*

The proof is immediately if one suppose by absurd that system (1) is instable.

This result can be sharpened in the following sense:

T h e o r e m 4. *If the constants* $\alpha_i>0$, $(i=1,\ldots,n)$, *exist, so that the following inequalities*

$$-a_{ii} > \frac{1}{\alpha_i} \sum_{\substack{j=1 \\ j \neq i}}^{n} |a_{ij}| \alpha_j, \quad (i = 1, \ldots, n),$$

are satisfied, then system (1) *is asymptotically stable.*

Proof. Let be

$$A_\alpha \triangleq [\operatorname{diag}(\alpha_1, \ldots, \alpha_n)]^{-1} A \operatorname{diag}(\alpha_1, \ldots, \alpha_n) = (\alpha_i^{-1} a_{ij} \alpha_j)_{1 \leqslant i, j \leqslant n}, \tag{13}$$

a similar matrix with matrix A. The Gershgorin's discs, [3], of matrix A_α, are

$$|s - a_{ii}| \leqslant \frac{1}{\alpha_i} \sum_{\substack{j=1 \\ j \neq i}}^{n} |a_{ij}| \alpha_j, \quad (i = 1, \ldots, n, \ s \in C). \tag{14}$$

In accordance with (12) all these discs are placed in the half plane $\mathcal{R}e\ s < 0$ of the complex plane C. But the discs (14) contain all the eigenvalues of the matrices A_α and A, which have the same spectrum. Consequently system (1) is asymptotically stable.

3. Conclusion

The discs (14) were used from Parodi [3], for a better spectrum location, or as positivness criterion of matrix A. Clearly, by using this discs one can formulate sufficient conditions for stability and asymptotic stability as theorems 3 and 4. The new fact, in connection with theorem 2, is that of the accurate significance of the constants $\alpha_i > 0$, $(i = 1, \ldots, n)$, namely, their values, for which inequalities (5) or (12) are satisfied, define the state evolution interval S for system (1).

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CONSTRÎNGERI DE STARE ȘI STABILITATEA ASIMPTOTICĂ A SISTEMELOR DINAMICE LINIARE CONSTANTE

(Rezumat)

Se formulează condițiile necesare și suficiente care permit determinarea mulțimii matricelor de evoluție pentru un sistem dinamic liniar constant, astfel încît constrîngerile de stare $|x_i| \leqslant \alpha_i$, $(\alpha_i > 0)$, $(i = 1, \ldots, n)$, să fie satisfăcute. Aceste condiții pun în evidență o relație precisă între ipoteza unei condiții suficiente de stabilitate asimptotică și constrîngerile de stare.

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APARAT PENTRU MĂSURAREA DIRECTĂ A DEFAZAJELOR ȘI A FACTORULUI DE PUTERE

DE

ADRIAN ÇREȚU

1. Introducere

Neajunsurile importante ale fazmetrelor și cosfimetrelor electrodinamice logometrice (fiabilitate redusă, game înguste de variație a tensiunii, curentului și frecvenței de lucru, consum propriu ridicat etc.), fac ca aceste aparate să fie înlocuite pe scară largă cu diferite tipuri de fazmetre și cosfimetre electronice, analogice sau numerice [3]...[6]. Majoritatea fazmetrelor electronice măsoară defazajul dintre două semnale sinusoidale oarecare prin transformarea lor prealabilă în semnale dreptunghiulare și determinarea ulterioară, analogică sau numerică, a diferenței de timp dintre trecerile prin zero ale impulsurilor dreptunghiulare astfel obținute. Schemele lor cuprind, în principiu, două canale de semnal (unul de referință și unul de măsurare), pe cît posibil identice, constituite din mai multe blocuri funcționale succesive ca: preamplificatoare, limitatoare sau comparatoare de tensiune, multivibratoare, circuite poartă etc. Principalele neajunsuri ale acestor fazmetre constau în prețul de cost relativ ridicat, scara liniară în fază dar neliniară în $\cos\varphi$, ambiguitate în indicarea semnului defazajului, necesitatea transformatoarelor de separare a semnalelor aplicate celor două canale, necesitatea surselor auxiliare de curent continuu, de regulă stabilizate, pentru alimentarea blocurilor funcționale etc.

Unele dintre aceste dezavantaje sînt eliminate în cazul fazmetrului—cosfimetrului electronic liniar propus și analizat în cele ce urmează.

2. Schema electrică

Schema electrică a aparatului (fig. 1) cuprinde în principal un redresor sensibil la fază electronic (RSFE), de concepție originală [2], cu bornă comună între circuitele sale de comandă și de măsurare, constituit din tranzistoarele complementare T_1 și T_2 și diodele semiconductoare D_1 și D_2, asociat cu un dispozitiv de măsurare magnetoelectric, DM, cu scara unilaterală sau bilaterală, în funcție de limitele extreme ale domeniului de măsurare impus. Comanda stării tranzistoarelor T_1 și T_2 este efectuată de un semnal practic rectangular, obținut la bornele grupului de diode D_3, D_4

conectate antiparalel, parcurse de curentul, i, din receptorul $\underline{Z}$ (curentul i_M care se ramifică prin circuitul de măsurare al RSFE este neglijabil în raport cu i) ; grupul diodelor D_3, D_4 constituie „circuitul de curent" al aparatului.

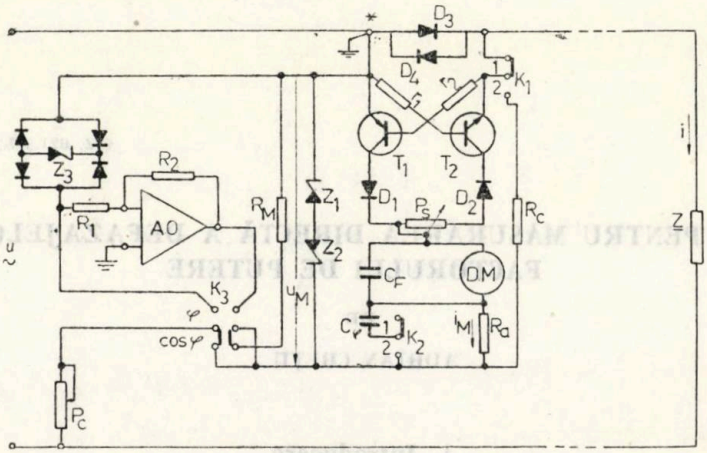

Fig. 1

„Circuitul de tensiune" al aparatului cuprinde, în afara joncţiunilor colector-emitor ale tranzistoarelor T_1 şi T_2, a diodelor D_1 şi D_2, a dispozitivului magnetoelectric DM şi a rezistenţei adiţionale R_a, de valoare relativ mare — care alcătuiesc împreună circuitul de măsurare al RSFE — un divizor de tensiune format din rezistenţa fixă R_M şi potenţiometrul de calibrare P_c. Acesta permite funcţionarea aparatului într-o gamă foarte largă de variaţie a tensiunii, u, de la bornele receptorului (generatorului).

Potenţiometrul P_s serveşte pentru simetrizarea schemei RSFE în raport cu semnalul de măsurat, u_M, ajustarea sa efectuîndu-se numai la construirea aparatului şi, eventual, în cadrul verificărilor de întreţinere.

Protecţia RSFE se realizează prin intermediul diodelor Zener Z_1 şi Z_2 conectate în serie-opoziţie între ele şi paralel cu circuitul de măsurare al acestuia.

Schema electrică din fig. 1 mai cuprinde : trei comutatoare bipoziţionale K_1, K_2 şi K_3, din care primele două au cîte un singur contact, iar K_3 — două, un condensator C_F destinat filtrării armonicilor superioare ale semnalului i_M, un condensator C_φ pentru determinarea semnului defazajului sau factorului de putere măsurat şi un dispozitiv amplificator-limitator simetric care intervine numai în cazul măsurării directe, liniare, a defazajelor. Acest dispozitiv este construit dintr-un amplificator operaţional, AO, la intrarea căruia se aplică un semnal alternativ, practic dreptunghiular (simfazic cu tensiunea u), a cărui amplitudine este limitată la valoarea necesară cu ajutorul diodei Zener Z_3 conectată în diagonala unei punţi redresoare simetrice cu patru diode.

3. Ecuațiile de funcționare

3.1. Măsurarea directă, liniară, a factorului de putere

Fie :

$$(1) \qquad i=\sqrt{2}I \sin \omega t, \quad u=\sqrt{2}U \sin (\omega t+\varphi),$$

semnalele sinusoidale care caracterizează receptorul oarecare $\underline{Z}$.

Pentru măsurarea factorului de putere, cos φ, al receptorului $\underline{Z}$, comutatoarele K_1 și K_2 se trec pe poziția *1* (măsurare), iar comutatorul K_3 — pe poziția cos φ (cosfimetru). În această situație, comanda RSFE este efectuată de un semnal (Si), practic dreptunghiular, simfazic cu curentul, i, din receptorul $\underline{Z}$; cu S s-a notat rezistența directă a șuntului neliniar alcătuit din grupul diodelor D_3 și D_4. Totodată, circuitului de măsurare al RSFE i se aplică semnalul sinusoidal u_M (simfazic cu tensiunea u), obținut la bornele rezistenței R_M care, împreună cu potențiometrul P_c constituie divizorul de tensiune al aparatului în cazul măsurării factorului de putere.

Funcționarea aparatului propus în cazul măsurării directe, liniare, a factorului de putere cos φ, poate fi urmărită cu ajutorul fig. 2, în care s-au reprezentat variațiile în timp ale semnalelor u, i, u_M și i_M care intervin în schema din fig. 1 în acest caz.

Întrucît RSFE este cu redresare simplă [2] (tranzistoarele complementare T_1 și T_2 sînt în conducție simultan și numai în alternanța pozitivă a semnalului de comandă Si, dioda D_1, numai în alternanța pozitivă a semnalului u_M, iar dioda D_2, numai în alternanța negativă a acestuia), deviația α a dispozitivului magnetoelectric DM, direct proporțională cu valoarea medie a curentului, i_M, din circuitul de măsurare al RSFE, calculată pe o perioadă a semnalului de comandă Si, riguros simfazic cu curentul, i, va rezulta din :

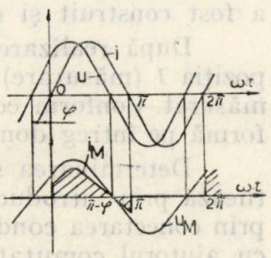

Fig. 2

$$(2) \qquad \alpha=S_I I_{M\,\text{med.}}=S_I \frac{1}{T}\int_0^T i_M \, dt = S_I \frac{1}{T}\int_0^T \frac{u_M}{R_{at}} \, dt,$$

în care S_I reprezintă sensibilitatea dizpozitivului DM iar în rezistența R_{at} s-au inclus toate rezistențele din circuitul de măsurare al RSFE.

Întrucît semnalul $u_M=\sqrt{2}U_M \sin (\omega t+\varphi)$ este în fază cu tensiunea u, dată de (1), iar :

$$i_M = \begin{cases} \dfrac{u_M}{R_{at}} \text{ în intervalul } \omega t \in (0,\ \pi) \text{ și} \\ 0 \text{ în intervalul } \omega t \in (\pi,\ 2\pi), \end{cases}$$

din (2), după efectuarea unor calcule simple, rezultă :

$$(3) \qquad \alpha \approx S_I \frac{\sqrt{2}}{\pi} \cdot \frac{U_M}{R_{at}} \cos \varphi.$$

Pentru ca deviația α să depindă liniar numai de factorul de putere, $\cos\varphi$, se impune ca valoarea efectivă, U_M, a semnalului u_M să rămînă practic aceeași, independent de variațiile tensiunii U de la bornele receptorului (generatorului). În acest scop măsurarea propriu-zisă a factorului de putere, $\cos\varphi$, trebuie precedată totdeauna de „calibrarea" cosfimetrului. Această operație constă în următoarele etape simple:

a) Înainte de cuplarea sub tensiune a aparatului, potențiometrul P_c se reglează pe rezistența sa maximă, comutatorul K_2 se conectează pe poziția *1* (măsurare), comutatorul K_3, pe poziția $\cos\varphi$, iar comutatorul K_1 se trece pe poziția *2* (calibrare). În această poziție a comutatorului K_1, datorită rezistenței R_c, de valoare convenabilă, mult mai mare decît R_a, tranzistorul T_1 și dioda D_1 vor fi în conducție numai în alternanța pozitivă a tensiunii u_M, iar tranzistorul T_2 și dioda D_2 vor fi blocate tot timpul, întrucît semnalul de comandă, *Si*, al RSFE este decuplat (de fapt, în acest caz funcționarea RSFE este similară cu cea din cazul în care semnalele sale de comandă și de măsurat sînt riguros în fază).

b) Se conectează tensiunea de alimentare și se reglează potențiometrul P_c pînă cînd deviația α a dispozitivului DM atinge valoarea sa maximă, $\alpha_{max} = S_I \frac{\sqrt{2}}{\pi} \cdot \frac{U_M}{R_{at}}$, corespunzătoare factorului de putere $\cos\varphi = 1$, situație în care valoarea efectivă a semnalului u_M ajunge la valoarea U_M pentru care a fost construit și etalonat aparatul.

După realizarea „calibrării", simpla trecere a comutatorului K_1 în poziția *1* (măsurare) permite citirea directă a valorii factorului de putere măsurat. Conform ecuației de funcționare (3), scara cosfimetrului este uniformă pe întreg domeniul de măsurare (0 ; 1) al aparatului.

Determinarea semnului defazajului (inductiv sau capacitiv) se efectuează prin introducerea unui mic defazaj suplimentar, capacitiv, obținut prin conectarea condensatorului C_φ în paralel cu rezistența adițională R_a, cu ajutorul comutatorului K_2 care, după măsurarea valorii factorului de putere, se trece în poziția *2* (+cap. ; — ind.). Evident, dacă deviația α crește în raport cu valoarea sa măsurată anterior comutării lui K_2, defazajul φ este capacitiv, iar dacă ea scade, φ este inductiv.

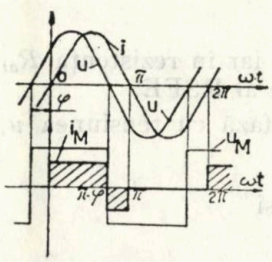

Fig. 3

3.2. Măsurarea directă, liniară, a defazajelor

În scopul măsurării directe, liniare, a defazajului φ dintre semnalele u și i, în raport cu cazul tratat mai sus, al măsurării factorului de putere, $\cos\varphi$, este suficientă trecerea comutatorului K_3 în poziția φ (fazmetru). Prin această operație se modifică numai semnalul aplicat circuitului de măsurare al RSFE, semnalul de comandă, *Si*, rămînînd neschimbat. Noul semnal de măsurat, u_M, obținut la ieșirea amplificatorului-limitator AO, are forma dreptunghiulară și amplitudinea U_M practic constantă într-un domeniu foarte larg de variație a amplitudinii tensiunii u aplicate receptorului $\underline{Z}$.

Funcționarea aparatului în cazul măsurării directe a defazajului φ poate fi urmărită cu ajutorul fig. 3, în care s-au reprezentat variațiile în timp ale semnalelor u, i, u_M și i_M care intervin în schema din fig. 1 în noua situație.

Deviația α a dispozitivului magnetoelectric DM în noua situație rezultă tot din (2), în care de această dată semnalul dreptunghiular i_M are valorile:

$$(4)\quad \begin{cases} i_M = \dfrac{u_M}{R_{at}} = +\dfrac{U_M}{R_{at}}, & \text{în intervalul } \omega t \in (0\ ;\ \pi - \varphi). \\ i_M = \dfrac{u_M}{R_{at}} = -\dfrac{U_M}{R_{at}}, & \text{în intervalul } \omega t \in (\pi - \varphi\ ;\ \pi), \\ i_M \approx 0 & \text{în intervalul } \omega t \in (\pi\ ;\ 2\pi). \end{cases}$$

Ținînd cont de (4), din (2), după efectuarea tuturor calculelor, se obține în final următoarea ecuație de funcționare:

$$(5)\quad \alpha = S_I \frac{U_M}{R_{at}} \left(\frac{1}{2} - \frac{\varphi}{\pi} \right).$$

Cînd defazajul φ ia valori în gama $(0, \pi/2)$ — mai des utilizată în practică — deviația α variază liniar între limitele $\alpha_{max} = \dfrac{1}{2} S_I \dfrac{U_M}{R_{at}}$ și 0.

În cazul măsurării directe a defazajului φ operația de „calibrare" este practic eliminată. Într-adevăr, datorită dispozitivului amplificator-limitator simetric utilizat, semnalul alternativ dreptunghiular, u_M, obținut la ieșirea sa, își menține amplitudinea U_M constantă într-o gamă largă de variație a tensiunii u, din care o mică fracțiune este aplicată la intrarea amplificatorului AO. Această fracțiune se aplică prin intermediul divizorului de tensiune constituit, de această dată, din potențiometrul P_c și circuitul limitator cu diodă Zener Z_3.

4. Realizare practică

Performanțele (precizia de măsurare și îndeosebi gamele de variație ale tensiunii, curentului și frecvenței de lucru) aparatului posibil de construit pe baza schemei date în fig. 1 sînt determinate, în principal, de parametrii dispozitivului magnetoelectric DM și de cei ai componentelor electronice utilizate.

Încercările experimentale s-au efectuat cu un prototip, proiectat și realizat în conformitate cu schema din fig. 1, în care: T_1 și T_2 sînt tranzistoare cu germaniu, complementare, AC 180, respectiv AC 181; D_1 și D_2 — diode cu germaniu EFD 110; D_3 și D_4 — diode cu siliciu RA 120; Z_1 și Z_2 — diode Zener PL 12, iar Z_3 — PL3; AO — amplificator operațional βA 741; DM — microampermetru 1 MH 96 de 100 μA; $R_M = 10$ kΩ, $R_a = 33$ kΩ, $R_1 = 10$ kΩ, $R_2 = 100$ kΩ, $r_1 = r_2 = 330$ Ω, $P_s = 5$ KΩ, $P_c = 500$ kΩ, $R_c = 511$ kΩ, $C_\varphi = 22$ nF, $C_F = 100$ μF.

Cu acest fazmetru-cosfimetru s-au măsurat, cu erori mai mici de 1...2,5%, defazajul, respectiv factorul de putere, în limitele $(0, \pm\pi/2)$, respectiv $(0, \pm 1)$, cînd tensiunea de lucru ia valori între 25 ... 500 V; curentul de lucru, între 0,2 ... 5 A; frecvența de lucru variază în gama 0 ... 1 000 Hz, iar factorul de distorsiune este mai mic decît 5%.

Extinderea limitelor superioare ale tensiunii și curentului se poate realiza cu ajutorul transformatoarelor de măsură. Pentru extinderea limitei superioare a frecvenței de lucru se vor utiliza tranzistoare și diode de comutație adecvate ; de asemenea, în acest caz este necesară folosirea dispozitivelor magnetoelectrice cu suspensie pe fire (cu reactanța neglijabilă în raport cu R_a).

5. Concluzii

1. Se propune un nou aparat electronic, destinat măsurării directe a defazajelor și factorului de putere (brevetat în R.S.R. [1]), bazat pe asocierea unui redresor sensibil la fază [2] cu un dispozitiv magnetoelectric cu suspensie pe lagăre.

2. Principalele avantaje ale acestui aparat sînt următoarele :

a) Scara sa este uniformă, atît în cazul măsurării defazajului cît și a factorului de putere, limitele domeniilor de măsurare fiind $(0, \pm\pi/2)$, respectiv $(0, \pm 1)$.

b) Gamele de variație ale tensiunii, curentului și frecvenței de lucru, în care măsurarea defazajului, respectiv factorului de putere, se efectuează practic cu aceeași precizie, sînt mult mai largi decît în cazul unor aparate similare cunoscute.

c) Schema propusă nu necesită transformatoare de „decuplare" a semnalelor aplicate ; de asemenea, în varianta cu calibrare (cosfimetru), nu necesită nici sursă de energie auxiliară.

d) Comparativ cu fazmetrele electronice cunoscute, aparatul propus are un preț de cost mai mic și poate fi construit numai cu componente produse în țară.

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APPAREIL DE MESURE DIRECTE DES DÉPHASAGES ET DU FACTEUR DE PUISSANCE

(Résumé)

On propose et on étudie un nouvel appareil électronique de mesure directe des déphasages et du facteur de puissance. Le schéma de l'appareil utilise l'association d'un redresseur électronique, sensible à la phase, de conception originale, à un mécanisme magnetoélectrique à suspension aux paliers. Le nouveau appareil a l'échelle linéaire, tant pour la mesure des déphasages, que pour celle du facteur de puissance, les limites de mesure étant $(0, \pm\pi/2)$, respectif $(0, \pm 1)$. Les domaines de variations admissibles de la tension, du courant et de la fréquence sont relativement larges (25...500 V ; 0,2...5 A ; 0...1 000 Hz, avec possibilités d'extension des limites supérieures) ; les erreurs de mesure ne dépassent pas 2,5%.

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C. Z. 621.317.31

ASUPRA UNUI PROCEDEU DE MĂSURARE A CURENŢILOR ELECTRICI SINUSOIDALI

DE

P. LEONTE şi ADRIAN BARABOI

1. În [1] se prezintă un nou procedeu de măsurare a amplitudinii intensităţii curentului electric sinusoidal, care constă în vizualizarea pe osciloscop a modificării curbei de variaţie în timp a t.e.m. induse, $e(t)$, considerată drept semnal de ieşire al unui dispozitiv electromagnetic conţinînd o bobină neliniară cu miez magnetizat pe direcţii ortogonale (BNMMO), avînd schema electrică reprezentată grafic în fig. 1, în momentul în care curentul continuu, I_0, reglat în sens crescător de la valoarea zero, egalează amplitudinea I_m a curentului sinusoidal $i(t)$, supus măsurării.

Metoda se dovedeşte avantajoasă în special pentru măsurarea amplitudinii curenţilor sinusoidali de mare intensitate şi, prin extindere, poate fi utilizată pentru măsurarea valorilor instantanee ale unor semnale periodice [2], [3].

În această Notă se studiază, pe cale analitică, procedeul menţionat, urmărindu-se în acelaşi timp evidenţierea unor posibilităţi de ameliorare a performanţelor acestuia.

Se poate arăta [4], [5] că în ipoteza utilizării unui miez feromagnetic omogen şi izotrop, caracteristica raportată la direcţia de comandă a unei BNMMO este în mod satisfăcător aproximabilă printr-o funcţie de forma :

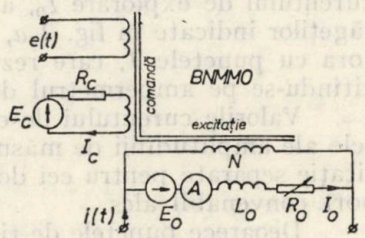

Fig. 1

$$\psi_c(\theta_c, \theta_e) = a^c_{10}\theta_c - a^c_{12}\theta_c\theta_e^2 - a^c_{30}\theta_c^3, \tag{1}$$

unde ψ_c reprezintă fluxul magnetic corespunzător direcţiei de comandă, θ_c — solenaţia produsă de curentul care parcurge înfăşurarea de comandă, θ_e — solenaţia produsă de înfăşurările dispuse după direcţia de excitaţie, ortogonală celei de comandă, a^c_{10}, a^c_{12}, a^c_{30} fiind constante pozitive.

Ţinînd seama că, în schema electrică reprezentată în fig. 1 solenaţia de comandă, θ_c, este constantă, iar solenaţia de excitaţie, θ_e, poate fi scrisă sub forma :

$$\theta_e = \theta_m \sin \omega t - \theta_0, \tag{2}$$

unde :

$$\theta_m = NI_m, \quad \theta_0 = NI_0, \tag{3}$$

N reprezentînd numărul de spire al înfășurării de excitație, I_m — amplitudinea intensității curentului sinusoidal $i(t)$, iar I_0 — curentul continuu de explorare, pentru semnalul de ieșire $e(t)$, dat de relația :

$$e(t) = -\frac{d\psi_e}{dt}, \tag{4}$$

rezultă expresia :

$$e(t) = 2a_{12}^2 \theta_c \theta_m \omega (\theta_m \sin \omega t - \theta_0) \cos \omega t. \tag{5}$$

2. În ipoteza :

$$\theta_0 < \theta_m, \tag{6}$$

fazele ωt_0 pentru care semnalul de ieșire, $e(t)$, al dispozitivului, se anulează, au expresiile :

$$\omega t_0 = \begin{cases} (2k+1)\frac{\pi}{2}, \\ (-1)^k \arcsin\frac{\theta_0}{\theta_m} + k\pi, \quad (k = 0, \pm 1, \pm 2, \ldots), \end{cases} \tag{7}$$

variația în timp a t.e.m., $e(t)$, dată de relația (5) în ipoteza (6), fiind reprezentată grafic în fig. 2 *a*. În fig. 2 *b* este reprezentată curba t.e.m. $e(t)$ pentru $\theta_0 \geq \theta_m$, cînd ecuația $e(t) = 0$ admite doar soluții de forma (7_1).

Măsurarea amplitudinii I_m a curentului sinusoidal $i(t)$ constă în urmărirea pe osciloscop, simultan cu creșterea de la valoarea zero a intensității curentului de explorare I_0, a deplasării punctelor de tipul *2* și *3* în sensul săgeților indicate în fig. 2 *a*, pînă în momentul obținerii coincidenței acestora cu punctele *1*, care rezultă pentru $I_m = I_0$, valoarea amplitudinii I_m citindu-se pe ampermetrul din schema electrică dată în fig. 1.

Valorile curentului de explorare I_0 pot fi demultiplicate în raport cu cele ale amplitudinii de măsurat, I_m, prin utilizarea unor înfășurări de excitație separate pentru cei doi curenți, avînd numerele de spire într-un raport convenabil ales.

Deoarece punctele de tipul *1*, *2*, *3* coincid pentru $\theta_0 \geq \theta_m$, vizualizarea la limită a acestei coincidențe este relativ dificilă și introduce erori nedorite în rezultatele măsurării. Erorile procedeului pot fi reduse, prin efectuarea măsurărilor la valori $\theta_0 < \theta_m$, pentru care intervalele *1—2*, *1—3* (fig. 2 *a*) pot fi determinate precis pe ecranul osciloscopului.

Astfel, de exemplu, dacă prin reglarea valorilor curentului de explorare, I_0, se măsoară pe osciloscop :

$$\omega t_{12} = \omega t_{13} = \frac{\pi}{3} \text{ rad.}, \tag{8}$$

ținînd seama de (7), rezultă :

$$I_m = 2I_0. \tag{9}$$

Se remarcă faptul că, în condițiile (8), (9), pe lîngă creșterea preciziei procedeului, consumul propriu al dispozitivului scade, prin reducerea la jumătate a valorilor curentului de explorare, I_0.

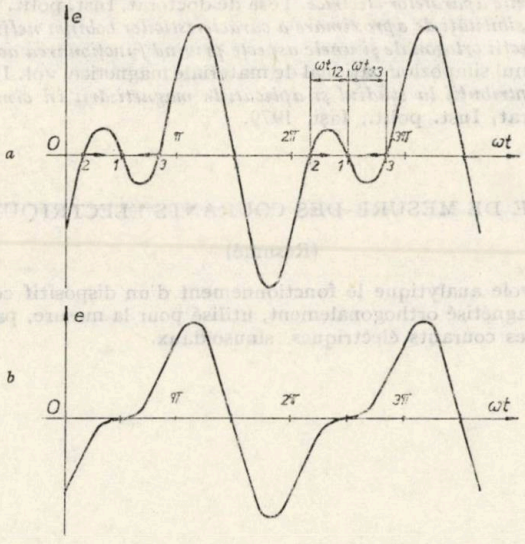

Fig. 2

În [3] se analizează în detaliu erorile de metodă, făcîndu-se precizări referitoare la modul de realizare a elementelor constructive ale dispozitivului de măsurare, în vederea obținerii preciziei maxime.

Concluzii

1. Se studiază pe cale analitică un nou dispozitiv de măsurare a amplitudinii curentului electric sinusoidal, funcționînd pe baza unei scheme electrice conținînd o BNMMO.

2. Justificîndu-se pe cale analitică principiul procedeului de măsurare, se evidențiază unele posibilități de creștere a preciziei acestuia, care conduc simultan și la reducerea consumului propriu.

3. Schema electrică reprezentată grafic în fig. 1 și principiul metodei de măsurare analizate permit determinarea pe cale experimentală a coeficienților de forma a_{12}^{c}, a_{12}, care intervin în expresiile funcțiilor de aproximare ale caracteristicilor unei BNMMO, [4], [5].

4. Dispozitivul analizat poate fi utilizat ca element de măsurare în construcția unor aparate electrice de protecție și automatizare.

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SUR UNE MÉTHODE DE MESURE DES COURANTS ÉLECTRIQUES SINUSOÏDAUX

(Résumé)

On étudie par voie analytique le fonctionnement d'un dispositif contenant une bobine nonlinéaire à noyau magnétisé orthogonalement, utilisé pour la mesure, par une méthode originale, de l'amplitude des courants électriques sinusoïdaux.

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LA CONDITION POUR L'ACTION SÛRE DE L'ÉLECTRO-AIMANT DE COURANT ALTERNATIF

PAR

EUGEN HNATIUC

1. Introduction

On considère un électro-aimant de courant alternatif de type U-I avec une armature mobile translante (Fig. 1) en tant qu'un système électro-mécanique de conversion d'énergie à deux degrés de liberté [2], [3], [6] ; à l'aide des principes de la mécanique-analytique on peut écrire les équations :

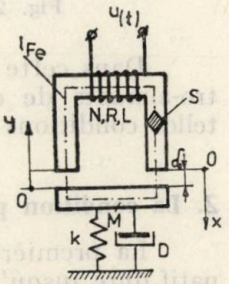

Fig. 1

$$(1)\quad \begin{cases} M\ddot{x}(t)+D\dot{x}(t)+k[x(t)-b]+\dfrac{1}{2L'}\psi^2(t)=F_R, \\ \dot{\psi}(t)+\dfrac{R}{L'}[x(t)+a]\psi(t)=u(t), \end{cases}$$

où : M est la masse de l'armature ; $x(t)$ — la position de l'armature mobile, $x(0)=\delta_0$; D — la constante de l'amortissement du mouvement du l'armature mobile dans l'air, très faible ; k — la constante élastique du ressort de rappel, pas tensionné pour $x=b$; $L(x)$ — l'inductance de la bobine de l'électro-aimant, qu'on peut approximer par la relation :

$$(2)\qquad L(x)=\frac{L'}{x(t)+a},$$

avec :

$$(3)\qquad L'=\frac{N^2\mu_0 S}{2}, \qquad a=\frac{1}{2}\cdot\frac{\mu_0}{\mu_{Fe}}l_{Fe},$$

deux constantes qu'on peut calculer en connaissant le nombre de spires de la bobine, N, la perméabilité magnétique de l'air respectivement du noyau magnétique μ_0 et μ_{Fe}, la longueur moyenne du noyau magnétique, l_{Fe}, respectivement la section transversale du noyau, S ; $\psi(t)$ — le flux

magnétique total produit par le courant électrique $i(t)$ qui passe par les spires de la bobine :

$$\psi(t) = L(x)\, i(t) \; ; \tag{4}$$

F_R — la force résistante résultante actionnant sur l'armature mobile ; R — la résistance électrique de la bobine ; $u(t)$ — la tension de la source d'alimentation, d'amplitude U, de phase initiale θ et de pulsation Ω :

$$u(t) = U \sin(\Omega t + \theta). \tag{5}$$

On peut signaler que la dépendance de la valeur de l'inductance L en fonction de la valeur de l'entrefer δ est représentée par la courbe de la Fig. 2.

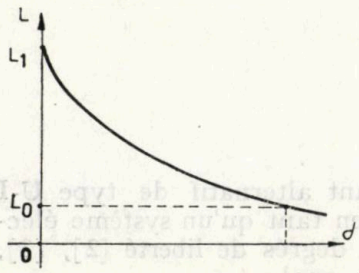

Fig. 2

D'habitude on peut considérer que pour actionner les électroaimants il est nécessaire à parcourir trois étapes, à savoir ; a) le régime transitoire d'augmentation du flux magnétique total jusqu'à la valeur de démarrage ψ_p, l'armature mobile en restant en position initiale ; b) le régime transitoire de déplacement de l'armature mobile de l'électroaimant jusqu'à la valeur minimum de l'entrefer ; c) la stabilisation du régime permanent dans le circuit électrique de la bobine dont l'armature mobile reste en position „fermée", $\delta \approx 0$.

Dans cette Note on se propose à analyser le comportement des électro-aimants de courant alternatif pendant la deuxième étape dans des telles conditions que leurs action soit sûre.

2. La condition pour l'action sûre des électroaimants de courant alternatif

La première étape de l'action des électro-aimants de courant alternatif dure jusqu'au moment lorsque le flux magnétique $\psi(t)$ atteint la valeur de démarrage, ψ_p, qu'on peut calculer à partir de l'équation (1_1) à l'aide de la relation :

$$\psi_p = \sqrt{2L'[F_R - k(\delta_0 - b)]}. \tag{6}$$

Pendant la première étape de l'action on peut écrire l'équation (1_2) sous la forme :

$$\dot{\psi}(t) + A_0 \psi(t) = u(t), \tag{7}$$

avec la solution :

$$\psi(t) = \frac{U}{\sqrt{A_0^2 + \Omega^2}} [\sin(\Omega t + \theta - \alpha) - \sin(\theta - \alpha) e^{-A_0 t}] \leqslant \frac{2U}{\sqrt{A_0^2 + \Omega^2}}, \tag{8}$$

où :

$$(9) \qquad A_0 = \frac{R(\delta_0 + a)}{L'} = \frac{R}{L_0}, \qquad \alpha = \operatorname{arctg} \frac{\Omega L_0}{R}.$$

La durée de la première étape de l'actionnement sera donc la solution de l'équation :

$$(10) \qquad \psi(t_p) = \psi_p,$$

dans laquelle il faut tenir compte des relations (6) et (8).

Il faut souligner que la condition de „démarrage" de l'armature mobile :

$$(11) \qquad \psi(t) \geqslant \psi_p,$$

est habituellement acceptée comme une condition d'actionner quoiqu'elle soit seulement nécessaire mais pas suffisante. Aussi on peut ajouter que la dispersion magnétique corrige les valeurs ψ_p de la manière indiquée par la relation :

$$(12) \qquad \psi_p' = \sigma(x)\, \psi_p, \quad \sigma(x) > 1,$$

ou $\sigma(x)$ est le facteur de dispersion magnétique.

La deuxième étape de l'action d'électro-aimant de courant alternatif commence au moment $t = t_p$ et dure pendant le déplacement de l'armature mobile jusqu'à la valeur minimum de l'entrefer.

Pour estimer le comportement du système électro-mécanique pendant cette deuxième étape on propose le changement de variables :

$$(13) \qquad x(t) = \delta_0 - y(t), \ \psi(t) = \psi_p + \varphi(t) \ \text{où} \ L(\delta_0) = L_0, \ L(0) = L_1,$$

et on considère le moment t_p comme l'origine du temps.

Dans ces conditions le système d'équations (1) peut être écrit sous la forme :

$$(14) \qquad \begin{cases} \ddot{y}(t) + 2h\dot{y}(t) + \omega_0^2 y(t) = B\varphi(t) + \mu C\varphi^2(t), \\ \dot{\varphi}(t) + A_0\varphi(t) = Gy(t) + \mu(t + t_p) - u(t_p) + \left.\dfrac{d\psi}{dt}\right|_{t=t_p} + \varepsilon \dfrac{A_0}{\delta_0} y(t)\varphi(t), \end{cases}$$

où sont utilisées les notations suivantes :

$$(15) \qquad \frac{D}{M} = 2h, \ \omega_0^2 = \frac{k}{M}, \ B = \frac{\psi_p}{ML'}, \ C = \frac{1}{ML'}, \ G = \frac{A_0\psi_p}{\delta_0 + a}, \ \mu = \frac{1}{2}, \ \varepsilon = \frac{\delta_0}{\delta_0 + a}.$$

On peut, approximativement, estimer le comportement de l'électro-aimant de courant alternatif, pendant cette deuxième étape, en tenant compte seulement du système linéaire qui résulte des équations (14) pour $h \approx 0$:

$$(16) \qquad \begin{cases} \ddot{y}(t) + \omega_0^2 y(t) - B\varphi(t) = 0, \\ \dot{\varphi}(t) + A_0\varphi(t) - Gy(t) = u(t + t_p) - u(t_p) + \left.\dfrac{d\psi}{dt}\right|_{t=t_p}, \end{cases}$$

dont l'équation caractéristique est :

$$(17) \qquad \begin{vmatrix} p^2 + \omega_0^2 & -B \\ -G & p + A_0 \end{vmatrix} = 0 \ \text{ou} \ p^3 + A_0 p^2 + \omega_0^2 p + A_0\omega_0^2 - BG = 0.$$

Le comportement stable du système électro-mécanique implique des valeurs positives pour les coefficients de l'équation (17_2) et aussi la relation :

$$a_1a_2-a_0a_3>0 \text{ ou } A_0\omega_0^2-A_0\omega_0^2+BG=BG>0. \tag{18}$$

Compte tenu que la condition (18) est toujours satisfaite le comportement stable de l'électro-aimant est assuré si :

$$A_0\omega_0^2-BG>0, \quad a_3>0. \tag{19}$$

Bien sur l'instabilité du système, donc l'action de l'électro-aimant suppose par contre :

$$A_0\omega_0^2-BG\leqslant 0. \tag{20}$$

La relation (20) représente justement la condition pour l'action sûre des électro-aimants de courant alternatif, en étant plus complète que la condition de „démarrage" (11).

3. Observations concernant l'action sûre des electro-aimants de courant alternatif

En tenant compte de relations (6), (15), la condition (20) devient :

$$\psi_p\geqslant(\delta_0+a)\sqrt{kL_0}, \tag{21}$$

donc l'action sûre d'un électro-aimant de courant alternatif sera possible pour des valeurs de la constante élastique du ressort qui satisfait l'inégalité:

$$k\leqslant\frac{2F_R}{3\delta_0+a-2b}. \tag{22}$$

Vu que pour les électro-aimants de courant alternatif la plus grande valeur de flux de démarrage est :

$$\psi_{p\max}\leqslant\frac{2U}{\sqrt{A_0^2+\Omega^2}}=\frac{2L_0}{\sqrt{R^2+\Omega^2L_0^2}}U, \tag{23}$$

l'amplitude de la tension d'alimentation $u(t)$, pour laquelle en résulte leur action sûre, peut être calculée avec la relation :

$$U\geqslant\frac{\delta_0+a}{2}\sqrt{\frac{k(R^2+\Omega^2L_0^2)}{L_0}}, \tag{24}$$

où l'on a tenu compte de (21) et (23).

À partir de l'expression (3_2) de la constante a on peut aussi calculer la valeur de l'entrefer δ_0 pour l'action sûre des électro-aimants de courant alternatif :

$$\delta_0\leqslant\frac{U}{2}\sqrt{\frac{L_0}{k(R^2+\Omega^2L_0^2)}}-a \text{ ou } \delta_0\leqslant U\frac{L_1-L_0}{2L_1}\sqrt{\frac{L_0}{k(R^2+\Omega^2L_0^2)}}. \tag{25}$$

Afin de réaliser une action sûre, pour une valeur minimum de l'amplitude de la tension $u(t)$, (δ_0=const.), ou, pour une valeur maximum de l'entrefer δ_0 (U=const.) il faut ajouter les conditions d'optimum :

$$\frac{d\delta_0}{dL_0}=0\,; \qquad \frac{dU}{dL_0}=0. \tag{26}$$

En utilisant les relations (24)...(26) il en résulte l'équation :

$$\Omega^3L_0^3+\Omega^2L_1L_0^2+3R^2L_0-L_1R^2=0. \tag{27}$$

Avec les notations :

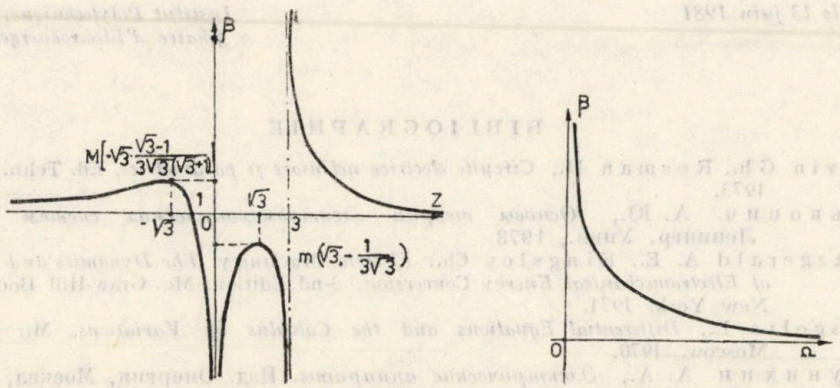

Fig. 3

$$z=\frac{L_1}{L_0}>1, \qquad \beta=\frac{R^2}{\Omega^2L_1^2}=\beta(\Omega)>0, \tag{28}$$

l'équation (27) peut être écrite sous la forme équivalente :

$$\beta z^3-3\beta z^2-z-1=0 \tag{29}$$

et donc

$$\beta(z)=\frac{z+1}{z^2(z-3)}. \tag{30}$$

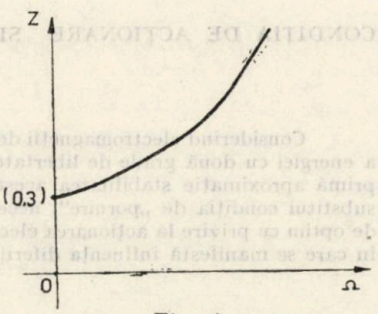

Fig. 4

Á l'aide de la représentation graphique des fonctions $\beta(z)$ et $\beta(\Omega)$ (Fig. 3) on peut obtenir la courbe $z(\Omega)$ (Fig. 4), qui met en évidence des valeurs z supérieures a la valeur 3 pour les électro-aimants de courant alternatif et aussi l'influence des valeurs de la pulsation Ω de la source d'alimentation. On confirme la valeur $L_1/L_0=3$, comme un optimum pour les électro-aimants de courant continu [6], On confirme aussi les valeurs usuelles du rapport $z=10...15$ [5], en précisant en outre, l'influence des différents facteurs et la condition d'optimum.

4. Conclusions

1. Pour l'électro-aimant de courant alternatif, considéré comme un système électro-mécanique de conversion de l'énergie à deux degrés de liberté, on obtient la condition qui assure une action sûre et qui peut remplacer la condition de „démarrage", nécessaire mais pas suffisante.

2. À partir de cette condition on peut calculer les valeurs de l'amplitude U de la tension d'alimentation et de l'entrefer initial, confirmées par voie expérimentale avec des erreurs moindres que 10%.

3. On peut calculer aussi les valeurs optima du rapport z pour assurer une action sûre des électro-aimants de courant alternatif.

4. On confirme les valeurs usuelles de ce rapport et on met en évidence l'influence des différents facteurs et la condition d'optimum.

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CONDIŢIA DE ACŢIONARE SIGURĂ PENTRU ELECTROMAGNEŢII DE CURENT ALTERNATIV

(Rezumat)

Considerînd electromagneţii de curent alternativ ca sisteme electromecanice de conversie a energiei cu două grade de libertate, descrise de ecuaţii de tip Lagrange şi analizînd în primă aproximaţie stabilitatea acestora se deduce o condiţie de acţionare sigură care poate substitui condiţia de „pornire", necesară dar nu şi suficientă. Se deduc, de asemenea, condiţii de optim cu privire la acţionarea electromagneţilor de curent alternativ şi se evidenţiază modul în care se manifestă influenţa diferiţilor factori.

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L'EFFET DE PINCEMENT SUR L'ARC ÉLECTRIQUE INTRODUIT DANS LA CHAMBRE D'EXTINCTION

PAR

ADRIAN BARABOI

1. La chambre d'extinction représente un élément constructif indispensable au fonctionnement de l'appareillage de commutation à basse tension, l'étude des processus produits dans une telle chambre en offrant des possibilités multiples en vue d'améliorer les performances des équipements de commutation.

Un grand nombre de chercheurs analyse dans leurs travaux les divers aspects concernant l'étouffement de l'arc, en utilisant dans ce but la théorie du modèle cylindrique de celui-ci [3], [4], les résultats obtenus en concordant d'une manière satisfaisante avec ceux acquis dans la pratique. Vu que dans la chambre d'extinction, la forme géometrique de l'arc électrique n'est pas cylindrique, l'étude de sa tenue dans ces conditions fournit des information assez intéressantes.

Dans ce que suit sont analysés et illustrés par des exemples les principaux aspects concernant l'effet de pincement sur l'arc électrique introduit dans la chambre d'extinction.

2. Soit une colonne d'arc électrique ayant en combustion libre la section transversale circulaire de rayon r (Fig. 1), introduite dans une chambre d'extinction de largeur $2a$, où celle-ci présente une section transversale limitée par les parois diélectriques *2*, *4* de la chambre et les arcs *1*, *3*, de rayon R. L'angle α (et la rayon R) résultent comme conséquences de la conservation de la densité de courant J dans la colonne de l'arc, à l'aide des équations :

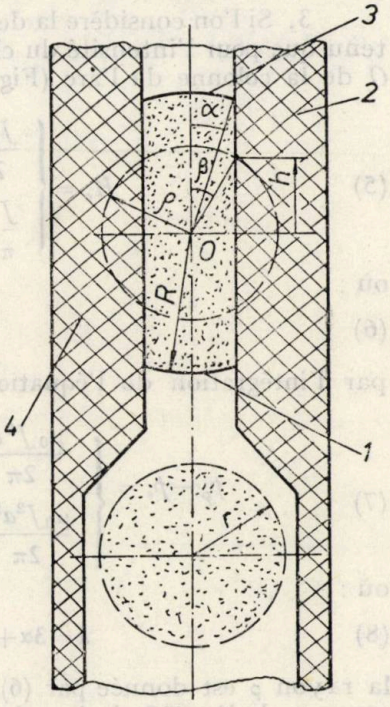

Fig. 1

$$(1)\quad \pi r^2 = 2a^2(\alpha + \alpha\,\mathrm{ctg}^2\alpha + c\,\mathrm{tg}\alpha),\quad R = a\,\mathrm{cosec}\alpha.$$

Dans la colonne de plasma de l'arc électrique on met en évidence la force de L o r e n t z (dans l'unité de volume) :

$$(2)\qquad \mathbf{F} = \mathbf{J} \times \mathbf{B},$$

due à l'interaction entre la densité J de courant et l'induction magnétique propre, **B** ; vu que les parois latéraux de la chambre ne sont pas ferromagnétiques, la relation (2) peut être écrite encore sous la forme :

$$\mathbf{F} = \mu_0(\mathbf{J} \times \mathbf{H}), \tag{3}$$

où $\mu_0 = 4\pi.10^{-7}$ H/m représente la perméabilité magnétique de l'air et **H** — l'intensité du champ.

Si l'on considère le tracé de l'arc comme rectiligne, la force de L o r e n t z est dirigée toujours vers l'axe *O* de celui-ci et comme effet elle produit une striction de la colonne de l'arc ; en régime stationnaire cette force est compensée complètement par le gradient de pression sur une direction radiale $|\beta| \leqslant \alpha$:

$$\operatorname{grad} p_\rho = -\mathbf{F}, \tag{4}$$

où p_ρ représente la pression sur une direction β, à la distance ρ de l'axe *O*. Sur les directions $\alpha \leqslant \beta \leqslant \pi/2$, la force de L o r e n t z compense partiellement le gradient de pression, le reste étant compensé par la résistance des parois de la chambre.

3. Si l'on considère la densité de courant, *J*, comme constante, compte tenu que pour l'intensité du champ magnétique H_ρ, à la distance ρ de l'axe *O* de la colonne de l'arc (Fig. 1), on obtient les expressions :

$$H_\rho = \begin{cases} \dfrac{J\rho}{2}, & 0 \leqslant \rho < a, \\ \dfrac{J}{\pi}(\rho\beta + a\cos\beta), & a \leqslant \rho \leqslant R, \end{cases} \tag{5}$$

où :

$$\rho = a \operatorname{cosec} \beta, \tag{6}$$

par l'intégration de l'équation (4), il en résulte :

$$p_\rho - p_e = \begin{cases} \dfrac{\mu_0 J^2 a^2}{2\pi}(k-\pi) - \dfrac{\mu_0 J^2 \rho^2}{4}, & 0 \leqslant \rho < a, \\ \dfrac{\mu_0 J^2 a^2}{2\pi}(k - 3\beta - \beta \operatorname{ctg}^2\beta - 3\operatorname{ctg}\beta), & a \leqslant \rho \leqslant R, \end{cases} \tag{7}$$

où :

$$k = 3\alpha + \alpha \operatorname{ctg}^2\alpha + 3\operatorname{ctg}\alpha ; \tag{8}$$

la rayon ρ est donnée par (6) et p_ρ, p_e représentent les pressions à la distance ρ de l'axe *O* de la colonne de l'arc, respectivement sur les surfaces cylindriques *1* et *3*.

En considérant que pour les surfaces extérieures *1*, *3* on peut écrire [1], [4] :

$$p_e \approx 0, \tag{9}$$

de (7_1), pour $\rho=0$, on obtient l'expression de la pression p_0 sur l'axe O de la colonne :

$$p_0=\frac{\mu_0 J^2 a^2}{2\pi}(k-\pi). \tag{10}$$

Les relations (6), (7_2), compte tenu de l'hypothèse (9), permettent aussi la détermination de la pression p_p, qui solicite les parois latéraux de la chambre d'extinction. L'expression de la pression $p_p(h)$ résulte de la forme :

$$p_p(h)=\frac{\mu_0 J^2 a^2}{2\pi}\left(k-3\,\text{arccotg}\frac{h}{a}-\frac{h^2}{a^2}\,\text{arccotg}\frac{h}{a}-3\frac{h}{a}\right),\quad h\in[0,R\cos\alpha]. \tag{11}$$

La valeur maximum de cette pression, p_{pm}, résulte pour $h=0$, ayant l'expression :

$$p_{pm}=\frac{\mu_0 J^2 a^2}{2\pi}\left(k-\frac{3\pi}{2}\right). \tag{12}$$

4. Si l'on note par p_{00} la pression sur l'axe O de la colonne de l'arc électrique ayant la section transversale circulaire de rayon r [3] :

$$p_{00}=\frac{\mu_0 J^2 r^2}{4}, \tag{13}$$

compte tenu des relations (1), (10), (12), on obtient

$$\frac{p_0}{p_{00}}=\frac{k-\pi}{k-2\alpha-2\,\text{cotg}\,\alpha}=f_1\left(\frac{a}{r}\right),\quad \frac{p_{pm}}{p_{00}}=\frac{2k-3\pi}{2(k-2\alpha-2\,\text{cotg}\,\alpha)}=f_2\left(\frac{a}{r}\right). \tag{14}$$

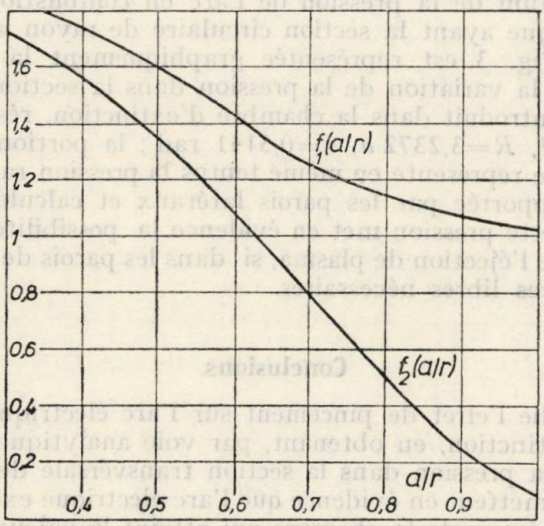

Fig. 2

Dans la Fig. 2 sont représentées graphiquement les courbes $f_1(a/r)$, $f_2(a/r)$, qui mettent en évidence que les valeurs de la pression sur l'axe O de

la colonne de l'arc introduit dans la chambre d'extinction de largeur $2a$ sont supérieures par rapport aux mêmes valeurs calculées pour l'arc en combustion libre, ayant la section transversale circulaire, de rayon r ; les processus de déionisation ont une intensité augmentée, pour des valeurs décroissan-

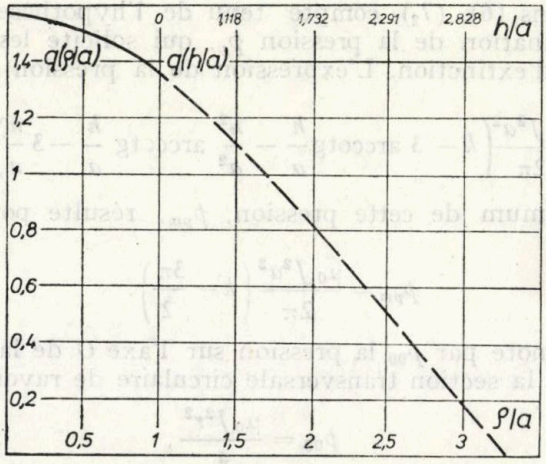

Fig. 3

tes du rapport a/r. Plus encore, la courbe $f_2(a/r)$ met en évidence que pour des valeurs $a/r<0{,}6$ la pression de la colonne de plasma est supérieure à la valeur maximum de la pression de l'arc en combustion libre, dans un volume cylindrique ayant la section circulaire de rayon a.

Dans la Fig. 3 est représentée graphiquement la courbe $q(\rho/a)=p_\rho/(\mu_0 J^2 a^2)$, de la variation de la pression dans la section transversale de l'arc électrique introduit dans la chambre d'extinction, résultée par calcul, pour $a/r=0{,}4967$, $R=3{,}2372\,a$, $\alpha=0{,}3141$ rad ; la portion tracée par une ligne interrompue représente en même temps la pression rapportée, $q(h/a)=p_p/(\mu_0 J^2 a^2)$ supportée par les parois latéraux et calculée à l'aide de la relation (11). Cette pression met en évidence la possibilité de l'apparition du phénomène de l'éjection de plasma, si dans les parois de la chambre sont prévus les espaces libres nécessaires.

Conclusions

1. On étudie l'effet de pincement sur l'arc électrique introduit dans la chambre d'extinction, en obtenant, par voie analytique, l'expression de la variation de la pression dans la section transversale de celui-ci ; les relations résultées mettent en évidence que l'arc électrique exerce une pression sur les parois latéraux de la chambre qui atteint la valeur maximum dans le plan de l'axe de la colonne de plasma.

2. Les relations obtenues mettent en évidence que la pression sur l'axe de l'arc introduit dans la chambre d'extinction a toujours des valeurs

supérieures par rapport à la même pression considérée pour l'arc électrique en combustion libre.

3. La pression exercée sur les parois latéraux de la chambre d'extinction offre la possibilité de l'utilisation du phénomène de l'éjection de plasma en vue d'une déionisation supplémentaire de l'arc électrique et donc de faciliter son étouffement.

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EFECTUL PINCH ASUPRA ARCULUI ELECTRIC INTRODUS ÎN CAMERA DE STINGERE

(Rezumat)

Se studiază efectul de stricţiune produs de forţele Lorentz asupra coloanei arcului electric amorsat în camera de stingere cu fantă îngustă, determinîndu-se pe cale analitică repartiţia presiunii în secţiunea transversală a acesteia. Rezultatele obţinute evidenţiază rolul deionizant pe care îl are acest efect asupra arcului electric.

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ÜBER DIE MESSUNG QUALITATIVER NACHRICHT

VON

VALERIU MUNTEANU, M. LUCANU, M. ANTONESCU, I. CLEJU und GH. ZAHARIA

1. Einleitung

Bei der Ausarbeitung des Nachrichtenbegriffs einbezieht man, im klassischen Sinne, nur die quantitative Seite, während die qualitative vernachlässigt wird. Zu diesm Zweck dienen ausschließlich die Wahrscheinlichkeiten der zufälligen Ereignisse.

In der üblichen Praxis dient die übertragene Nachricht einem bestimmten Zweck. Die ausschließliche Berücksichtigung der probabilistischen Abhängigkeiten zwischen den übertragenen Mitteilungen ist ungenügend denn die Übertragungswirksamkeit hängt auch von der Wahl jener Mitteilungen ab, die dem gewünschten Ziel entsprechen. Aus diesem Grunde muß man in jedem Übertragungssystem auch die Qualität der übertragenen Signale schätzen.

Im allgemeinen wird also die übertragene Nachricht sowohl von der probabilistischen Verteilung der Mitteilungen, als auch von der Bedeutung, der Nützlichkeit, dem Preis u.s.w. dieser abhängen, in einem Wort also, von der Qualität der Mitteilungen.

Die Arbeit nimmt sich die Messung der qualitativen Nachricht die mittels gelieferten Mitteilungen bestimmter Wahrscheinlichkeiten übertragen ist, vor.

2. Die qualitative, durchschnittliche Nachricht einer diskreten, völligen, speicherfreien Quelle

Es sei eine diskrete, völlige, speicherfreie, mit der Verteilung:

$$S:\begin{pmatrix} s_1 & s_2 \dots s_n \\ p_1 & p_2 \dots p_n \end{pmatrix} \tag{1}$$

Quelle gegeben, wobei s_i, $(i=\overline{1,n})$, die Mitteilungen der Quelle, bzw. p_i, $(i=\overline{1,n})$, die Lieferungswahrscheinlichkeiten sind.

Es sei:

$$u_i : S \to \mathbf{R}^+ \tag{2}$$

gegeben, wobei u_i, die Nützlichkeiten der Mitteilungen s_i, bedeuten. Aus der Annahme der diskreten, völligen, speicherfreien Quelle S, folgt die Unabhängigkeit jeder zwei Mitteilungen.

Falls :

$$s_k = s_i \cap s_j, \quad (i \neq j, \ i = \overline{1, n}, \ j = \overline{1, n}), \tag{3}$$

dann :

$$u_k = u_i + u_j, \tag{4}$$

wobei sich unter u_k die Nützlichkeit der Mitteilung s_k versteht.

Die qualitative Nachricht, $I(s_k)$, wird eine Funktion F der Nützlichkeit u_k sein, nämlich :

$$I(s_k) = F(u_k), \tag{5}$$

oder, wenn man die Beziehung (4) beachtet:

$$I(s_k) = F(u_i + u_j). \tag{6}$$

Genauso wie im Falle der probabilistischen Nachrichtenbestimmung, [1], [2], nimmt man auch für die qualitative Nachricht an, daß die von zwei unabhängigen Ereignissen gelieferte Nachricht gleich mit der Summe der von den einzelnen Ereignissen gewonnenen Nachricht ist. Es folgt also :

$$I(s_k) = I(s_i) + I(s_j). \tag{7}$$

Unter Ersetzung von (7) in (6) und Einbeziehung von (5) folgt :

$$F(u_i + u_j) = F(u_i) + F(u_j). \tag{8}$$

Die Lösung der funktionellen Gleichung (8) ist der Form :

$$I(s_i) = a u_i, \quad (i = \overline{1, n}), \tag{9}$$

wobei a eine willkürliche Konstante ist.

Aus physikalischen Gründen muß die mit einem Ereignis verknüpfte Nachricht größer (oder gleich) Null sein, es ist also nötig, daß $a \in \mathbf{R}^+$.

Die mit der Beziehung (9) bestimmte Nachricht $I(s_i)$ ist eine zufällige Variable, deren Werte mit den Wahrscheinlichkeiten p_i erscheinen. Die durchschnittliche, qualitative Nachricht einer Mitteilung s_i wird dann von der Beziehung :

$$\overline{I(s_i)} \overset{\text{not.}}{=} H(S) = \sum_{i=1}^{n} p_i I(s_i), \tag{10}$$

gegeben, oder, unter Beachtung der Beziehung (9) :

$$H(S) = a \sum_{i=1}^{n} p_i u_i. \tag{11}$$

Ist U die Nützlichkeit der ganzen, völligen, diskreten, speicherfreien Quelle, dann offenbar :

$$U=\sum_{i=1}^{n} u_i. \tag{12}$$

Es läßt sich beweisen, [2], daß die durchschnittliche, probabilistische Nachricht einen Spitzenwert erreicht wenn :

$$p_i=\frac{1}{n}, \quad (i=\overline{1, n}). \tag{13}$$

In dieser Situation hat die durchschnittliche, qualitative Nachricht den Wert :

$$H(S)=\frac{aU}{n}. \tag{14}$$

Denselben Wert gewinnt man für die durchschnittliche, qualitative Nachricht, unabhängig von den Lieferungswahrscheinlichkeiten der Mitteilungen, falls jede Mitteilung dieselbe Nützlichkeit besitzt, also :

$$u_i=\frac{aU}{n}. \tag{15}$$

Im Falle einer beliebigen, diskreten, völligen, speicherfreien, mit der Nützlichkeit U Quelle, gewinnt man die maximale durchschnittliche, qualitative Nachricht falls man den mit den größten Wahrscheinlichkeiten gelieferten Mitteilungen die größten Nützlichkeiten zuteilt und umgekehrt.

Tatsächlich, es sei eine diskrete, völlige, speicherfreie, mit der Verteilung :

$$S:\begin{pmatrix} s_1 & s_2 \dots s_n \\ p_1 & p_2 \dots p_n \\ u_1 & u_2 \dots u_n \end{pmatrix} \tag{16}$$

Quelle gegeben, für die man die Ordnungsbeziehungen :

$$p_1 \geqslant p_2 \geqslant \dots \geqslant p_n \tag{17}$$

und :

$$u_1 \geqslant u_2 \geqslant \dots \geqslant u_n \tag{18}$$

annimmt. In diesem Falle wird die durchschnittliche, qualitative Nachricht von der Beziehung (11) gegeben.

Nehmen wir jetzt an, daß der Mitteilung mit der Wahrscheinlichkeit p_k, $(k=\overline{1, n})$, die Nützlichkeit u_{k+1}, bzw. der Mitteilung mit der Wahrscheinlichkeit p_{k+1} die Nützlichkeit u_k zugeteilt wird. Die durchschnittliche, qualitative Nachricht läßt sich in diesem Falle mit der Beziehung:

$$H'(S)=a\left(\sum_{i=1}^{k-1} p_i u_i+p_k u_{k+1}+p_{k+1}u_k+\sum_{i=k+2}^{n} p_i u_i\right) \tag{19}$$

berechnen. Aus der Beziehungen (11) und (19) folgt dann, daß :

$$H(S)-H'(S)=a(p_k-p_{k+1})\,(u_k-u_{k+1})\geqslant 0. \tag{20}$$

Also, falls man die von der Beziehungen (17) und (18) gegebene Ordnung nicht beachtet, wird sich immer kleinere durchschnittliche, qualitative Nachricht ergeben.

3. Die durchschnittliche, qualitative Nachricht der Ausdehnung einer diskreten, völligen, speicherfreien Quelle

In vielen Quellenmodellen ist es vorteilhaft zu annehmen, daß die Quelle Mitteilugsgruppen, statt einzelnen Mittei ungen, liefert.

Im allgemeinen, kann man von einer diskreten, völligen und speicherfreien Quelle S, die die Mitteilungen s_i, $(i=\overline{1, n})$, mit den Warscheinlichkeiten p_i, liefert, zu einer anderen Quelle — die man die Ausdehnung m-er Ordnung nennt — übergehen, falls diese letzte, mit S^m aufgezeichnete Quelle, Gruppen von m Mitteilungen der Quelle S liefert.

Die Ausdehnung m-er Ordnung enthält n^m zusammengestellten Symbolen, σ_i, der Form :

$$\sigma_i = s_{i_1} s_{i_2} \dots s_{i_m}, \quad (i=\overline{1, n^m}), \tag{21}$$

wobei s_{i_1}, s_{i_2},..., s_{i_m} Mitteilungen der Quelle S sind.

Weil die Quelle S speicherfrei angenommen worden ist, folgt daß :

$$p(\sigma_i) = p(s_{i_1})\, p(s_{i_2}) \dots p(s_{i_m}). \tag{22}$$

Entsprechend der Beziehung (9), die qualitative Nachricht des Symbols σ_i läßt sich mit der Beziehung :

$$I(\sigma_i) = a(u_{i_1} + u_{i_2} + \dots + u_{i_m}) \tag{23}$$

berechnen, wobei u_{i_k} die Nützlichkeiten der Mitteilungen s_{i_k}, $(k=\overline{1, n})$, bedeuten. Der durchschnittliche Wert der qualitativen Nachricht wird dann:

$$\overline{I(\sigma_i)} \overset{\text{not.}}{=} H(S^m) = \sum_{i=1}^{n^m} p(\sigma_i) I(\sigma_i) \tag{24}$$

oder :

$$H(S^m) = a \sum_{i=1}^{n^m} p(\sigma_i)\, (u_{i_1} + u_{i_2} + \dots + u_{i_m}). \tag{25}$$

Im Falle der Ausdehnung zweiter Ordnung, $m=2$, folgt :

$$H(S^2) = a \sum_{i_1=1}^{n} \sum_{i_2=1}^{n} p(s_{i_1})\, p(s_{i_2})\, (u_{i_1} + u_{i_2}), \tag{26}$$

oder, wenn man die Tatsache, daß die Quelle S völlig ist berücksichtigt :

$$H(S^2) = 2H(S). \tag{27}$$

Angenommen, die Beziehung :

$$H(S^m) = mH(S) \tag{28}$$

gültig wäre, nehmen wir uns vor zu beweisen, daß sie auch für $m+1$ gültig ist. Es sei dafür :

(29) $$\sigma_{i_k}=\sigma_i s_{i_k}$$

gegeben, wobei das Symbol σ_i von der Beziehung (21) gegeben ist und s_{i_k} eine Mitteilung der Quelle S ist.

Unter der Annahme die Quelle wäre speicherfrei, folgt :

(30) $$u(\sigma_{i_k})=u(\sigma_i)+u(s_{i_k}).$$

Also :

(31) $$H(S^{m+1})=\sum_{i=1}^{n^{m+1}} p(\sigma_i)p(s_{i_k})[u(\sigma_i)+u(s_{i_k})].$$

Unter der weiteren Annahme die Quelle S wäre völlig, folgt :

(32) $$H(S^{m+1})=mH(S)+H(S).$$

4. Die durchschnittliche, qualitative Nachricht einer ergodischen, diskreten, speicherbesitzenden Quelle

Eine diskrete, mit einem m-er Ordnung Speicher Quelle, ist mittels der Tatsache kenngezeichnet, daß die Lieferung einer Mitteilung von der m vorher gelieferten Mitteilungen bedingt ist, wobei m eine Ganzzahl ist.

Unter Zustand in einem gegebenen Zeitpunkt einer diskreten, mit einem Speicher m-er Ordnung Quelle, versteht sich die Reihenfolge letzter m gelieferten Mitteilungen vorherig dem berücksichtigten Zeitpunkt.

Eine diskrete Quelle mit Speicher m-er Ordnung wird also n^m Zustände haben.

Die diskrete, mit Speicher m-er Ordnung Quelle ist als ergodisch betrachtet, falls die Wahrscheinlichkeiten der n^m Zustände zeitinvariant sind.

Es sei die ergodische, diskrete, mit Speicher m-er Ordnung Quelle gegeben, mit der Verteilung (16) und dem Zustand S_i von folgende Mitteilungsreihenfolge kenngezeichnet :

(33) $$S_i \rightarrow s_{i_1} s_{i_2} \dots s_{i_m}.$$

Man zeichnet mit :

(34) $$p(s_j/s_{i_1}s_{i_2} \dots s_{i_m}),$$

die Lieferungswahrscheinlichkeit der Mitteilung s_j im Zustand S_i und mit :

(35) $$u(s_j/s_{i_1}s_{i_2} \dots s_{i_m}),$$

die Nützlichkeit der Mitteilung s_j, falls die Mitteilungsnützlichkeiten die den Zustand S_i bestimmen bekannt sind, auf.

Die bei der Lieferung der Mitteilung s_j aus dem Zustand S_i gewonnene qualitative Nachricht ist der Form :

(36) $$I(s_j/s_{i_1}s_{i_2} \dots s_{i_m})=au(s_j/s_{i_1}s_{i_2} \dots s_{i_m}).$$

Aus dem Zustand S_i läßt sich jede Mitteilung s_j, $(j = \overline{1, n})$, mit einer bedingten Wahrscheinlichkeit liefern (eventuell gleich Null falls die Lieferung einiger Mitteilungen aus jenem Zustand unmöglich ist).

Es folgt also, daß die qualitative, von dem Zustand S_i gelieferte Nachricht, sich mit der Beziehung :

$$I(s_{i_1}s_{i_2} \dots s_{i_m}) = \sum_{j=1}^{n} p(s_j/s_{i_1}s_{i_2} \dots s_{i_m})\, I(s_j/s_{i_1}s_{i_2} \dots s_{i_m}), \tag{37}$$

berechnen läßt, oder :

$$I(s_{i_1}s_{i_2} \dots s_{i_m}) = a \sum_{j=1}^{n} p(s_j/s_{i_1}s_{i_2} \dots s_{i_m}) u(s_j/s_{i_1}s_{i_2} \dots s_{i_m}). \tag{38}$$

Es seien p_k, $(k = \overline{1, n^m})$, gegeben, die die Zuständewahrscheinlichkeiten der diskreten, ergodischen, mit m-er Ordnung Speicher Quelle bedeuten.

Die durchschnittliche, qualitative Nachricht dieser Quelle wird dann :

$$\overline{I(S)} \overset{\text{not.}}{=} H(S) = \sum_{k=1}^{n^m} p_k I(s_{i_1}s_{i_2} \dots s_{i_m}), \tag{39}$$

oder :

$$H(S) = a \sum_{k=1}^{n^m} \sum_{j=1}^{n} p_k p(s_j/s_{i_1}s_{i_2} \dots s_{i_m}) u(s_j/s_{i_1}s_{i_2} \dots s_{i_m}). \tag{40}$$

5. Beziehungen zwischen den durchschnittlichen, qualitativen Nachrichten

Zwecks Mitteilungsübertragungen über verschiedene Übertragungkanäle werden oft Blockkoden vewendet. In diesem Falle teilt man jeder Mitteilung s_i ein Kodewort x_i zu, das aus einer Reihe von N vom Übertragungskanal akzeptierten Symbolen besteht. Offenbar, die Kodewörter werden am Eingang des Übertragungskanals mit denselben Wahrscheinlichkeiten wie die Mitteilungen der Quelle S geliefert.

Weil der Übertragungskanal störungsunfrei ist, an seinem Ausgang werden die Wörter y_j empfangen, die sich von den gesandten Wörte unterscheiden können.

Angenommen, die Kanäle seien binär und die Störungen additiv, werden höchst :

$$m = 2^N \tag{41}$$

Wörter empfangen.

Um die Fehlerentzerrung möglich zu machen, die Kodewörter werden passend gewählt, aber unter Beachtung der Bedingung :

$$n < m. \tag{42}$$

Die Blockschaltung des Übertragungskanals ist in der Abb. 1 dargestellt, wobei : $[X] = [x_1 x_2 \dots x_n]$ — das Kodewörterfeld, $[Y] = [y_1 y_2 \dots y_m]$ — das Feld empfangener Wörter, C — den Übertragungskanal und P — die additiven Störungen auf dem Kanal bedeuten.

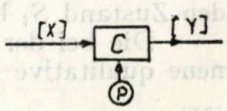

Abb. 1

Es sei x_i das Kodewort am Eingang des Übertragungskanals und y_j das empfangene Wort gegeben. Ist die Wahrscheinlichkeit der Verwircklichung dieses Ereignisses $p(x_i \cap y_j)$, dann läßt sich die Nützlichkeit des Ereignisses $x_i \cap y_j$ mit einer folgender Beziehungen berechnen :

(43) $u(x_i \cap y_j) = u(x_i) + u(y_j/x_i)$, oder : (44) $u(x_i \cap y_j) = u(y_j) + u(x_i/y_j)$,

wobei die Aufzeichnungen folgende Bedeutungen haben : $u(x_i)$ — die Nützlichkeit des Wortes x_i, die gleich mit der Nützlichkeit der Mitteilung s_i ist ; $u(y_j/x_i)$ — die vom Wort y_j aufgewiesene Nützlichkeit, falls die Nützlichkeit des Wortes x_i bekannt ist ; $u(y_j)$ — die Nützlichkeit des Wortes y_j ohne jede zusätzliche Nachricht ; $u(x_i/y_j)$ — die vom Wort x_i aufgewiesene Nützlichkeit, falls die Nützlichkeit des Wortes y_j bekannt ist.

Infolge der Beziehung (9), läßt sich die bei der Verwircklichung des Ereignisses $x_i \cap y_j$ gewonnene Nachricht mit den Beziehungen :

$$i(x_i \cap y_j) = a[u(x_i) + u(y_i/x_i)], \tag{45}$$

oder :

$$i(x_i \cap y_j) = a[u(y_j) + u(x_i/y_j)] \tag{46}$$

berechnen.

Die durchschnittliche, qualitative Nachricht des $[X]$ mit $[Y]$ vereinigten Felder ist dann :

$$\overline{i(x_i \cap y_j)} \overset{\text{not.}}{=} H(X, Y) = \sum_{i=1}^{n} \sum_{j=1}^{m} p(x_i \cap y_j)\, i(x_i \cap y_j), \tag{47}$$

oder, unter Berücksichtigung von (45) und (46) :

$$H(X, Y) = a \sum_{i=1}^{n} \sum_{j=1}^{m} p(x_i \cap y_j)\, [u(x_i) + u(y_j/x_i)], \tag{48}$$

oder :

$$H(X, Y) = a \sum_{i=1}^{n} \sum_{i=1}^{m} p(x_i \cap y_j)\, [u(y_j) + u(x_i/y_j)]. \tag{49}$$

Wir nehmen weiter an, daß der Empfang des Wortes y_j erfolgte. Infolge der Störungsanwesenheit auf dem Kanal, besteht eine Unsicherheit die die gesandten Wörter betrifft. Diese können x_1, x_2 oder jeder der Wörter des Feldes $[X]$ sein. Falls $p(x_i/y_j)$ die Wahrscheinlichkeit der Sendung des Wortes x_i ist wenn das Wort y_j empfangen wurde, die durchschnittliche, qualitative Nachricht die man beim Empfang des Wortes y_j über das Feld $[X]$ gewinnt, ist :

$$\overline{i(x_i/y_j)} = a \sum_{i=1}^{n} p(x_i/y_j)\, u(x_i/y_j). \tag{50}$$

Wenn man alle Empfangsmöglichkeiten der Wörter y_j, $(j = \overline{1, m})$, einbezieht, die durchschnittliche, qualitative Nachricht über das Feld $[X]$, wenn das Feld $[Y]$ bekannt ist, hat den Wert :

$$(51) \qquad \sum_{j=1}^{m} p(y_j)\overline{i(x_i/y_j)} \overset{\text{not.}}{=} H(X/Y),$$

wobei $p(y_j)$ die Empfangswahrscheinlichkeit des Wortes y_j darstellt.

Falls man die Beziehung (50) beachtet, folgt:

$$(52) \qquad H(X/Y)=a\sum_{i=1}^{n}\sum_{j=1}^{m} p(x_i \cap y_j)u(x_i/y_j).$$

Auf änlicher Weise läßt sich die durchschnittliche, qualitative Nachricht über das Feld $[Y]$ wenn das Feld $[X]$ bekannt ist, deffinieren:

$$(53) \qquad H(Y/X)=a\sum_{i=1}^{n}\sum_{j=1}^{m} p(x_i \cap y_j)u(y_j/x_i).$$

Die durchschnittlichen, qualitativen Nachrichten der Felder $[X]$ und $[Y]$ lassen sich mit den Beziehungen:

$$(54) \quad H(X)=a\sum_{i=1}^{n} p(x_i \cap y_j)u(x_i), \quad \text{bzw.:} \quad (55) \quad H(Y)=a\sum_{j=1}^{m} p(x_i \cap y_j)u(y_j),$$

berechnen.

Die Nachprüfung folgender Beziehungen zwischen den durchschnittlichen, qualitativen, mittels der Beziehungen (48), (49), (52) und (55) eingeführten Nachrichten, läßt sich einfach durchführen:

$$(56) \quad H(X, Y)=H(X)+H(Y/X), \qquad (57) \quad H(X, Y)=H(Y)+H(X/Y).$$

6. Schlußfolgerungen

In der Arbeit wurden die Modellen der diskreten, in der Praxis oft vewendeten Quellen betrachtet. Neben den Lieferungswahrscheinlichkeiten der Mitteilungen, wurden diesen auch Nützlichkeiten zugeteilt. Auf dieser Weise wurden Berechnungsbeziehungen für die durchsnittlichen, qualitativen Nachrichten der Mitteilung gewonnen.

Unter Verwendung für die Nachrichtenübertragung der binären, mit additiven Störungen Kanäle, erfolgt die Bestimmung der Beziehungen zwischen den durchschnittlichen, qualitativen Nachrichten des Systems.

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ASUPRA MĂSURĂRII INFORMAŢIEI CALITATIVE

(Rezumat)

Considerîndu-se numai caracterul semantic (calitativ) al unui mesaj, apreciat printr-o anumită utilitate, s-a definit informaţia calitativă.

Pe baza acestei noţiuni s-au calculat entropiile calitative ale surselor discrete cu şi fără memorie, stabilindu-se relaţii de legătură între acestea.

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DER AUFBAU DES IN EINEM ELEKTROMAGNETISCHEN LEVITATIONSYSTEM VERWENDETEN ENDVERSTÄRKERS

VON

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1. Einleitung

In den letzten Jahren zeigten mehrere Ländern ein großes Interesse für die Herstellung einiger Fahrzeuge die die elektromagnetische Levitation verwenden. Dieses Federungsverfahren ist als eine zukunftige Lösung für die Fahrgästebeförderung angesehen [1]. Selbstverständlich sind die Erfordernisse des Levitationsystems verschieden und vom Anwendungsgebiet des Fahrzeuges abhängig. So sind die Problemen der Systemen hoher Geschwindigkeit, die im interurbanen Verkehr dienen, viel schwieriger als derjeniger die für den städttischen Verkehr bestimmt sind [1], [2].

Der in den Levitationsystemen verwendeten Endverstärker muß den Mittelwert des Stromes ändern der den Elektromagnet der die Tragkraft schafft durchfließt. Die Änderung soll von einer von dem automatischen Luftspaltsteuerungsystem gelieferten Steuerspannung abhängen. Wegen der hohen Stärke dieses Stromes (die mehrere Amperzehnten betragen kann) wird der Endverstärker in Form eines Leistungszerhackers realisiert.

Im Aufbau der Verstärkern der schon verwirklichten Levitationsystemen werden sowohl Transistoren [2], als auch Thyristoren verwendet [1]. Wegen der hohen Betriebsfrequenz dieser Zerhackern, erscheint die Anwendung von Transistoren vorteilhafter. Dazu trägt auch die Tatsache bei, daß man zur Zeit Transistoren mit denen man Leistungen von 10...15 kVA steuern kann zur Verfügung stehen.

In der vorliegenden Arbeit beschreibt man einen mit Transistoren realisierten Endverstärker aus einem elektromagnetischen Levitationsystem.

2. Die Blockschaltung des Verstärkers

In der Abb. 1 ist die Blockschaltung des Verstärkers dargestellt. Sie besteht aus folgenden Stufen: NK — nichtstabile Kippschaltung die den LVS steuert ; LVS — linear variabler Spannungsgenerator ; K — Komparatorstufe ; V_1 — Gleichstromverstärker der die von dem automatischen Luftspaltsteuerungsystem gewonnener Steuerspannung U_c verstärkt ; V_2 — Vorverstärker ; LZ — Leistungszerhacker ; SQ — stabilisierte Spannungsquelle ; LQ — Leistungsquelle die den Elektromagnet speist ; EM — Elek-

tromagnet der die Tragkraft des elektromagnetischen Levitationsystems schafft.

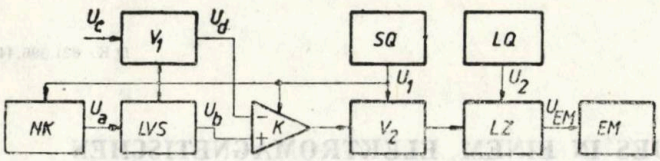

Abb. 1

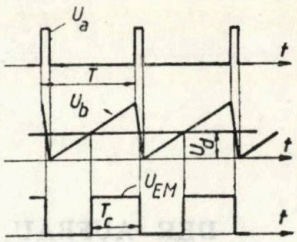

Abb. 2

In der Abb. 2 erscheinen einige Verstärkerwellenformen aus denen man die Funktionierung des Verstärkers ersehen kann. So lange die Ausgangsspannung U_b des LVS höher als der Ausgangsspannung U_d des V_1 ist, schaltet der Leistungszerhacker LZ die Quelle LQ an den Klemmen des Elektromagnets.

Wird der Luftspalt größer (das System zeigt also eine Ausfalltendenz), dann steigt das Steuersignal U_c, die Spannung U_d setzt herab, die Zeitspanne T_c während die Quelle LQ an den Klemmen des Elektromagnets angeschlossen ist nimmt zu und der Mittelstrom der den Elektromagnet durchfließt nimmt ebenfalls zu. Infolgedessen erhöht sich die Tragkraft des Elektromagnets was den Luftzwischenraum zum vorgeschriebenen Wert herabsetzt.

3. Die elektronische Schaltung des Verstärkers

In der Abb. 3 ist die volkommene elektronische Schaltung des realisierten Verstärkers dargestellt. Sie beruht auf der Blockschaltung aus der Abb. 1. Die Transistoren Q_1, Q_2 bilden die NK. Der Konstantstromgenerator Q_4 sichert eine linear steigende Spannung an den Klemmen des Kon-

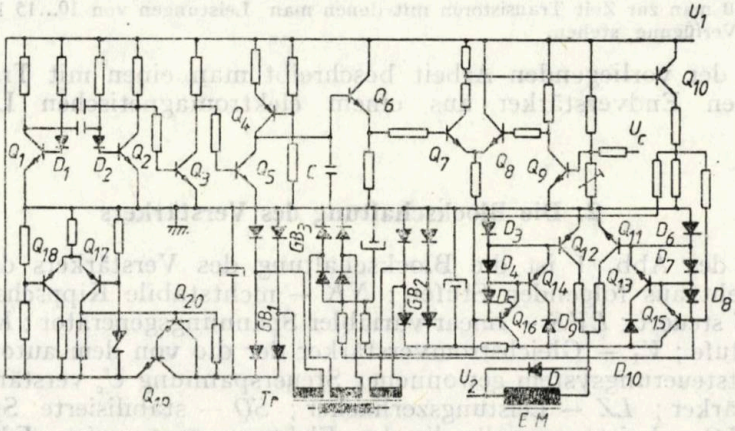

Abb. 3

densators C. Die Entladung des Kondensators erfolgt über Q_5, der vom Q_3 gesteuert ist. Die Spannung an den Klemmen des Kondensators ist von dem Emitterfolger Q_6 nachgebildet. Die Transistoren Q_7, Q_8 bilden den Komparator, Q_9 ist der Gleichstromverstärker, während der Vorverstärker V_2 mit Q_{10} realisiert ist.

Die stabilisierte Quelle SQ ist aus der Gleichrichterbrücke GB_1 und den Transistoren $Q_{17}...Q_{20}$ zusammengesetzt. Der Endverstärker ist mit den Transistoren $Q_{11}...Q_{16}$ verwirklicht. Bemerke man, daß nur die Transistoren Q_{11}, Q_{12} an der Quelle LQ, die eine hohe Spannung liefert (60 ..120 V), angeschlossen sind. Die Transistoren $Q_{13}...Q_{16}$ sind von der Gleichrichterbrücke GB_2 mit einer niedriger Spannung gespeist, so daß man keine besondere Transistoren, auf hohen Spannung Rücksicht nehmend, mehr braucht.

Von der Gleichrichterbrücke GB_3 gewinnt man die negative Vorspannung für die Transistorenbasen aus dem Leistungsverstärker während des Zeitintervalls in dem diese gesperrt sind. Die Dioden $D_3...D_8$ vermeiden die hohen Gegenspannungen an den Basis—Emitter Verbindungen und dienen gleichzeitig einer guten Umschaltung.

Die Entlandungsdiode, D, vermeidet die Erscheinung Überspannungen an den Transistoren Q_{11}, Q_{12} und sichert ebenfalls die Erhaltung des Stromes durch die Elektromagnetspule während der Zeitspanne in der die Transistoren nicht leitend sind.

4. Gesichtspunkte über die Schaltungsprojektierung

Die Projektierung der Stufen die den Endverstärker bilden ist bekannt, so daß wir diesen Absatz der Wahl der Betriebsfrequenz der NK und der Dimensionierung der Entladungsdiode, D, in parallel mit der Spule widmen werden.

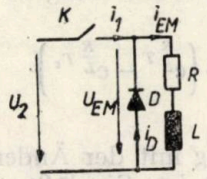

Abb. 4

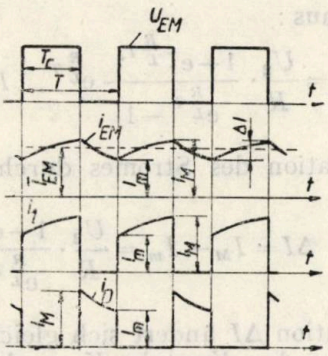

Abb. 5

Für die folgende Analyse ist in der Abb. 4 die equivalente Schaltung der Leistungsstufe dargestellt, in der die Transistoren $Q_{11}...Q_{16}$ mittels dem Kontakt K und die Elektromagnetspule EM mittels seiner Widerstand R und seiner Induktivität L ersetzt sind.

In der Abb. 5 erscheinen die Wellenformen der Spannung u_{EM} an den Elektromagnetklemmen, des Stromes i_{EM} durch den Elektromagnet, des Stromes i_1 durch den Kontakt K (also durch die Transistoren Q_{11}, Q_{12}) und des Stromes i_D durch die Entladungsdiode.

Nach dem Schließen des Kontaktes K gewinnt man den Strom i_{EM} aus der Integrierung der differentiellen Gleichung:

$$L\frac{di'_{EM}}{dt}+Ri'_{EM}=U_2, \tag{1}$$

mit der Anfangsbedingung $i'_{EM}(0)=I_m$. Also:

$$i'_{EM}(t)=\frac{U_2}{R}+\left(I_m-\frac{U_2}{R}\right)e^{-\frac{Rt}{L}},\ t\in[t_1,t_2]. \tag{2}$$

Während der Zeitspanne T_c, wo der Kontakt K geschlossen ist, $i'_{EM}(t)=i_1(t)$. Nach der Öffnung des K fließt der Strom i_{EM} durch die Diode D, $(i''_{EM}(t)=i_D(t))$. Diesen berechnet man aus der Gleichung:

$$L\frac{di''_{EM}}{dt}+Ri''_{EM}=0, \tag{3}$$

mit der Anfangsbedingung $i''_{EM}(0)=I_M$. Also:

$$i''_{EM}(t)=I_Me^{-\frac{R}{L}},\quad t\in[t_2,t_3]. \tag{4}$$

Setzt man in (2) $t=T_c$ und in (4) $t=T-T_c$, für den stationären Vorgang folgt:

$$I_M=\frac{U_2}{R}+\left(I_m-\frac{U_2}{R}\right)e^{-\frac{R}{L}T_c},\quad I_m=I_Me^{-\frac{R}{L}(T-T_c)} \tag{5}$$

und von hier aus:

$$I_M=\frac{U_2}{R}\cdot\frac{1-e^{-\frac{R}{L}T_c}}{e^{\frac{R}{L}T}-1}e^{\frac{R}{L}T},\quad I_m=\frac{U_2}{R}\cdot\frac{1-e^{-\frac{R}{L}T_c}}{e^{\frac{R}{L}T}-1}e^{\frac{R}{L}T_c}. \tag{6}$$

Die Variation des Stromes durch den Elektromagnet ist:

$$\Delta I=I_M-I_m=\frac{U_2}{R}\cdot\frac{1-e^{-\frac{R}{L}T_c}}{e^{\frac{R}{L}T}-1}\left(e^{\frac{R}{L}T}-e^{\frac{R}{L}T_c}\right). \tag{7}$$

Die Variation ΔI ändert sich gleichzeitig mit der Änderung des Zeitintervalls T_c wo der Kontakt K geschlossen ist. Sie läßt ein Maximum zu für eine Zeitspanne T_c (die man aus der Bedingung $d\Delta I/dT_c=0$ gewinnt):

$$T_c=\frac{T}{2}. \tag{8}$$

Setzt man (8) in (7), dann folgt:

$$(9) \qquad (\Delta I)_{max} = \frac{U_2}{R} \cdot \frac{e^{\frac{R}{2L}T} - 1}{e^{\frac{R}{2L}T} + 1} .$$

Wird die Variation $(\Delta I)_{max}$ vorgeschrieben, aus der Beziehung (9) läßt sich die minimale Betriebsfrequenz der NK berechnen:

$$(10) \qquad \frac{1}{f} = T = \frac{2L}{R} \ln \frac{U_2 + R(\Delta I)_{max}}{U_2 - R(\Delta I)_{max}} .$$

Für die Dimensionierung der Diode D interessiert die Bestimmung des maximalen Mittelwertes des Stromes der sie durchfließt. Für praktische Berechnungen erhält man genügend exakte Beziehungen falls man annimt, daß der Elektromagnet von einem konstanten Strom durchflossen ist:

$$(11) \qquad i_{EM} \approx kt = \bar{I}_{EM} = \frac{T_c}{T} \cdot \frac{U_2}{R} .$$

Unter dieser Voraussetzung ist der Mittelwert des Stromes der die Entladungsdiode D durchfließt:

$$(12) \qquad \bar{I}_D = \frac{T - T_c}{T} \bar{I}_{EM} = \frac{T_c}{T} \cdot \frac{T - T_c}{T} \cdot \frac{U_2}{R} .$$

Das Maximum für $\bar{I}_D$ erzielt man für $T_c = T/2$:

$$(13) \qquad \bar{I}_{Dmax} = \frac{U_2}{4R} .$$

5. Schlußvolgerungen

Es wird der in einem elektromagnetischen Levitationsystem verwendeten Endverstärker beschrieben, der die Rolle des Wirkgliedes in dem automatischen Luftspaltsteuerungsystem spielt.

Die Betriebsfrequenz des Leistungszerhackers läßt sich mit der Beziehung (10) berechnen. Im allgemeinen wird sie aber höher als 2...3 KHz gewählt um das Rauschen herabzusetzen. Auf dieser Betriebsfrequenz Rücksicht nehmend ist die Realisierung des Zerhackers mit Transistoren empfehlenswerter als mit Thyristoren.

Die mit dem Elektromagnet in parallel geschalteten Entladungsdiode muß man für einen Strom gleich ein Viertel von dem Strom der den Elektromagnet durchfließt dimensionieren.

Bei der Dimensionierung der Wärmestrahlern der Transistoren Q_{11}, Q_{12} aus dem Leistungsverstärker muß man die Verlustleistung auf den Transistoren während der Umschaltung besonders in Acht nehmen.

Es sind realisiert und mit guten Ergebnissen experimentiert worden vier Endverstärkern die der Schaltung aus der Abb. 3 entsprechen und die Betriebsfrequenz gleich 3 KHz haben. Jeder Verstärker speist einer von

den vier Elektromagneten des Levitationsystems. In diesem Fall verwendete man eine einzige nichtstabile Kippschaltung die alle vier Verstärkern synchron steuerte.

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Technische Hochschule, Jassy,
Lehrstuhl für Elektronik und Rechenanlagen

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CONSTRUCŢIA AMPLIFICATORULUI FINAL FOLOSIT ÎNTR-UN SISTEM DE LEVITAŢIE ELECTROMAGNETICĂ

(Rezumat)

Se prezintă un amplificator final, folosit pentru un sistem de levitaţie electromagnetică, realizat cu tranzistoare. Se expune modul de alegere a frecvenţei de lucru a chopperului din componenţa amplificatorului şi dimensionarea diodei conectate în paralel cu electromagnetul care realizează forţa portantă.

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D.C. 621.394.9

SYNTHESIS OF A BI-PHASE *M* CODER WITH A SYMBOLIC METHOD

BY

D. ALEXANDRU and MARIA-LILIANA ALEXANDRU

1. Introduction

The bi-phase (BiΦ) *M* code belongs to the family of „split-phase" codes [1]. The NRZ-coded data signal (x) together with the BiΦ-*M* coded signal (Y) are represented in Fig. 1.

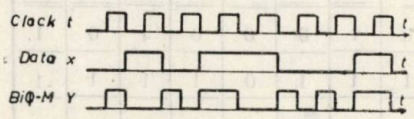

Fig. 1

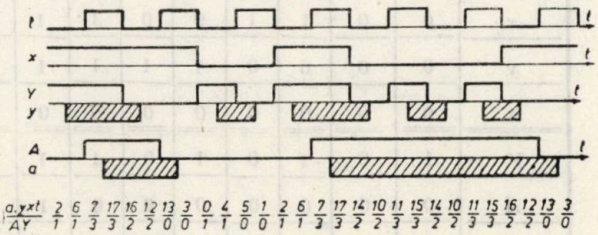

Fig. 2

A transition occurs at the beginning of every simbol period. One is represented by no second transition. Zero is represented by a second transition one-half simbol period later.

2. Synthesis

The synthesis was made using the symbolic method elaborated in [2]. The variables describing the operation of the BiΦ-*M* coder are illustrated in Fig. 2 for a given data sequence.

We denote the data sequence by x, the clock signal by t and the output sequence by Y.

The synthesis implied the choice of an auxiliary variable A as the states describing the coder operation should all be independent, which means that all the fractions describing the operation of the circuit having the same numerator should have the same denominator.

As the output variable A when applied at the input appears after a certain delay, it was denoted by a and represented accordingly in Fig. 2.

This choice was determined by the fact that in the BiΦ-M signar waveform the phase of the f_{bit} signal (clock) is preserved if between successive zeros there is an even number of ones and a change of 180° occurs fol an odd number of ones.

The variable y corresponding to the output variable Y when applied to the input was also introduced.

So, the input variables were chosen as a, y, x and t and the output variables as A and Y. The states occuring in the operation of the circuit synthesizing the BiΦ-M coded signal are described with the help of these binary variables in the octal code.

As an example we consider the situation represented in Fig. 2 : $t=1$, $x=1$, $y=1$, $a=1$ and $A=1$, $Y=1$, The fraction $a.yxt/AY$ describing the circuit operation becomes 1.111/11, which in the octal code represents 17/3.

The states describing the operation of the circuit to be synthesized are extracted from Fig. 2 and rallied in the Table 1.

Table 1

t	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
x	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
y	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
a	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
Y	1	0	1	0	1	0	1	1	0	1	0	0	0	1	0	1
A	0	0	0	0	0	0	0	1	1	1	1	0	1	1	1	1
$\frac{a.yxt}{AY}$	$\frac{0}{1}$	$\frac{1}{0}$	$\frac{2}{1}$	$\frac{3}{0}$	$\frac{4}{1}$	$\frac{5}{0}$	$\frac{6}{1}$	$\frac{7}{3}$	$\frac{10}{2}$	$\frac{11}{3}$	$\frac{12}{2}$	$\frac{13}{0}$	$\frac{14}{2}$	$\frac{15}{3}$	$\frac{16}{2}$	$\frac{17}{3}$

The synthesis will be performed with help of the logic functions OR. The permitted ($A=1$, $Y=1$) and forbidden ($A=0$, $Y=0$) states are extracted from Table 1 and we get :

$$A = U^{a.yxt}_{1:\,(7,10,11,12,14,15,16,17)\ 0:\,(0,1,2,3,4,5,6,13)}$$

$$Y = U^{a.yxt}_{1:\,(0,2,4,6,7,11,15,17)\ 0:\,(1,3,5,10,12,13,14,16).}$$

The states are grouped according to their adjacencies and the implicants are found as follows :

A 7|17 7 has adjacency 10 with 17.

The number 7 is written in the octal code as 0.111 and the bit corresponding to the adjacency is suppressed like $\emptyset$.111 which implies $a.yxt = yxt = y.x.t$ So,

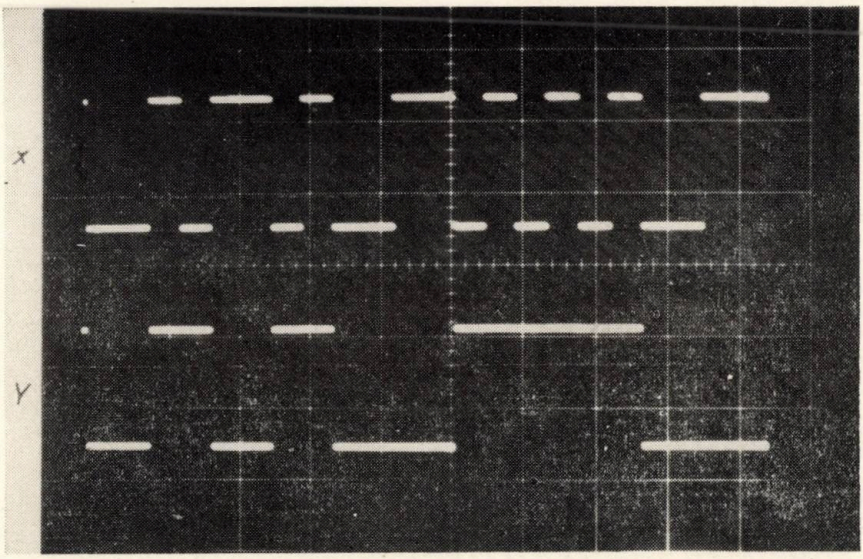

Fig. 7

$$A \quad \overset{10}{7|17} \qquad \varnothing.111 \qquad yxt$$

$$\overset{1}{10}|\overset{4}{11}|14,15 \qquad 1.\varnothing 0\varnothing \qquad a\bar{x}$$

$$\overset{2}{10}|\overset{4}{12}|14,16 \qquad 1.\varnothing\varnothing 0 \qquad a\bar{t}$$

It results :

$$A = y.x.t \cup a.\bar{x} \cup a.t.$$

Similarly, by grouping the permitted states of Y, according to their adjacency, we get :

$$Y \quad \overset{2}{0}|\overset{4}{2}|4,6 \qquad 0.\varnothing\varnothing 0 \qquad \bar{a}.\bar{t}$$

$$\overset{10}{7}|17 \qquad \varnothing.111 \qquad y.x.t$$

$$\overset{4}{11}|15 \qquad 1.\varnothing 01 \qquad a.\bar{x}.t$$

It results :

$$Y = \bar{a}.\bar{t} \cup y.x.t \cup a.\bar{x}.t.$$

The subsequent graph as plotted in Fig. 3 results, based on the observation that the output variables A and Y are obtained from the input variables t and x, and a and y result from A and Y by applying them to the circuit input.

Applying the de M o r g a n's theorems from the Boolean algebra we get :

$$A = \bar{\bar{A}} = \overline{y.x.t.a.\bar{x}.a.t}, \quad Y = \bar{\bar{Y}} = \overline{\bar{a}.\bar{t}.y.x.t.a.\bar{x}.t}.$$

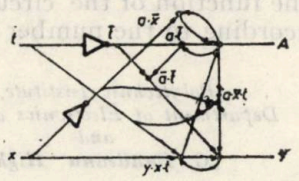

Fig. 3

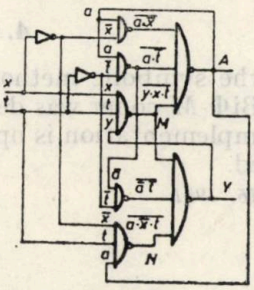

Fig. 4

The circuit represented in Fig. 4 results.

The variables A and a can be obtained from the output of a T flip-flop which counts modulo 2 the bites 1 in the data sequence, as shown in Fig. 5.

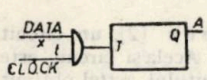

Fig. 5

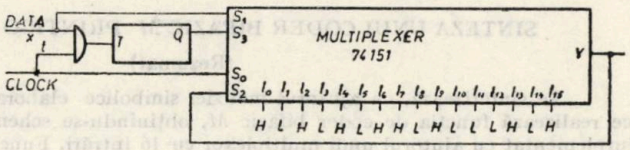

Fig. 6

If the Table 1 is transcribed in the decimal code, Table 2 is obtained and the coder can be implemented with a multiplexer, as shown in Fig. 6.

The signals t, x, y and a are applied on the control inputs S_0, S_1, S_2 and S_3 and the data inputs I_0, I_1, ..., I_{15} are set to the L or H level, according to the Table 2.

Table 2

t	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
x	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
y	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
a	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
Y	1	0	1	0	1	0	1	1	0	1	0	0	0	1	0	1
A	0	0	0	0	0	0	0	1	1	1	1	0	1	1	1	1
$a.yxt$	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
AY	1	0	1	0	1	0	1	3	2	3	2	0	2	3	2	3

3. Test Results

The validity of this synthesis is supported by experimental evidence. Fig. 7 illustrates a plot of data (x) and of BiΦ-M coded signal (Y) versus time for the coder represented in Fig. 4.

4. Conclusions

Using the symbolic method elaborated by N. Reus [2], a simple scheme of a BiΦ-M coder was derived. The function of the circuit is reliable and the implementation is optimal according to the number of the logic gates involved.

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SINTEZA UNUI CODER BIFAZIC M PRINTR-O METODĂ SIMBOLICĂ

(Rezumat)

Se sintetizează, cu ajutorul metodei simbolice elaborată de N. Reus [2], un circuit ce realizează funcția de coder bifazic M, obținîndu-se schema sa logică. Același circuit este implementat cu ajutorul unui multiplexor cu 16 intrări. Funcționarea circuitului astfel obținut este verificată experimental.

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D.K. 621.395.34

EIN UNTERSAGUNGSKREIS FÜR FERNGESPRÄCHE

VON

M. ANTONESCU, VALERIU MUNTEANU und M. LUCANU

1. Einleitung

Für verschiedene Gründe kann die Notwendigkeit der Einschränkung der Ferngespräche von einiger Teilnehmeranschlüße erscheinen. Diese Situation wird — auf Wunsch des Teilnehmers — gelösst unter Verwirklichung bestimmter Schaltungsänderungen im Selbstwählfernsprechamt dem der Teilnehmer angehört. Falls man nicht eine völlige Untersagung, sondern nur die Herabsetzung der Ferngespräche beabsichtigt, muß man ziemlich häufig die vorhererwähnte Modifizierung unternehmen, Tatsache die unannehmbar für das Personal des Fernsprechamtes und unangenehm für den Teilnehmer ist.

In der Arbeit wird ein am telephonischen Gerät schaltbarer Kreis vorgeschlagen, der die üblichen örtlichen Gespräche erlaubt, aber den Zutritt zum Fernsprechnetz verbietet. Die Ferngespräche sind doch möglich falls man mit einem Schlüssel den Kreis abschaltet, so daß das Telephon zum üblichen Betrieb zurückkehrt.

2. Die Blockschaltung

Die Blockschaltung des Kreises ist in der Abb. 1 dargestellt. Das Telephon T ist an der Telephonleitung L_1L_2 an deren Klemmen dauernd die Gleichspeisespannung des Mikrophons vorliegt, angeschlossen.

Vom Telephongabelkontakt TGK gewinnt man Steuersignal für die Generierung, im Resetkreis (RK) II, eines Resetimpulses am Anfang und Ende jedes Gesprächs.

Vom Scheibenkontakt SK erlangt man Impulse die gezählt werden müßen. Weil sie mittels mechanischen Kontakten erzeugt sind, sind sie störungsunfrei. Man kann sie nur störungslos verwenden und zu diesem Zweck dient der Impulsbildungskreis (IBK) I.

Die Verwircklichung eines Ferngesprächs benötigt eine Vorwahl, deren erste Ziffer — in unserem Lande — eine 9 ist. Also, die Impulse der ersten

Ziffer müßen, mittels des Impulszählers (*IZ*) *IV*, gezählt werden und das Bildungsende dieser Ziffer, mit dem Diskriminator (*D*) *III*, erfasst werden. Nach diesem Zeitpunkt untersagt der Diskriminator den Zugang zum Impulszähler und entziffert, mit dem Dekoderkreis (*DK*) *V*, das Zählerinhalt. War die Ziffer eine 9, erzeugt der Dekoderkreis ein Steuersignal für den Abschaltungskreis der Teilnehmehrleitung (*AKTL*) *VI*. Im widrigen Fall fehlt dieses Signal und die anderen Ziffern der städtischen Nummer lassen sich normal wählen.

In der Abb. 2 ist ein Teil eines gewöhnlichen Schemas eines Telephons dargestellt. Daraus folgt die Anschließungsmöglichkeit des Untersagungskreises.

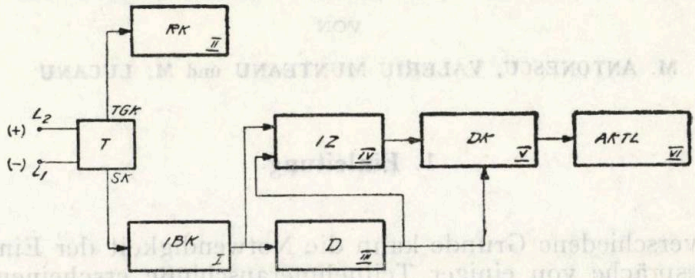

Abb. 1

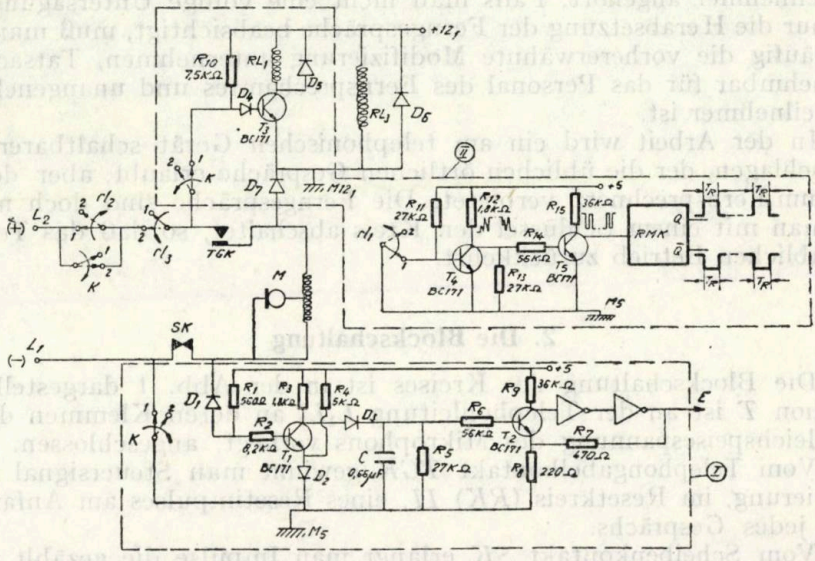

Abb. 2

rl_2 und rl_3 sind Relaiskontakte der Relais RL_2 (Abb. 3) und RL_3 (Abb. 2) die mit ununterbrochener Linie im Ruhezustand gezeichnet worden sind. RL_2 ist das Relais, daß im Falle des Ferngesprächs (erste Ziffer 9)

betätigt ist und die Teilnehmerleitung vom Wählerfernsprechamt abtrennt. Falls der Untersagungskreis nicht gespeist ist, macht RL_3 jedes Gespräch unmöglich. K ist ein mehrfacher Kontakt, der mit einem Schlüssel die Ausschließung des Untersagungskreises ermöglicht (die Kontakte in der Stellung *2*) und das Ferngespräch erlaubt.

3. Der Resetkreis

Der Resetkreis *II* ist in der Abb. 2 in Einzelheiten dargestellt. Zwischen den Gesprächen ist der Telephongabelkontakt *TGK* geöffnet, was die Sättigung des Transistors T_3 veranlaßt. Die Diode D_7 schließt in dieser Situation die direkte Anschaltung der Mikrophongleichspannung (mit der in der Abb. 2 angenommenen Polarität) an der Emitor—Basis Verbindung aus. Das Relais RL_1 ist gespeist, sein Kontakt rl_1 befindet sich in der Stellung *2* und infolgedessen ist T_4 gesättigt, bzw. T_5 gesperrt.

Bei der Aufhebung des Mikrotelephons schließt sich der Telephongabelkontakt, was die Sperre des Transistors T_3 bestimmt (die Spannungsabnahmen auf D_7 und D_4 gleichen sich aus). rl_1 geht in die Stellung *1* über, Tatsache die die Sperre des Transistors T_4, bzw. die Sättigung des Transistors T_5 herbeibringt.

Der Telephongabelkontakt öffnet sich am Gesprächende und man kommt zum anfänglichen Zustand, mit T_3 und T_4 gesättigt und T_5 gesperrt, zurück.

Weil die Transistorsteuerung mittels mechanischen Kontakten erfolgt, jeder Übergang zwischen zwei stabilen Zuständen ist von mehreren Störungsschaltungen verfolgt, was die rl_1 Kontaktprellung herbeizieht und damit eine Reihe mehrerer Störimpulse im Kollektor des Transistors T_4 veranlasst. Die letzten sind mit dem Transistor T_5 verstärkt so daß, sowohl am Schließen, als auch am Öffnen des Telephongabelkontaktes *TGK*, positive Impulse mit Sicherheit gewinnt werden (Abb. 4 *a*), Impulse die die Kippschwelle des Schmittrigers, der die Eingangstufe einer integrierten, für Reset bestimmten monostabilen Kippschaltung *RMK* bildet, überschreiten. Als Folge, sowohl am Anfang, als auch am Ende eines Gesprächs wird ein Resetimpuls, mit einer Dauer T_R von etwa 100 mS erzeugt (Abb. 4 *b*), der die anderen Störsteuerungen vermeidet und das Schema in den anfänglichen Zustand bringt.

4. Der Impulsbildungskreis

Der Impulsbildungskreis *I* ist auch in der Abb. 2 dargestellt. Ist der Scheibenkontakt geschlossen, dann sind die Transistoren T_1 gesperrt (die Spannungsabnahme auf D_1 gleicht sich mit der auf D_2 aus), bzw. der Transistor T_2 gesättigt. Die Diode D_1 trennt die Mikrophonspeisespannung von der Emitor—Basis Verbindung des Transistors T_1 ab am Öffnen des Scheibenkontaktes. Mit dem geöffneten Scheibenkontakt sind T_1 gesättigt, bzw. T_2 gesperrt. Zwischen den zwei stabilen Zuständen des T_1 sind — wie vorher für T_3, T_4 und T_5 gezeigt wurde — einige Störungsübergänge vorhan-

den infolge der Kontaktwiderstandschwankungen des Scheibenkontaktes mit einer maximalen Ausdauer in der Höhenordnung von mS. Um diese unerwünschten Übergänge zu beseitigen, die völligen oder teilweisen Kollektorspannungübergängen des Transistors T_1 sind, mittels dem Siebekreis CR_5R_6, in umgekehrten Übergängen der Kollektorspannung des Transistors T_2 verwandelt, deren Amplitude kleiner als die Einsatzspannung der zwei Umsteller ist. Erst als der Kontaktübergangsprozeß vorbei ist, ist die Einsatzspannung in entweder einer, oder enderen Richtung überschritten, die Umsteller wenden in den entgegengesätzten Zustand und beschleunigen unter positiven Rückwirkung über R_7 das Umschalten des Transistors T_2 in den Sättigung oder Sperrezustand. Als Folge, bei der wiederholten Öffnen und Schließen des Scheibenkontaktes gewinnt man im Punkt E eine Impulsreihe wie in der Abb. 4 *c*, mit $\Delta \approx 51 \ldots 73$ mS und $\delta \approx 31 \ldots 48$ mS (der Eifachheithalber wurde in der Abb. 4 Δ=kt. und δ=kt. angenommen ; s. auch § 5).

B e m e r k u n g. Die Massen der 5 V Quelle (M_5), bzw. der ersten 12 V Quelle (M_{12_1}) sind getrennt um das Mikrotelephon nicht kurzzuschließen. Dafür die Masse der 5 V Quelle (M_5) und die der zweiten 12 V Quelle (M_{12_2}), von der das Abtrennungsrelais RL_2 gespeist wird, sind gemeinsam (Abb. 3).

5. Der Diskriminator des Bildungsende der ersten Ziffer

Der Auswahlbefehl besteht aus mehreren Öffnungen des Scheibenkontaktes deren Zahl gleich mit dem Wert jeder Ziffer ist. Weil das Antriebsystem dieses Kontaktes mechanisch ist, ist eine Schwankung der Scheibenimpulsperiode in den Grenzen 90 ... 110 mS zugelassen. Dasselbe gilt auch für das Pause—Impuls Verhältnis, das sich in den Grenzen 1,3 ... 1,9 befinden muß. Anders gesagt, während einer Zifferbildung ist der Scheibenkontakt etwa 51 ... 73 mS geöffnet, bzw. 31 ... 48 mS geschloßen, was dieselben Werte für Δ und δ hineinzieht (Abb. 4 *c*). Unglücklicherweise schwankt die Impulsperiode und das Pause—Impuls Verhältnis nicht nur von einem Telephon zu anderem, sondern auch bei demselben Telephon während einer Zifferbildung, Tatsache die das Erfassen des Zifferbildungsende erheblich erschwert.

Der Diskriminator des Bildungsende der ersten Ziffer ist in der Abb. 3 dargestellt. Seine Funktionierung läßt sich mit der Hilfe der Zeitdiagrammen aus der Abb. 4 verfolgen.

Nach dem Zeitpunkt Z_2, alle bistabile Kippschaltungen und der binäre Zähler befinden sich im „0" Zustand infolge des Resetimpulses beim Aufheben des Mikrotelephons. Die Bildung der ersten Ziffer beginnt in Z_3, wo die negative Flanke des Impulses von E' (Abb. 4 *d*), die bistabile Kippschaltung BK_1 in den Zustand „1" wendet (Abb. 4 *e*). Die monostabile Kippschaltung MK_{20}, die Impulse mit einer Dauer von 20 mS erzeugen kann, befindet sich vorläufig in Ruhezustand, so daß im Punkt A, am Ausgang des ODER Gatters O_1, das Signal in Z_3 ebenfalls von „1" zu „0" schaltet (Abb. 4 *f*). Auf dieser Weise wird die monostabile Kippschaltung MK_{200} gesteuert, an deren Q-Ausgang ein Impuls von 200 mS erscheint, (Abb. 4 *g*). Für die Dauer dieses Impulses ist die UND Gatter U_1 validiert,

was den Zugang der Impulse von Punkt E zum Eingang des asynchronen, binären, mit zwei Zellen aus bistabilen Kippschaltungen BK_2 und BK_3 gebildeten Zähler, ermöglicht, (Abb. 4 h).

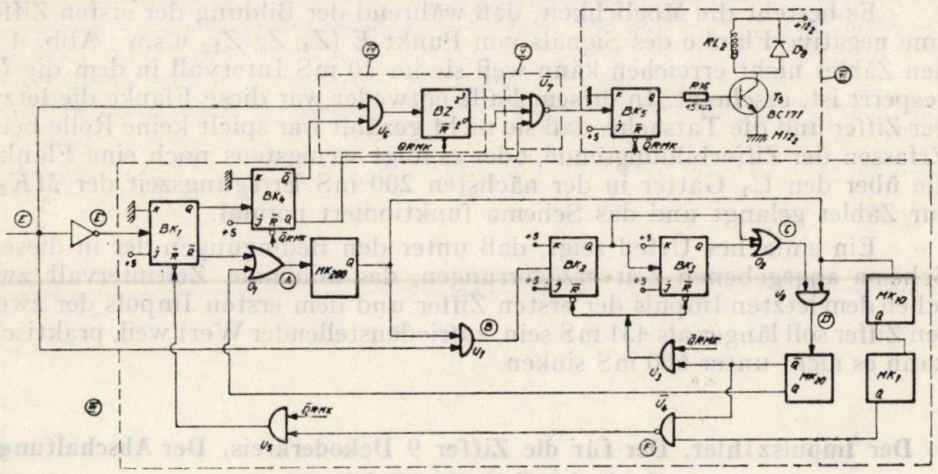

Abb. 3

In einem Zeitintervall von 200 mS können den Zähler BK_2-BK_3 höchstens zwei Impulse erreichen. Er kann aber, sicherheitshalber, auch drei Impulse zählen. Als Folge der ersten negativen, in Z_4 gezählten Flanke, kommutiert der Ausgang des ODER Gatters O_2 in „1" (Abb. 4 i).

Bemerkung. In Z_6, QBK_2 schaltet von „1" zu „0". Dieser Übergang zieht, nach einer Fortpflanzungszeit von 10 ... 20 nS, das Schalten des QBK_3 von „0" zu „1" hinein. Im Punkt C erscheint also eine „0" für 10 ... 20 nS, die die Funktionierung des Diskriminators stören kann. Um so etwas zu vermeiden genügt ein kleiner Kondensator am Ausgang des Gatters O_2 der diesen Ausgang am „1" erhaltet auch wenn sich beide Eingänge für kurze Zeit (10 ... 20 nS) am „0" befinden.

Unter diesen Umständen hat die Signal entwicklung in D das Auftreten aus der Abb. 4 j. Auf der negativen Flanke in Z_8 des Signals in D wird die monostabile Kippschaltung MK_{20} für 20 mS erregt, (Abb. 4 k), was das Reset des Zählers BK_2-BK_3 und das Wiederanfahren in Z_{11} der MK_{200} hineinzieht.

Andererseits, die von der QMK_{200} in Z_8 gewonnene negative Flanke wird die monostabile Kippschaltung MK_{10} für 10 mS erregen (Abb. 4 m), die in Z_9 die monostabile Kippschaltung MK_1 für 1 mS steuern wird (Abb. 4 n).

Die Entwicklung des Schemas folgt weiter wie vorher beschrieben ; bei der zweiten Erregung der MK_{200} wird ein einziges Impuls gezählt (in diesem Beispiel wurde als erste Ziffer eine 3 angenommen) ; bei der dritten Erregung bleibt der Zähler in „0", MK_{20} kann nicht mehr gesteuert werden, aber es funktionieren wie vorher MK_{10} und MK_1. Als Folge gewinnt man

im Punkt F im Zeitintervall $Z_{18}-Z_{19}$, „0" logisches Signal für 1 mS, was das Reset der BK_1 mit sich bringt. Gleichzeitig, die negative Flanke vom QBK_1 steuert jetzt die bistabile Kippschaltung BK_4 in „1" Zustand, was das Bildungsende der ersten Ziffer bedeutet.

Es besteht die Möglichkeit, daß während der Bildung der ersten Ziffer eine negative Flanke des Signals von Punkt E (Z_4, Z_6, Z_{12} u.s.w., Abb. 4 *c*) den Zähler nicht erreichen kann weil sie im 20 mS Intervall in dem die U_1 gesperrt ist, erscheint. In diesem Falle entweder war diese Flanke die letzte der Ziffer und die Tatsache, daß sie nicht gezählt war spielt keine Rolle beim Erfassen der Zifferbildungsende, oder es folgt wenigstens noch eine Flanke, die über den U_1 Gatter in der nächsten 200 mS Erregungszeit der MK_{200} zur Zähler gelangt und das Schema funktioniert normal.

Ein einfaches Urteil zeigt, daß unter den Bedingungen der in diesem Schema angegebenen Zeitverzögerrungen, das minimale Zeitintervall zwischen dem letzten Impuls der ersten Ziffer und dem ersten Impuls der zweiten Ziffer soll länger als 431 mS sein, zufriedenstellender Wert weil, praktisch, kann es nicht unter 600 mS sinken.

6. Der Impulszihler. Der für die Ziffer 9 Dekoderkreis. Der Abschaltungskreis der Teilnehmehrleitung

Auch diese Kreise erscheinen in der Abb. 3.

Der Zähler ist vom binärer, asynchroner Typus, mit vier Zellen. Der Zugang der Impulse aus Punkt E zum Zählereingang ist nur während der Bildung der ersten Ziffer möglich, infolge der Anwesenheit des UND Gatters U_6 und der BK_4.

Der Dekoder für die Ziffer 9 ist mit dem $\overline{\text{UND}}$ Gatter $\overline{U}_7$ verwirklicht, an deren Eingänge die Ausgänge 2^0 und 2^1 angekoppelt sind. Dieser Gatter ist gesperrt während der Bildung der ersten Ziffer, bzw. geöffnet nachher. Also, falls die erste Ziffer eine 9 war, am Ausgang des Dekoders gewinnt man den Übergang „1" →„0" zum Zeitpunkt des BK_4 Schaltens. Für jede andere Ziffer bleibt der Ausgang am „1".

Der Abschaltungskreis der Teilnehmehrleitung besteht aus der vom Dekoder gesteuerten bistabilen Kippschaltung BK_5 und dem Relais RL_2, dessen Kontakt rl_2 in der Leitung L_2 eingereiht ist.

7. Schlußfolgerungen

In der Arbeit ist ein Untersagungskreis für Ferngespräche beschrieben, der an jedem Telephongerät angeschaltet werden kann.

Von seinem Kreisen ist von besonderer Interesse der Diskriminator für das Erfassen des Bildungsende der ersten Ziffer. Er kann, mit entsprechenden Zeitverzögerrungen, zum Erfassen des Impulsreihenende einer Impulsreihe mit schwankender Periode und Impuls—Pause Verhältnis in verschiedenen anderen Anwendungen dienen.

Das Schema wurde mit gewöhnlichen *TTL* integrierten Schaltungen verwircklicht und funktioniert ausgezeichnet.

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Lehrstuhl für Elektronik und Rechenanlagen

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CIRCUIT DE INTERZICERE A ACCESULUI LA INTERURBAN

(Rezumat)

Din diferite motive poate apare necesitatea restricționării convorbirilor interurbane ale unui post de abonat. Este descris un asemenea circuit, atașabil la un telefon obișnuit, care permite convorbirile urbane, dar le interzice pe cele interurbane. Acestea se pot totuși efectua dacă se dispune de o cheie cu care circuitul de supraveghere poate fi exclus.

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ANALYSIS AND SYNTHESIS FOR A STURM—LIOUVILLE PROBLEM WITHIN THE FRAMEWORK OF ELECTRIC DRIVES

BY

TRAIAN GIOSAN and SIMONA CABA

1. Introduction

An analysis and a synthesis are drawn up having a S t u r m-L i o u v i l l e problem in the centre for the linear systems with distributed or concentrate parameters. In this way, was fixed a special comprehensive classes of systems in Physics and Mathematics subjected to all kinds of restrictions.

The authors intended to draw attention on the validity of this S t u r m-L i o u v i l l e problem for many branches of technics and especially for electric drives. Of course can be added many other theoretical and applied interpretations.

Within the framework of the analysis, the equation is given and the solution is deduced thus. The process is given and its evolution is determined.

Within the framework of the synthesis, the solution will be given and the corresponding equation must be determined ; it means that the evolution of the process is given and we must determine the corresponding equation.

In fact, it means that a technical equipment is given and must be determined its dynamical behaviour and respectively the behaviour is given following the deduction of the corresponding installation.

Using a theoretical applicative theme we tried to solve the problem of causality, i.e. the cause and effect problem.

2. Theoretical development

a) We define two H i l b e r t real spaces Σ and $L_2(l_2, \tau)$ and a unitary transformation U and we consider the dense linear operator defined on Σ through :

$$(Ly)(t, \alpha) = \{p(t, \alpha) y\alpha(t, \alpha)\}_\alpha, \quad (t, \alpha) \in \Delta, \tag{1}$$

where Σ there is the space of all measurable functions on Δ, finite with regard to the induced norm by the inner product :

$$(2) \qquad <x, y> = \int_{t_0}^{t_1} \int_{0}^{b} x(t, \alpha) y(t, \alpha) \, d\alpha \, dt, \quad x, y \in \Sigma,$$

and $L_2(l_2, \tau)$ consists from all real vector functions with values from l_2.

With orthonormal basis $\{\omega_i\}$ from $L_2(l_2, \tau)$ and a basis of coordinates $\{l_i\}$ from l_2 results that transformation $U : \Sigma \to L_2(l_2, \tau)$ leads to $[Ux, Uy] = <x, y>$, i.e. Σ and $L_2(l_2, \tau)$ are congruent. Generally, if H_1 and H_2 are two congruent Hilbert spaces, then $<Ux, Uy>_{H_2} = <x, y>_{H_1}$ for every x, y from H_1.

Because the Hilbert space of functions with many variables, describing the answer and the input of the system, is congruent with the Hilbert space of functions with single variable, we may say that the transition from the systems with distributed parameters to ones with concentrate parameters is made in the same way as the transition from dimensional infinite systems to finite dimensional ones.

Thus if we have :

$$(3) \qquad y_t(t, \alpha) = (Ly)(t, \alpha) + f(t, \alpha), \quad t, \alpha \in \Sigma,$$

then we must write down the equation

$$(4) \qquad \dot{\mathbf{y}}(t) = \mathbf{A}(t)\, \mathbf{y}(t) + \mathbf{f}(t)$$

from $L_2(l_2, \tau)$ where $\mathbf{y} = Uy$, and $\mathbf{f} = Uf$, with $\mathbf{A}(t)$ infinite dimensional and with the elements $a_{ij}(t)$.

Consequently if the transformed system is optimal in the meaning of space $L_2(l_2, \tau)$ the initial system will be optimal in the meaning of space Σ.

Naturally, for the selection of the set $\{\omega_i\}$ we apply, if possible, the technique of the separation of variables associated with the Sturm-Liouville problem [3].

b) With this justification we consider the equation with distributed parameters :

$$(5) \qquad \frac{\partial}{\partial x}\left[p(x)\frac{\partial u}{\partial x}\right] - q(x)u = r(x)\frac{\partial^2 u}{\partial t^2},$$

where $r(x)$, $p(x)$ and $q(x)$ are continuous functions with $r(x) > 0$, $p(x) > 0$ and $q(x) \geqslant 0$ and with the limite conditions :

$$(6) \qquad \alpha U(0, t) + \beta \frac{\partial u(0, t)}{\partial x} = 0, \quad \gamma U(l, t) + \delta \frac{\partial u(l, t)}{\partial x} = 0$$

and the initial ones

$$(7) \qquad u(x, 0) = \varphi_0(x), \quad \frac{\partial u(x, 0)}{\partial t} = \varphi_1(x), \quad 0 \leqslant x \leqslant l.$$

First of all we determine a solution from of equation (5) fulfilling the conditions (6) under the form of the product :

$$(8) \qquad u(x, t) = T(t)\, y(x)$$

After the separation of the variables results the equation :

$$(9) \qquad \frac{d}{dx}\left[p(x)\frac{dy}{dx}\right] + [\lambda r(x) - q(x)]y = 0,$$

respectively :

$$T''(t)+\lambda T(t)=0, \tag{10}$$

where λ is an arbitrary constant.

Because $T(t)\neq 0$, in order that the function (8) satisfy the limite conditions it is necessary and enough to fulfill the conditions :

$$\alpha y(0)+\beta y'(0)=0, \quad \gamma y(l)+\delta y'(l)=0. \tag{11}$$

In this may we have arrived to the limite value problem for which we should find those proper values λ_n for which there exist a solution of the equation (9) different from 0 and which should satisfy the conditions at the limite value (11), solutions called the proper functions [6].

c) A similar problem with (9) is reproduced by the heterogenous equation with discontinuities :

$$L(y)=\frac{\mathrm{d}}{\mathrm{d}x}[p(x)y']-q(x)y=-f(x), \tag{12}$$

where we presume that the function $f(x)$ is null allover the interval $[a, b]$ except a small interval $[\xi-\varepsilon, \xi+\varepsilon]$ where the condition :

$$\int_{\xi-\varepsilon}^{\xi+\varepsilon} f(x)\mathrm{d}x=1 \tag{13}$$

is fulfilled. For $\varepsilon\to 0$, the point x coincides with ξ. We presume that the solution $y_\varepsilon(x)$ satisfies the limite value :

$$\alpha_1 y(a)+\alpha_2 y'(a)=0, \quad \beta_1 y(b)+\beta_2 y'(b)=0, \tag{14}$$

where α_k and β_k are given numbers. At the same time we suppose that at least one of the numbers α_k and β_k are different from 0.

Integrating (12) and taking into account (13) we get the solution :

$$y(x)=\int_a^b G(x, \xi)\, f(\xi)\mathrm{d}\xi, \tag{15}$$

where $G(x, \xi)$ is the Green function of the operator $L(y)$ with limite values (14).

Hence it follows the posibility to reduce a problem of limite value to an integral equation.

d) It follows to determine the processes coresponding to functional:

$$I(y)=\int_0^l [p(x)y'^2+q(x)y^2+2f(x)y]\mathrm{d}x \tag{16}$$

under the influence of the restrictions :

$$y(0)=a, \quad y(l)=b, \quad x\in[a, b], \tag{17}$$

with $f(x)$, $p(x)$ and $q(x)$ continuous functions into a closed interval $[0, l]$ and $p(x)$ having continuous derivative with $p(x)>0$, $q(x)\geq 0$.

The Euler equation for this functional:

$$\frac{d}{dx}[p(x)y']-q(x)y=f(x) \tag{18}$$

has an unique solution in the interval $[0, l]$, the conditions (17) being fullfiled.

If $f(x)=0$, we aquire a Sturm-Liouville equation with the operator:

$$L(y)\equiv\left\{\frac{d}{dx}\left[p(x)\frac{d}{dx}\right]-q(x)\right\}y=0, \quad y(0)=a, \quad y(l)=b, \tag{19}$$

wich corresponds to the homogeneous process:

$$p(x)\frac{d^2y}{dx^2}+p'(x)\frac{dy}{dx}-q(x)y=0. \tag{20}$$

As a consequence of the substitutions $p(x)=a_0(x)$, $q(x)=-a_2(x)$, $p'(x)=a_1(x)$, $a_1(x)=a_0'(x)$, Eq. (20) gets the usual form:

$$a_0(x)\frac{d^2y}{dx^2}+a_1(x)\frac{dy}{dx}+a_2(x)\,y=0 \tag{21}$$

which generate the differential operators:

$$L(y)\equiv a_0(x)\frac{d^2y}{dx^2}+a_1(x)\frac{dy}{dx}+a_2(x)y, \tag{22}$$

$$L^*(z)\equiv\frac{d^2}{dx^2}[a_0(x)z]-\frac{d}{dx}[a_1(x)z]+a_2(x)z. \tag{23}$$

The Green's formula may be written:

$$zL(y)-yL^*(z)=\frac{dp(x)}{dx}, \tag{24}$$

where:

$$P(x)\equiv a_0(x)\,(zy'-yz')+[a_1(x)-a_0'(x)]yz \tag{25}$$

represents the conjunction of $y(x)$ and $z(x)$ with regard to L.

The condition $a_1(x)=a_0'(x)$ changes L into a selfadjunct Sturm-Liouville operator:

$$L\equiv-\frac{d}{dx}\left[p(x)\frac{d}{dx}\right]-q(x)\,; \quad L(y)=0. \tag{26}$$

The integration by sides guides to the generalized Green's formula:

$$\int_a^b[(z,\,L(y))-(L^*(z),\,y)]\,d\xi=-p(x)(zy'-yz')\Big|_a^b. \tag{27}$$

e) Particularly if in the selfadjunct S t u r m-L i o u v i l l e operator (26) we replace y by x and x by t and then we make $p(x)=1$, we get the usual equation from which we form a H i l l equation and particularly a M a t h i e u equation:

$$\frac{d^2x}{dt^2}+q(t)x=0. \tag{28}$$

In addition, if $q(t)$ is a periodic function with the period T then $q(t)$ may be developped into a F o u r i e r series, the equation (28) taking the form [7]:

$$\frac{d^2x}{dt^2}+[\delta+\varepsilon\psi(t)]x=0, \tag{29}$$

where δ and ε are some parameters and $\psi(t)$ a periodic function with the period T. Under this form the equation (29) represents the well known H i l l equation.

f) If in a special case:

$$q(t)=\frac{A_0}{2}+A_1\cos\frac{2\pi}{T}t+B_1\sin\frac{2\pi}{T}t \tag{30}$$

we replace:

$$A_1=A\cos\gamma,\quad B_1=A\sin\gamma, \tag{31}$$

where A and γ are some constants, then results:

$$\frac{d^2x}{dt^2}+\left[\frac{A_0}{2}+A\cos\left(\frac{2\pi}{T}t-\gamma\right)\right]x=0. \tag{32}$$

With convenable substitutions we obtain an equation of the M a t h i e u type as a special case of H i l l equation, namely:

$$\frac{d^2x}{dt^2}+(\delta+\varepsilon\cos\tau)x=0, \tag{33}$$

where:

$$\delta=\frac{T^2}{4\pi^2}\cdot\frac{A_0}{2},\quad \varepsilon=\frac{T^2}{4\pi^2}A. \tag{34}$$

3. Conclusions

For the analysis and synthesis of linear systems it is enough to know the coefficients $p(x)$, $q(x)$ and the function $f(x)$ to which is added from case to case the initial conditions and the limite values in order to solve the problem when the operator is one of S t u r m-L i o u v i l l e type.

In this sense if we know the solution, the equation may be formed and viceversa, if we know the equation, the solution may be written directly.

Therefore the problem has an easy solution in the both senses, that means a convenient fact for analysis and synthesis of linear systems.

Thus, for example, if we start from the M a t h i e u equation (33) in which are known the expresions for the coefficients, the H i l l equat-

ion (29) may be formed and it follows the equation (28) and through adequate substitutions and formal additions we aquire the selfadjunct Sturm-Liouville operator (26) with its characteristics.

With functions $p(x)$, $q(x)$ and $f(x)$ the functional (16) may be written, associated with Euler equation (18) having the form of the Sturm-Liouville operator, which is reduced to the homogeneous process (20), respectively (21), (22). If the function $f(x)$ performs conditions (13), then Green function may be deduced for the solution (8) which satisfy the equation (5). If the solution (4) is adopted we can start from (1), which in its turn is a component of the Sturm-Liouville operator.

From the technical point of view the process (5) represents an oscillation of a string loaded and supported at its extremities such as can be seen at the equipments of the cabbways with electric drive. The process (10) conveys a free oscillation. The solution (8) is able to modelling a structure of type (10) and (21) in which x is replaced by vt, where v is homologous to speed and t is the time ; under this form the process (21) can reproduce a system of tambours in regime of free oscillations winding stocks that is inertia, frictions and variable effects.

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ANALIZA ŞI SINTEZA PENTRU O PROBLEMĂ STURM-LIOUVILLE ÎN DOMENIUL ACŢIONĂRII ELECTRICE

(Rezumat)

Cadrul general al analizei funcţionale este potrivit pentru a trata probleme de analiză şi sinteză a sistemelor liniare cu parametri concentraţi sau uniform distribuiţi.

Problema controlului optimal al sistemelor cu parametri distribuiţi supuse la diferite tipuri de restricţii este, prin excelenţă, o problemă de sinteză extremală. Reducerea la o problemă Sturm-Liouville şi de aici determinarea funcţiei Green angajează cele mai diverse situaţii ce oferă o bază largă de interpretări. Prin aceasta se precizează un anumit cadru de analiză şi sinteză, aplicabil la cele mai cuprinzătoare clase de sisteme fizice şi matematice supuse la restricţii de toate genurile.

Modul de aplicare al metodei de analiză şi sinteză propusă pentru sisteme liniare supuse la restricţii este ilustrat prin cîteva exemple practice.

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ANALOGICAL MODELLING OF THE STEPPING MOTOR WORKING EQUATION

BY

CRISTEA PAL

1. Introduction

Stepping motor working, when controled to perform a single step or when it is in low quasi-stationary state is rather accurately depicted by a second-order differential equation. By linearizing the motor torque expression this equation becomes a linear one, with constant coefficients. Linearization may be performed around the static stable equilibrium and, in peculiar cases, it could be extended up to normalized step [1], [2].

Analytical solution or modelling with help of an analogical computer of the working equation involves a precise determination of the initial integrating conditions leading to getting unerring results.

In what follows, for the peculiar case of a permanent magnet stepper motor, some aspects regarding the determination of initial conditions and modelling of operating equation are studied.

2. Linearized Working Equation of Stepping Motor

For a stepping motor with a number n of excited phases covered by the current I_2 in steady state and with the rotor dephased with the electrical angle θ_e w.r.t. the first phase axis, the total torque in steady state will result as a sum of the n partial torques [2] :

$$M = \sum_{k=1}^{n} M_0 \sin [\theta_e + (k-1)\, \pi/n - \theta_0]. \tag{1}$$

In Fig. 1 is represented, in the $\theta_e/\theta_{Me} = x$ and M/M_0 co-ordinates the variation curves of M_1, M_2, M_3, M_4 (partial torques) and M (total torque), in the peculiar case of a permanent magnet stepper motor of original construction, with electrical step $\theta_{Me} = 45°$ and with a total number of eight phases whom four are excitated in steady state. In the same figure the linearized torque are drawn by an interrupted line.

The total torque, M, of the motor, for the case of a dynamical state of phases feeding is calculated taking into consideration the variation of

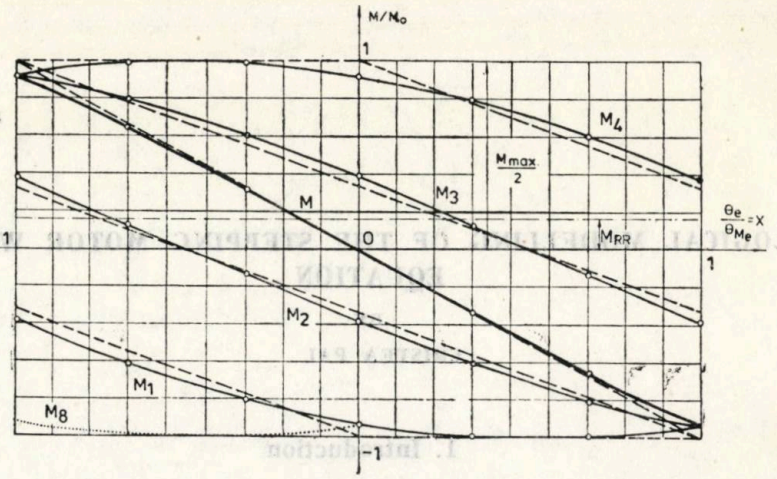


Fig. 1

the current $i(t)$ through the excitation winding, in the most cases an exponential function, limited at the nominal value, I_2 [3] (Fig. 2):

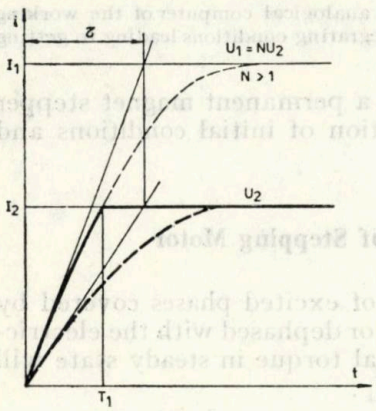


Fig. 2

$$(2)\qquad i(t)=\begin{cases} NI_2(1-e^{-t/\tau}), & \text{for } |\,i(t)\,|<I_2,\\ I_2, & \text{for } |\,i(t)\,|\geq I_2, \end{cases}$$

with N — the voltage multiplication factor and τ — the time constant of winding.

The motor working equation, in dynamical state of feeding and for a resistent torque, M_{RR}, reduced to the shaft and constant, is:

$$(3)\qquad \frac{d^2x}{dt^2}+2\xi\omega_0\frac{dx}{dt}+\omega_0^2x-\frac{\omega_0^2}{2}\cdot\frac{i(t)}{I_2}+\frac{\omega_0^2}{2}+\omega_0^2M_{RR}=0,$$

where ω_0 is the normal pulsation and ξ — the damping factor. Eq. (3) can also be written as follows:

$$(4)\qquad \frac{d^2x_1}{dt^2}+2\xi\omega_0\frac{dx_1}{dt}+\omega_0^2x_1=\frac{\omega_0^2}{2}\left[\frac{i(t)}{I_2}-1\right],$$

where $x_1=x+M_{RR}$.

In the studied motor working appears an important peculiarity which should be considered in determining the integration constants necessary to solve Eq. (4). The carrying out of the stepping motor advance with maximum starting torque, from $x_1=-1$ position to $x_1=0$ one (Fig. 1), involves the unexpected interruption of the current in phase 8 (the torque M_8 is represented in Fig. 1 by a dashed line) and the excitation of phase 4 with a current of opposite sign, in comparison with that of phase 8. These operations lead to changing of sign and value of partial torque from

$$M_8=-\frac{M_{max}}{2} \text{ to } M_4=\frac{M_{max}}{2}\cdot\frac{i(t)}{I_2}.$$

Thus, at any moment $t\leqslant -0$, $i(t)=-I_2$, and the rotor takes the position $x_1=-1$. At the moment $t\geqslant +0$ the sudden desexcitation of the phase 8 takes place and the excitation of phase 4 begin, that, in normal conditions, is achieved by a non-negligible time constant. Under the simultaneous action of torques M_1, M_2 and M_3, the rotor has the tendency to leave the position $x_1=-1$ and move to the position $x_1=-1/2$. Since the position $x_1=-1$ is considered as origin, one could produce an initial jump in origin, before to realize the excitation of phase 4 ; this jump should lead to change the initial integration conditions of Eq. (4).

3. The Determination of Initial Conditions for Solving the Working Equation of the Motor

The equation (4) of the stepping motor is identic with a equation of the form :

$$a_2\frac{d^2x_1}{dt^2}+a_1\frac{dx_1}{dt}+a_0x_1=b_2\frac{d^2u}{dt^2}+b_1\frac{du}{dt}+b_0u, \tag{5}$$

in which :

$$u(t)=\frac{1}{2}\left[\frac{i(t)}{I_2}-1\right] \text{ or } u(t)=\frac{1}{2}\left[N(1-e^{-t/\tau})-1\right], \tag{6}$$

and

$$a_2=1,\ a_1=2\xi\omega_0,\ a_0=\omega_0^2,\ b_2=0,\ b_1=0,\ b_0=\omega_0^2.$$

We suppose that at the moment when the cause $u(t)$ is applied, the system described by Eq. (5) had been developed under input action, in a certain manner, before the moment $t=+0$, from which the response of the element is of interest [4]. By applying unilateral direct Laplace transform to Eq. (5) and in accordance with the theorem of initial value one can write :

$$a_2\Delta x_{10}=b_2\Delta u_0, \tag{7}$$

with

$$\Delta x_{10}=x_1(+0)-x_1(-0) \text{ and } \Delta u_0=u(+0)-u(-0)\ ;$$

$$a_2\Delta\dot{x}_{10}+a_1\Delta x_{10}=b_2\Delta\dot{u}_0+b_1\Delta u_0, \tag{8}$$

with

$$\Delta\dot{x}_{10}=\dot{x}_1(+0)-\dot{x}_1(-0) \text{ and } \Delta\dot{u}_0=\dot{u}(+0)-\dot{u}(-0).$$

In Eq. (4), at $t=-0$, we have :

$$i(-0)=-I_2,\ x_1(-0)=-1,\ \dot{x}_1(-0)=0\ ; \tag{9}$$

$$u(-0)=-1, \qquad \dot{u}(-0)=0. \tag{10}$$

At the moment $t=+0$ we can write :

$$u(+0)=-\frac{1}{2}, \tag{11}$$

$$\dot{u}(t)=\frac{1}{2}\cdot\frac{N}{\tau}\,e^{-t/\tau},\ \dot{u}(+0)=\frac{1}{2}\cdot\frac{N}{\tau} \tag{12}$$

and taking into consideration (7) and (8) we obtain

$$\Delta u_0=\frac{1}{2}, \qquad \Delta\dot{u}_0=\frac{1}{2}\cdot\frac{N}{\tau}\cdot \tag{13}$$

The system of algebraic equations (7), (8) can be written under vectorial-matrix form, namely :

$$\begin{bmatrix} a_2 & 0 \\ a_1 & a_2 \end{bmatrix}\begin{bmatrix}\Delta x_{10} \\ \Delta\dot{x}_{10}\end{bmatrix}=\begin{bmatrix} b_2 & 0 \\ b_1 & b_2 \end{bmatrix}\begin{bmatrix}\Delta u_0 \\ \Delta\dot{u}_0\end{bmatrix}. \tag{14}$$

Since $a_2\neq 0$ it results that the triangular matrix (S t a n l e y) of the left member is irreversible and thus the solution of the system (14) is :

$$\begin{bmatrix}\Delta x_{10} \\ \Delta\dot{x}_{10}\end{bmatrix}=\begin{bmatrix} a_2 & 0 \\ a_1 & a_2 \end{bmatrix}^{-1}\begin{bmatrix} b_2 & 0 \\ b_1 & b_2 \end{bmatrix}\begin{bmatrix}\Delta u_0 \\ \Delta\dot{u}_0\end{bmatrix}. \tag{15}$$

Considering the relations (6), (13) it results :

$$\begin{bmatrix}\Delta x_{10} \\ \Delta\dot{x}_{10}\end{bmatrix}=\begin{bmatrix} 1 & 0 \\ 2\xi\omega_0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \frac{1}{2} \\ \frac{1}{2}\cdot\frac{N}{\tau} \end{bmatrix}, \tag{16}$$

with the solutions :

$$\Delta x_{10}=0, \qquad \Delta\dot{x}_{10}=0. \tag{17}$$

In accordance with (7) and (8) we obtain finally :

$$x_1(+0)=x_1(-0)=-1, \qquad \dot{x}_1(+0)=\dot{x}_1(-0)=0, \tag{18}$$

which represent the initial conditions for solving the working equation of stepping motor taken into consideration.

4. Modelling on the Analogical Computer of the Working Equation of a Stepping Motor

The modelling has been performed with help of a MEDA 43 HA computer for the permanent magnet stepper motor above mentioned, with

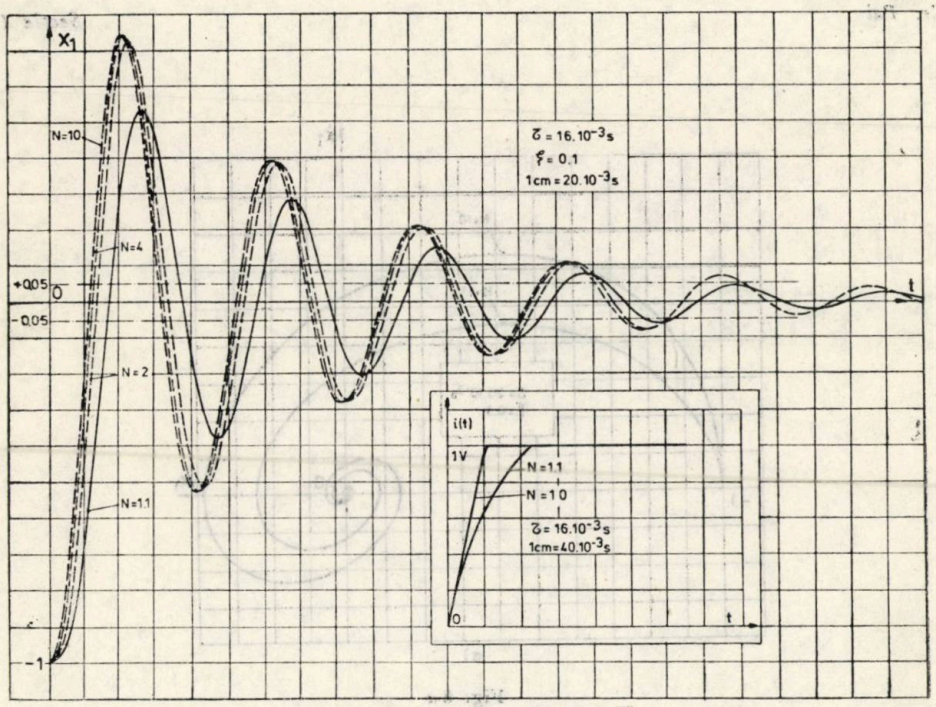

x1
N=10
N=4
N=2
N=1.1
+0.05
0
-0.05
-1
t
τ = 16.10⁻³s
ζ = 0.1
1cm = 20.10⁻³s
i(t)
1V
N=1.1
N=1.0
τ = 16.10⁻³s
1cm=40.10⁻³s
0
t

Fig. 4

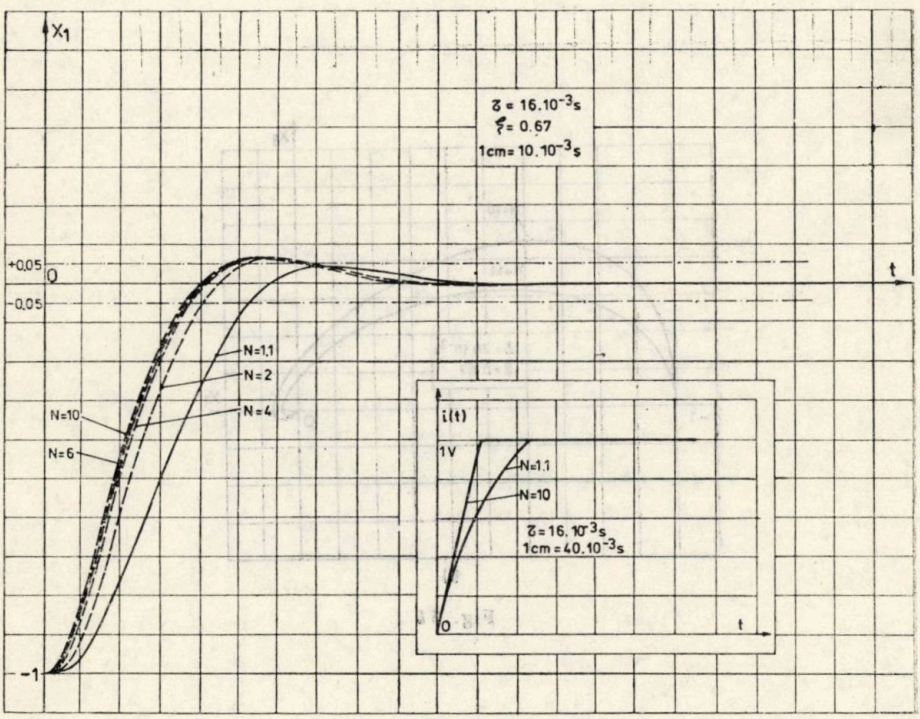
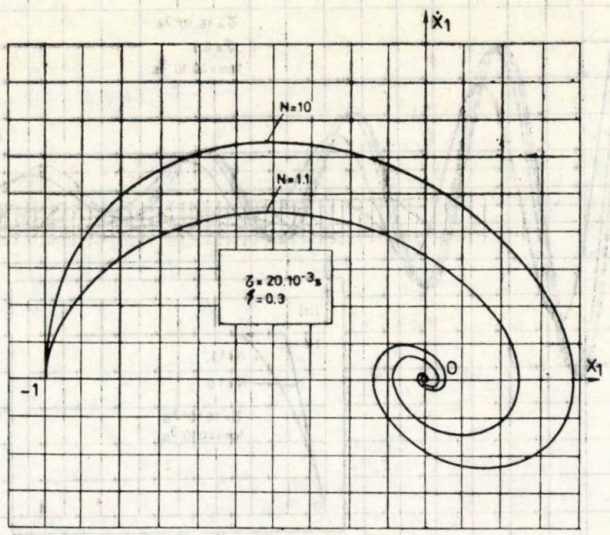
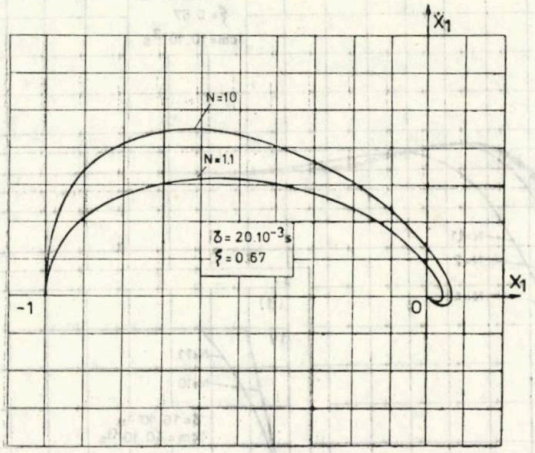

x1
τ = 16.10⁻³s
ζ = 0.67
1cm = 10.10⁻³s
+0.05
0
-0.05
-1
t
N=1.1
N=2
N=4
N=10
N=6
i(t)
1V
N=1.1
N=10
τ = 16.10⁻³s
1cm = 40.10⁻³s
0
t

Fig. 5



a)

Fig. 6 *a*



b)

Fig. 6 *b*

$\omega_0 = 75.4$ rad/s and $I_2 = 0.4$ A. The working equation with numerical coefficients is :

$$\frac{d^2x_1}{dt^2} + 150.8\,\xi\,\frac{dx_1}{dt} + 5\,685\,x_1 = 2\,842.5\,N(1 - e^{-t/\tau}) - 2\,842.5.$$

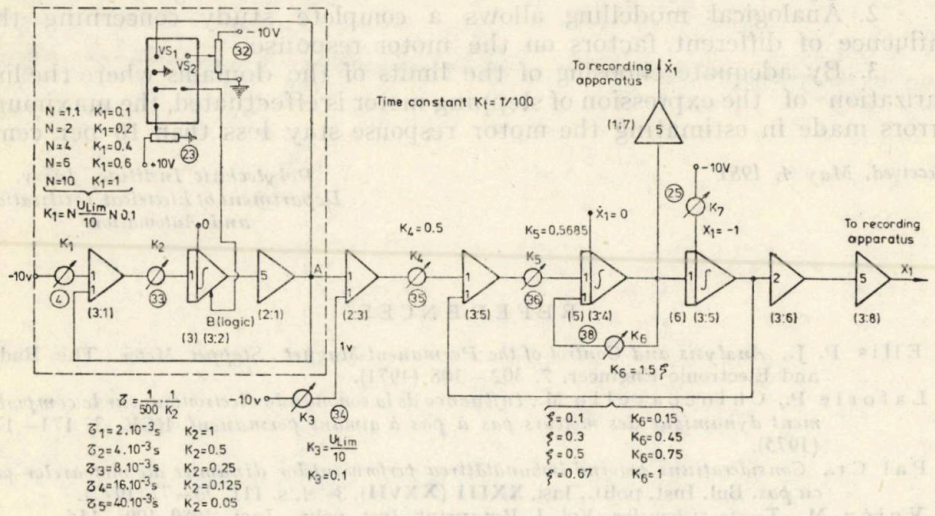


Fig. 3

În Fig. 3 the diagram of analogical modelling is represented. The diagram region defined by an interrupted line represents the block able to produce the right member of Eq. (19). At the moment when the signal with exponential variation from the point A reaches the value I_2, the comparator BC command, through a logical signal, the transition of the integrator (3:2) into memorization state and thus the signal from the point A is maintained constant, at the value I_2. Therefore relation (2) is satisfied. To study the influence of different factors on the motor response, the possibility of vary the values of τ, ξ and N was forecasted ($\tau = 2\times 10^{-3}...40\times$ $\times 10^{-3}$ s; $\xi = 0.1...0.67$; $N = 1...10$).

In Figs. 4 and 5 are represented the responses obtained by modelling with help of the analogical computer, for the cases $\tau = 16\times 10^{-3}$ s, $\xi = 0.1$ and $\tau = 16\times 10^{-3}$ s, $\xi = 0.67$, the multiplication factor, N, having values in the range 1.1...10. In the same figures are represented the current variation curves, $i(t)$, through the excitation winding in the cases $N = 1.1$ and $N = 10$.

In Fig. 6 *a*, *b* there are represented, in position—velocity co-ordinates (x_1, x_1) the motor responses for $\tau = 20\times 10^{-3}$ s, $\xi = 0.3$, with a multiplication factor $N = 1.1$ and 10. These curves allow the maximum velocity estimation in motor response.

The results achieved by modelling with help of an analogical computer were compared with the experimental ones obtained by laboratory experiments. One found that, in similar conditions, the differences between the normal responses and those achieved on analogical model stay non-significant (less than 10 per cent).

5. Conclusions

1. Analytical solution or results of modelling with help of analogical computer of working equation of stepping motor involve a good knowledge of initial conditions, that are to be defined for every motor, depending on the type of construction and on the command diagram structure.

2. Analogical modelling allows a complete study concerning the influence of different factors on the motor response.

3. By adequate choosing of the limits of the domains where the linearization of the expression of stepping motor is effectuated, the maximum errors made in estimating the motor response stay less than 10 per cent.

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MODELAREA ANALOGICĂ A ECUAŢIEI DE FUNCŢIONARE A MOTORULUI PAS CU PAS

(Rezumat)

Se prezintă unele aspecte privind stabilirea condiţiilor iniţiale pentru integrarea ecuaţiei diferenţiale de funcţionare, liniarizate, a unui motor pas cu pas, cu magneţi permanenţi. Se prezintă, de asemenea, schema modelului analogic şi rezultatele experimentale ale acestei modelări.

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SUR LES RÉGIMES OPTIMAUX DES RÉSEAUX INDUSTRIELS

PAR

AL. POEATĂ, GH. CÎRȚINĂ, D. IVAS et GH. GEORGESCU

1. Analyse des pertes d'énergie active dans les réseaux industriels

La réduction des pertes d'énergie active est l'un des problèmes actuels importants dans l'exploitation des réseaux industriels.

Il est connu que les pertes d'énergie active dans les réseaux des entreprises industrielles représentent quelques pourcentages de l'énergie totale consommée. L'établissement avec précision de la valeur des pertes, par calculs ou mesures, est très difficile et c'est pourquoi ces calculs et ces mesures on ne les effectue pas d'habitude.

Du point de vue de l'exploitation il est très important et stimulatif à disposer d'un moyen d'apprécier, même à titre indicatif, mais d'une façon expéditive, la valeur des pertes d'énergie et surtout les variations des pertes en fonction des différentes mesures adoptées dans la pratique.

Pour la mesure directe des pertes d'énergie par effet J o u l e-L e n z dans un élément de réseau radial on a utilisé les compteurs de pertes [1]. Relativement récemment on a montré que les compteurs de pertes peuvent être utilisés pour la mesure des pertes même dans les réseaux radiaux si les variations des charges des tous les usagers sont conformes.

Soit un réseau radial (Fig. 1) pour lequel on analyse les possibilités d'utilisation du compteur de pertes. Pour un régime de fonctionnement de ce réseau il est possible à calculer les courants dans chaque élément et à déterminer les pertes de puissance avec la relation,

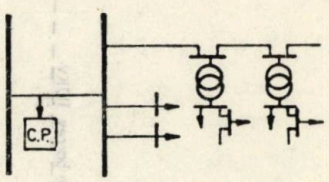

Fig. 1

$$\Delta P = 3 \sum_{i=1}^{n} R_i I_1^2, \tag{1}$$

où : n — le nombre des éléments du réseau (lignes et transformateurs) ; $R_i\,[\Omega]$, $I_i\,[A]$—la résistance, respectivement, le courant de l'élément i. Si l'on connait la valeur des pertes de puissance pour un régime donné, dont le courant total absorbé par le réseau est I, la résistance équivalente du réseau a pour expression

(2) $$R_e = \frac{\Delta P}{3I^2}.$$

Si toutes les charges vont se modifier proportionnelement, pour un autre régime du réseau la résistance équivalente restera constante. Il en résulte que pour chaque régime le réseau aura une autre résistance équivalente si les graphiques de charge sont différentes. Ainsi, les pertes d'énergie dans une période de 24 heures ont pour expression

(3) $$\Delta W_{24} = 3\sum_{j=1}^{24} R_{ej} I_j^2 = 3R_e \sum_{j=1}^{24} I_j^2,$$

où la somme $\sum_{j=1}^{24} I_j^2$ peut être remplacée par $\int_0^{24} I^2 \, dt$, mesurée avec un compteur de pertes et la valeur de la résistance équivalente du réseau a pour expression

(4) $$R_e = \frac{\sum_{j=1}^{24} R_{ej} I_j^2}{\sum_{j=1}^{24} I_j^2}.$$

Avec une précision relative, la valeur de la résistance équivalente, R_e, peut être déterminée à partir de 3 ... 4 régimes caractéristiques du réseau considéré.

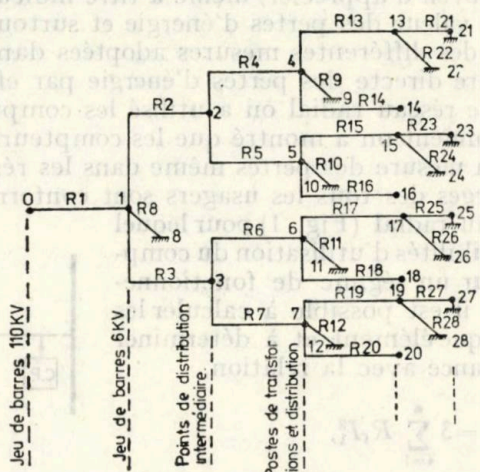

Fig. 2

En principe, la résistance équivalente, R_e, peut représenter aussi les récepteurs d'énergie électrique, les transformateurs, etc. Dans ce cas, pour

le calcul des pertes des puissances actives, ΔP, dans un réseau industriel, on considère le schéma radial, dont le graphique est donné dans la Fig. 2. Les noeuds sont numérotés de 0 à n ($n=28$, dans la Fig. 2) dans une succession convenable ; l'étiquette de chaque ligne est celle du noeud du côté droit.

Les paramètres longitudinaux des transformateurs (R, X) du réseau (s'il y a plusieurs niveaux de tension) sont inclus dans ceux des lignes (1, 4, 5, 6, 7). Les paramètres transversaux des transformateurs 110 KV/6 KV et 6 KV/0,4 KV (dus aux pertes ΔP_{Fe}, ΔQ_μ) sont considérés par les impédances des lignes 8, 9, 10, 11 et 12.

En ce qui concerne les récepteurs d'énergie électrique ils peuvent intervenir par leurs pertes actives. Par exemple, on suppose des moteurs asynchrones branchés à l'extrémité des lignes 13, 15, 17 et 19. À partir du schéma équivalent du moteur asynchrone on considère les impédances des stators inclus dans ceux des lignes 13, 15, 17 et 19 et les impédances des rotors (rapportées aux stators), 21, 23, 25 et 27, en paralèlle avec les impédances qui correspondent au fonctionnement à vide, 22, 24, 26 et 28.

Il faut souligner que l'impédance du rotor est une fonction de la charge mécanique et de la tension d'alimentation.

Du schéma établi ci-dessus on constate que lorsque la tension dans le réseau augmente, augmentent aussi les pertes dans le fer des transformateurs et des moteurs. Dans une certaine mesure les pertes dans les rotors diminuent avec l'augmentation de la tension.

Dans ce cas, pour le calcul de la résistance équivalente, dans la formule (2) ΔP contient, outre les pertes par effet J o u l e-L e n z dans les éléments du réseau (ΔP_{JL}), les pertes dans le fer des transformateurs (ΔP_{Fe}) et les pertes totales des moteurs asynchrones (ΔP_{ma}), donc

$$R_e = \frac{\Delta P_{JL} + \Delta P_{Fe} + \Delta P_{ma}}{3I^2}. \tag{5}$$

Dans la pratique les plus importantes sont les conséquences de chaque mesure visant la réduction des pertes.

En considérant $P(t)$, $Q(t)$ et $U(t)$ comme variables de temps aléatoires, $X(t)$, on peut les caractériser numériquement par la valeur moyenne $\overline{X}$ et par l'écart type σ_X. Entre ces grandeurs statistiques et la moyenne quadratique, $\overline{X^2}$, on a la relation

$$\frac{1}{m}\sum_{j=1}^{m} X_j^2 = \overline{X^2} = \overline{X}^2 + \sigma_X^2. \tag{6}$$

Ainsi les pertes d'énergie active dans une journée sont

$$\Delta W_{24} = \sum_{i=1}^{n}\sum_{j=1}^{24} \frac{P_{ij}^2 + Q_{ij}^2}{U_j^2} R_i = \frac{24R_e}{\overline{U}^2 + \sigma_U^2} [\overline{P}_\alpha^2 + \overline{Q}_\alpha^2 + \sigma_{\alpha P}^2 + \sigma_{\alpha Q}^2], \tag{7}$$

où :

$$\begin{cases} \alpha_j^2 = \dfrac{\overline{U^2}}{U_j^2} = \dfrac{\overline{U}^2 + \sigma_U^2}{U_j^2}, \quad (j=1, 2, \ldots, 24); \\ \overline{P}_\alpha = \alpha\overline{P}, \ \overline{Q}_\alpha = \alpha\overline{Q}, \ \sigma_{\alpha P}^2 = \overline{P_\alpha^2} - \overline{P}_\alpha^2, \ \sigma_{\alpha Q}^2 = \overline{Q_\alpha^2} - \overline{Q}_\alpha^2. \end{cases} \tag{8}$$

Dans les relations ci-dessus P et Q représentant les puissances actives et réactives absorbées par le réseau.

La relation (7) peut être encore transformée, si nécessaire, en tenant compte de principales propriétés des grandeurs statistiques, [2], comme suit :

$$(9)\qquad \overline{(X+Y)}=\overline{X}+\overline{Y}, \qquad (10)\qquad \sigma^2_{(aX+bY)}=a^2\sigma^2_X+b^2\sigma^2_Y,$$

où : a, b — constantes, X, Y — variables aléatoires indépendantes.

Il en résulte de la relation (6) la dépendence des pertes d'énergie en fonction du niveau de la tension ($\overline{U}$), des variations de la tension (σ^2_U), du niveau de la compensation de l'énergie réactive ($\overline{Q_\alpha}$) et de l'aplatissement des graphiques des charges actives et réactives ($\sigma^2_{\alpha P}$, $\sigma^2_{\alpha Q}$).

Si on considère la tension controlée par les moyens de réglage du réseau, on peut négliger les variations de la tension, ($\overline{U}^2=U_n^2$) et la relation (7) devient

$$(11)\qquad \Delta W_{24}=\frac{24R_e}{U_n^2}[\overline{P}^2+\overline{Q}^2+\sigma^2_P+\sigma^2_Q],\ \alpha_j\cong 1,$$

dont le minimum est :

$$(12)\qquad \Delta W_{24\min}=\frac{24R_e}{U_n^2}\overline{P}^2.$$

Ce minimum correspond à la situation où l'énergie réactive est complètement compensée et la puissance active est constante ($P_j=\overline{P}$, $j=1, 2, \ldots, 24$).

La contribution des autres termes qui interviennent dans la relation (11), ($\overline{Q}$, σ^2_P, σ^2_Q), à l'augmentation des pertes d'énergie, par rapport au niveau minimum, donné par la relation (12), peut être plus au moins grande. Généralement, σ^2_P et σ^2_Q peuvent contribuer à l'augmentation des pertes d'énergie maximum avec $10\,\%+10\%=20\,\%$.

2. L'analyse de la compensation de l'énergie réactive

Comme il est connu, le transit réactif apporte une importante contribution aux pertes totales d'énergie d'un réseau industriel. La situation actuelle de l'énergie dans le monde, l'importance qu'on prête aux diminutions des pertes d'énergie active imposent la réduction au minimum du transit réactif. Une économie d'énergie stable, tant soit que peu en pourcentage, peut assurer des gains assez importants. La réalisation de cet objectif est possible parce que la technologie des condensateurs et de l'équipement électrique correspondant a fait des grands progrès et le prix, par KVar, a beaucoup diminué et diminuera encore, dans l'avenir.

Dans ce sens on va installer des condensateurs fixes dans chaque noeud, à partir des usagers les plus éloignés, pour la compensation de l'énergie réactive, jusqu'à la limite d'éviter la surcompensation, c'est-à-dire

$$(13)\qquad Q_i^c=Q_i^{\min},\ (i=n, n-1, \ldots, 2, 1).$$

En utilisant (9) et (10) on constate que les termes $\sigma_{\bar{Q}}^2$ restent constants. Par la compensation jusqu'au niveau $Q_i^{\min}$ la réduction des pertes annuelles d'énergie est donnée par la relation [3] :

$$(14)\quad \delta(\Delta W_a) = \frac{24N}{U_n^2} \sum_{i=1}^{n} R_i Q_i^{\min}(2\bar{Q}_i - Q_i^{\min}) = \frac{24N}{U_n^2} R_e Q^{\min}(2\bar{Q} - Q^{\min}),\ N = T,$$

où :

$$(15)\quad \bar{Q}_i = \sum_{j \in M_i} \bar{Q}_j, \qquad (16)\quad Q_i^{\min} = \sum_{j \in M_i} Q_j^{\min}$$

et $M_1, M_2, \ldots, M_n$ représentent les ensembles des noeuds qui sont alimentés directement à partir des noeuds 1, 2, ..., n.

La compensation plus proche des usagers est facilitée compte tenu qu'il s'agit des puissances réactives importantes. La compensation est économique pour l'ensemble du réseau si l'inégalité

$$(17)\quad k_a \delta(\Delta W_a)\beta \geqslant Q_{\text{comp.}}(\gamma + k_a p_c T \beta)$$

est satisfaite, où : γ — le coût du KVar installé dans les condensateurs, p_c — les pertes spécifiques d'énergie active dans les condensateurs, β — le prix du KWh perdu, T — le nombre des heures de fonctionnement par année, $Q_{\text{comp.}} = \sum_{i=1}^{n} Q_i^{\min}$ — la puissance totale des condensateurs installés dans le réseau, k_a — un coefficient d'actualisation. Si la relation (17) n'est pas satisfaite, on cherche les condensateurs les moins économiques et on les élimine. Le schéma logique correspondant est présenté dans la Fig. 3.

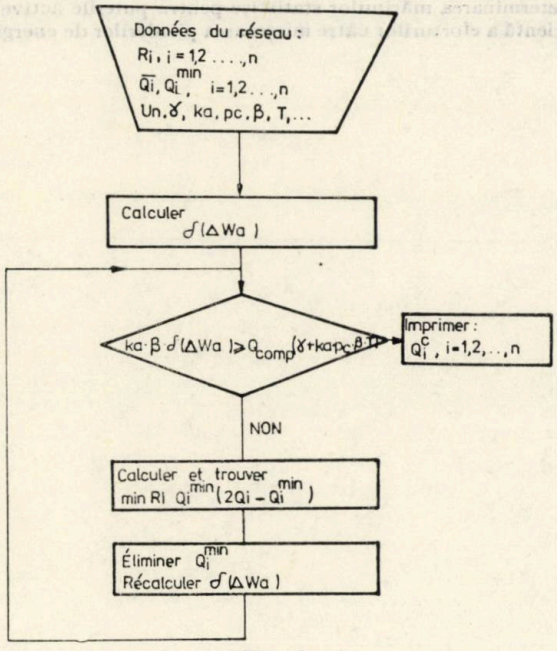

Fig. 3

3. Conclusions

Les grandeurs statistiques permettent à obtenir une image claire quant aux variations des variables correspondantes et l'efficacité des mesures pour la réduction des pertes d'énergie active.

L'utilisation du compteur de pertes dans un réseau industriel pour lequel on a établi la valeur de la résistance équivalente constitue une méthode expéditive pour l'evaluation quotidienne du niveau des pertes.

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ASUPRA EXPLOATĂRII OPTIMALE A REȚELELOR INDUSTRIALE

(Rezumat)

Folosind noțiunea de rezistență echivalentă a pierderilor se prezintă modul de utilizare a contoarelor de pierderi în scopul determinării pierderilor de energie activă pe ansamblul unei rețele industriale. Determinarea mărimilor statistice pentru puterile active, reactive și tensiuni permite dirijarea eficientă a eforturilor către micșorarea pierderilor de energie activă.

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RECENZII

BOOK REVIEWS РЕЦЕНЗИЙ

THOLL H., *Bauelemente der Halbleiterelektronik*. B. G. Teubner Verlag, Stuttgart, 1976. Bd. 1, 236 S, Bd. 2, 324 S.

This work deals with the operation principle, the caracteristics and the applications of the main semiconductor electronic devices. It is intended to students preparing in electronics.

The first chapter of the first volume presents briefly the band structure and the conductivity of the semiconductors.

At the beginning, the second chapter (p-n junction) deals with problems connected with the p-n junction : pontential barrier, static characteristic equation, frequency response, and temperature influence. Then it presents and analyses some rectifying circuits.

The third chapter deals with special diodes : breakdown (Z e n e r) diode, tunnel diode, backward diode, hot-carrier diode, variable capacitance diode, pin diode, impatt diode and gunn diode. For each of the diodes are described its operation principle and construction and presented its main applications.

The fourth chapter is intended to the bipolar transistor study. The operation principle, the static characteristics, the frequency response and the bias circuits are presented. Its dynamic behaviour is analysed using h and y parameters for each of the three main transistor configurations.

The first chapter of the second volume, entitled *Transistors with Special Properties*, deals with the field-effect transistors (FET), the unijunction transistors (U JT) and the avalanch transistors. For each type are shown its structure, described the operation and static characteristcs, presented its operation equations and given the application circuits.

The SCR's are presented in the second chapter. In addition to the common SCR, there are also presented the diac, the triac and the diode-thyristor.

The third chapter, entitled *Optoelectronic Devices*, deals, at the beginning, with the physical basis of the optoelectonics. Then are presented the photodetector devices (photoresistor, photodiode, phototransistor, photothyristor) and the electroluminiscent devices (light-emissive diode (LED), seven-segment display, 7×5 diodes indicator matrix). The presentation of each device is organised in the same way : construction — operation — characteristics — applications.

The fourth, the fifth and the sixth chapters deal with some rare devices : H a l l generators, voltage dependent resistors (VDR) and temperature dependent resistors (termistors).

Gh. Maxim

COLIN A., *Programming for Microprocessors*. Newnes-Butterworths, London, Boston, Sydney, Wellington, Durban, Toronto, 1979, VIII+206 pp.

The book intends to provide a simple and readable introduction to the programming for microprocessors.

Manny electronic engineers and students, when they start work on microprocessors, find that they have to learn new and unfamiliar techniques. All too often they spen a great deal of time rediscovering for themselves the principles of programming. This book help the students and engineers bypass this lengthly and unproductive process.

The work includes twelve chapters, namely : General Introduction (7 pp.) Representation of Data (10 pp.), Basic Structure of a Microprocessor (12 pp.), Assembly Language (11 pp.), Some Aspects of Microprocessor Design (10 pp.), Introduction to the Motorola 6800 (19 pp.), Some Programming Techniques (16 pp.), Control of Peripheral Devices (in two parts, 38 pp.), Comparing Microprocessors (7 pp.), High-level Languages (13 pp.), Practical Aids to Programming (16 pp.). The book contains also four appendix in wich are presented ASCII code, Motorola 6800 instruction repertoire, PL/F(6800 version) and PL/F sample program.

The subject matter is presented with a great competence.

The author deals with the subject both in a general way and and at the level of detail that is so essential to the successful design of μP systems. The book includes detailed discussion of topics such as number representation and program design, which are omitted from most manufacturers' manuals. The devices chosen as the main exemple is the Motorola 6800, but the book can be profitably read by intending users of any μP, since the programming techniques can easily be transferred.

C. Huţanu

SPORLEDER F., UNGER H. G., *Waveguide Tapers Transitions and Couplers* Peter Peregrinus Ltd., 1979.

This book, edited in the IEE Electromagnetic Waves Series, is intended to cover the domain of the techniques of the sub-centimetric electromagnetic waves, going up to the optical waves.

The development of the topic is performed on the basis of the general theory of the waves in anisotropical and unhomogeneous medii. This feature gives to the book a compact aspect and continuity.

The *Introduction* contents a brief description of the matter and includes a good general bibliography. The second chapter, devoted to the *Coupled Wave Equations*, describes, on the fifth part of the total number of pages, the basic theory of the subject involved. Dielectric media are also treated in some detail.

The following chapters deal with particular propagation cases in connection to the applications which are the most known : the horn structure, abrupt waveguide transitions, couplers, etc. In these sections the reader may find the exact descriptions of some most used structures. The explanations are clear, even didactical and they be easily followed. Some design procedures are also developped.

One of the most valuable feature of the book is that it includes the basical theory of the dielectric waveguides. However, I think that by adding only a few number of pages to detail this up-to-date kind of waveguides, the manual could become of greatest interest for many specialists in optoelectronics.

The book is well written, in a concise style, explaining all the basical problems encounted in the domain of the tapered waveguides. It is of interest for advanced students and for the specialists, not only for its scientific value but also as a compendium.

The reader is somewhat disappointed by the lack of a more complete bibliography. E.g., Chapter 9 contains only one reference, maybe not the most representative, on a topic yet developping. This is surely an important limitation from the point of view of the researchers. On the other hand, the production engineer is not encouraged to retain the book as an every day handbook because some modern and practical problems, like rugged horns, are not treated.

However, by its qualities, this manual remains an important reference fo all those interested by the domain of the waveguides.

H. N. Teodorescu

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